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By 10 A. A. Dmitriyevskiy

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By

A. A. Dmitriyevskiy

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Chapter VIII

STABILITY OF MOTION AND THE STABILIZATION OF ROCKETS AND OF PROJECTILES.

§1. Common/general/total concepts of stability of motion and of the stabilization of rockets and of projectiles.

The problem of stability of motion of bodies was solved for the first time by noted Russian scientists by N. Ye. Joukowski in work "The strength of motion" (1882) and by A. M. Lyapunov in work "common/general/total problem of stability of motion" (1892). At present stability theory is widely developed and improved, it examines not only questions of stability of motion of mechanical systems (aircraft, rockets, projectiles), but also questions of the stability of the systems of control, automatic control systems, etc.

During investigation the motion of the rockets and projectiles usually is subdivided into disturbed and that not disturbed. The undisturbed motion is called similar, which they would complete the

rocket or projectile in standard atmosphere or in the vacuum under the action previously provided, that were being subordinated to the specific laws, forces. The corresponding to the undisturbed motion trajectory is also called not disturbed or calculated (nominal).

However, under actual conditions the motion of rockets and projectiles occurs with the supplementary, random factors which usually during the calculation of nominal trajectories are not taken into account (deviation of temperature of air from normal law, wind and wind gusts, the pulsation of the engine thrust, nonprogrammed control forces, etc.). The actions of these factors, called disturbing, lead to the fact that the rocket and projectile will move not over nominal (calculated) trajectory, but differing from it more or less considerably depending on value and direction of disturbance/perturbations. The motion, the reflection effect of the perturbation factors, calls the disturbed motion, and the corresponding to it trajectory - by the disturbed trajectory.

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With the concept of the disturbed motion of rockets and projectiles, is organically-bound the concept of the stability of their motion.

In accordance with classical stability theory, the motion of solid body can be named stable in such a case, when it possesses property to return after the break-down of the perturbation factors to the interrupted by them not disturbed motion.

Let us explain this position with the aid of Fig. 8.1. On figure by solid line is depicted the trajectory, which corresponds to the undisturbed motion. Let on section AB of trajectory on rocket act some disturbance/perturbations, which will force it to move over the disturbed trajectory AB', which differs from the nominal. The motion of rocket will be stable, if after the break-down of disturbance/perturbations at point B' the disturbed trajectory will converge from nominal and it will coincide from the latter at certain point V.

But if this does not occur and rockets it will fly along the trajectory B'C', then its motion must be described as unstable. It is completely obvious that stability of motion in the sense, which corresponds to the given above definition, virtually it cannot have provided not only the unguided projectiles and the rockets, but also of rockets with the complex control system, since no real control system due to the which are inherent in it errors not in state to just as ideally parry disturbance/perturbations and consequences of their effect, in order always to accurately return rocket to nominal

trajectory. Based on this, we subsequently will understand under the stable motion of rockets similar, during which the deviation of real trajectory from the nominal under the action of short-term or prolonged disturbance/perturbations does not exceed the established/installed limits.

In Fig. 8.1 zones of the stable motion of the relatively nominal trajectory CC is isolated by dot-dash lines.

During the study of stability of motion of rockets and projectiles, are usually examined separately stability of motion of the center of mass along trajectory and the stability in their rotary motion relative to the center of mass. The provision for the latter is the necessary stability condition of the motion of the center of mass and it is inseparably connected with the concept of the angular stabilization of rockets and projectiles. Under stabilization is understood the totality of the measures which provide preservation/retention/maintaining by rocket or projectile in the trajectory of correct position relative to direction of motion.

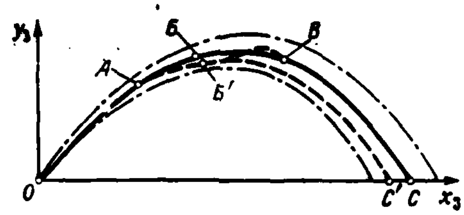

Fig. 8.1. Diagram to the determination of stability of motion of rockets and projectiles.

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Other conditions being equal, the drag will be smallest in the case when the axis of projectile coincides with direction of motion. However, in the process of flight, the velocity vector of the center of mass $\bar{v}$ is turned. Because of this stable rocket or projectile they must continuously change its angular position, being turned relative to the center of mass following by vector $\bar{v}$ in the manner that it is shown on Fig. 8.2.

If the longitudinal axis of the housing of projectile or finless rocket is deflected from direction of motion at least in insignificant angle, then the resulting aerodynamic force $\bar{F}$, which acts on rocket or projectile, will be applied in the center of

pressure (ts.d.) which is located between the apex/vertex of rocket and its center of mass (Fig. 8.3). This leads to appearance relative to the center of mass of the tilting moment $\bar{M}$, which during the motion of the nonstabilized rocket along trajectory will cause random movement relative to the center of mass and, as a result, is considerable the distortion of trajectory. Thus, the position of the housing of the projectile at which the axis of masses and apex/vertex, can be described as position of unstable equilibrium, since the least misalignment of rocket from vector $\bar{V}$ will cause the irreversible increase in this deviation. For preventing of this phenomenon and provision for a correct position in flight (by nose section forward) the rocket and projectile must be stabilized. In connection with the unguided rockets and projectiles, are utilized two different of the "passive" method of stabilization - spin stabilization and stabilization by tail assembly.

The stabilization of rockets and projectiles by tail assembly consists in the fact that on the tail section of the oblong housing are fastened the diverse in their structural/design forms and size/dimensions stabilizers. This leads to the fact that during the flow around housing of airflow at angle of attack α the character of pressure distribution along the length of rocket will change, as a result of which ts.d. shifts relative to the center of mass to the side of stabilizers. During the appropriate selection of the

size/dimensicns of statilizers, it is pcssitle tc attain this displacement/movement of ts.d. that it with respect to the apex/vertex of rocket or prcjectile will render/show behind the center of mass. In this case with $\alpha \neq 0$ the resulting aerodynamic force $\bar{R}$ will act in the manner that shown cn Fig. 8.4 (for motion in vertical plane).

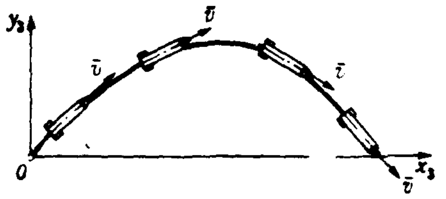

Fig. 8.2. Diagram of the consecutive positions of the fin-stabilized projectile in trajectory during correct flight.

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It is obvious that caused by force of $\bar{F}$ torque/moment M_z will attempt to decrease the angle α, give housing to such position in which the axis Ox_1 coincides with velocity vector $\bar{v}$, and torque/moment itself M_z will become equal to zero, i.e., it will turn out that unlike the preceding/previous case, the stabilizing (or restoring) effect. The position of housing, which is characterized by the value $\alpha=0$, relative to which torque/moment M_z stabilizes housing, will be the position of stable equilibrium.

If the phenomenon indicated is is examined only in statics, then for torque/moment M_z it is possible (with low α) to accept

dependence $M_z = Sqlm_z^\alpha \alpha$ (see Chapter II, §4). Based on this expression, we find the sign/criterion, which shows the character of action M_z on rocket or projectile.

For the tilting moment whose sign coincides with angle α, we have $m_z^\alpha > 0$; for stabilizing moment $m_z^\alpha < 0$. Derivative m_z^α is connected with lift coefficient by the relationship/ratio

$$m_z^\alpha \approx c_y^\alpha \frac{l_{ц.м} - l_{ц.д}}{l}, \quad (8.1)$$

where l - a overall length of rocket.

$l_{ц.м}$ and $l_{ц.д}$ - respectively distance from the apex/vertex of rocket to the center of masses and center of pressure (see Fig. 8.3 and 8.4).

Since $c_y^\alpha > 0$, then, obviously, the sign of derivative coincides with the sign of difference $l_{ц.м} - l_{ц.д}$. Based on this, the conditions of the static stability of rocket and projectile can be formulated as follows:

$l_{ц.м} - l_{ц.д} < 0$ - rocket or the projectile "statically" stable;

$l_{ц.м} - l_{ц.д} > 0$ - rocket or projectile are unstable;

$l_{ц.м} - l_{ц.д} = 0$ - rocket or projectile they are located in the state of neutral equilibrium.

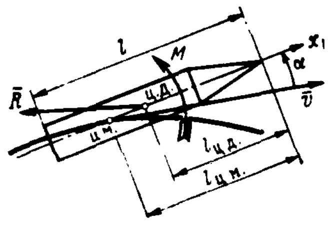

Fig. 8.3.

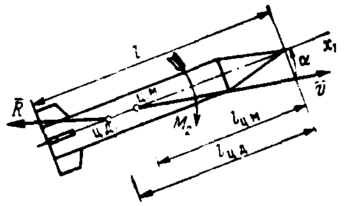

Fig. 8.4.

Fig. 8.3. Pattern of the action of aerodynamic moment on finless rocket (projectile).

Fig. 8.4. Pattern of action of aerodynamic moment on fin-stabilized rocket (projectile).

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During the motion of projectile along trajectory, the velocity of its flight and orientation relative to velocity vector continuously change, which leads to a change in the position of ts.d. relative to housing. Furthermore, on powered flight trajectory as a result of the high fuel consumption in the engine operation the center of mass of the rocket also is displaced from its initial position. These reasons can cause a considerable change in value $l_{ц.м} - l_{ц.д}$ and, therefore, in the derivative m_z^{α} that determines the stability level of rocket. Based on this, it follows that for

providing the angular stabilization of rocket in flight it is necessary that along an entire trajectory would be fulfilled condition $m_z^\alpha < 0$ inequality $|m_z^\alpha| \geq |m_{z\,\text{доп}}^\alpha|$.

In other words, stabilized by rocket fin or projectile must possess the so-called factor of "static" stability. The factor of "static" stability they usually characterize by expressed in percentages value $|m_z^{c_y}|$, which is located from the relationship/ratio

$$|m_z^{c_y}| = \frac{|m_z^\alpha|}{c_y^\alpha} = \frac{|l_{\text{ц.м}} - l_{\text{ц.д}}|}{l}\, 100\% = |c_{\text{ц.м}} - c_{\text{ц.д}}|\, 100\%, \qquad (8.2)$$

where $c_{\text{ц.д}}$ - a center-of-pressure coefficient;

$c_{\text{ц.м}}$ - the coefficient of the center of mass.

It is customary to assume that the unguided fin-stabilized rockets and projectiles are well stabilized, if they possess the factor of "static" stability, equal to $|m_z^{c_y}| = (10-15)\%$.

We investigate the character of the motion of the unguided "statically" stable projectile of its relatively center of mass, assuming that it is completed only in range plane. For simplicity let us examine the trajectory phase, during motion along which it is possible to consider constant m_z^α, and also θ=const. In this case without the account of damping, the equation of motion relative to the center of mass will be written in the form

$$\frac{d^2\alpha}{dt^2} + n^2\alpha = 0, \qquad (8.3)$$

where $n^2 = \frac{M_z}{J_z\alpha} = \frac{Sql}{J_z}|m_z^\alpha|$ - the coefficient which can be designed previously along the known trajectory of the center of mass of the fin-stabilized projectile.

Set/assuming within the limits of the small phase of trajectory n^2=const, for initial conditions when $t_н=0, \alpha=\alpha_н$ and $\dot{\alpha}=\dot{\alpha}_н$ we will obtain the solution of equation (8.3) in the form

$$\alpha=\alpha_н \cos nt + \frac{\dot{\alpha}_н}{n} \sin nt. \qquad (8.4)$$

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By simple conversions this solution can be given to the more convenient form

$$\alpha=\alpha_m \sin (nt+\varepsilon), \qquad (8.5)$$

where phase shift

$$\varepsilon=\operatorname{arctg} \frac{n\alpha_н}{\dot{\alpha}_н}. \qquad (8.6)$$

Solution shows that with the adopted assumptions the motion "statically" stable fin-stabilized projectile relative to the center of mass represents by itself the flat/plane harmonic oscillations, which are characterized by the amplitude

$$\alpha_m=\sqrt{\left(\frac{\dot{\alpha}_н}{n}\right)^2+\alpha_н^2} \qquad (8.7)$$

and by period $T=2\pi/n$.

The curve/graphs of a change in the angle α are in connection with the obtained solution represented in Fig. 8.5.

During the analysis of the oscillatory motion of finned "statically" stable rockets and projectiles, we did not consider damping moment M_{0z}. The nature of this torque/moment was examined earlier (chapter II, §4) and let us here note just action M_{0z} leads to the rapid attenuation of the oscillatory motion, caused by initial disturbances and torque/moment M_z. The character of a change in angle of α upon consideration of damping shows in Fig. 8.5 curve/graph, carried out by fine/thin line.

Thus, in the dense layers of the atmosphere fin-stabilized rockets and projectiles move over trajectory so that their longitudinal axis smoothly "follows" vector $\bar{V}$ until any disturbance/perturbations again excite the oscillations of rockets.

Let us examine attitude control by rotation. During motion along the trajectory of the projectile, which rapidly rotates relative to their longitudinal axis, aerodynamic forces, creating torque/moment, attempt to invert projectile, but it, as gyroscope, it is not inverted, but it moves stable. The longitudinal axis of projectile,

"fcllowing" the tangent tc trajectcry, cscillates in the process of mcving the relatively dynamic axis of equilibrium. Artillery prcjectile obtains rotaticn in bore during the motion of the driving band of projectile along screw-shaped threads. The rockets, which are stabilized by rotation, are called the spin-stabilized missiles (TRS); they rotate because of the outflcw of gas behind oblique nozzles.

The angular rate of rotation of artillery shell or TRS must be calculated so that during the motion along trajectory its longitudinal axis contirucusly "would fcllcw" the direction of the motion of the center of mass of projectile, differing from the latter within the limits of the permissible angles.

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The stabilization of the guided missiles and projectiles is realized because of the active contrcl forces, capable, besides the conduct of rocket along programmed trajectory or the trajectory of induction, to parry the complex system of the disturbance/perturbations, which act on rocket in flight. For producing of control forces, flight vehicles are supplied with controls (chapter II, §6). The guided missile can not possess the necessary steady-state stability factor, but because of the operation

of the complex of the control system be well stabilized and execute the predetermined trajectory of motion. The composite examination of the stability conditions of the motion of rocket and control system is the object/subject of the study of the dynamics of the control systems by unmanned flight vehicles [19, 36, 37].

During ballistic calculations the study of stability of motion of rockets and projectiles is conducted in two ways.

First, it is possible to comprise and to solve the complete system of differential equations, which describes the flight of flight vehicle taking into account the action of all forces, including disturbing, which can cause the incorrect flight of flight vehicle. After its solution by the obtained motion characteristics, they judge the stability of flight vehicle in flight. This path, although it is theoretically strict however not always is utilized in practice due to the impossibility to have the comprehensive data about all perturbation factors, for example, action of wind gusts, eccentricity of thrust, etc.

In the second place, an investigation for stability of motion can be carried out, after comprising the differential equations of the deviations of trajectory elements from the calculated and performing analysis of these equations, without examining directly

the action of the perturbation factors.

In the theory of the analysis of stability of flight vehicles wide distribution obtained the method of the slight disturbances. In this the method of the deviation of the parameters of the disturbed motion from that not disturbed they are accepted as so low that in the equations of the disturbed motion these parameters can be represented in the form of the sums, which contain the deviations of the parameter only to the first degree.

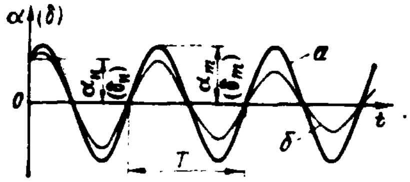

Fig. 8.5. Curve/graphs of a change in the angle of attack $\alpha(t)$ (or $\delta(t)$): a) without damping; b) during damping of angular motion.

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For example, the velocity, angle of attack and pitch angle in the disturbed motion can be represented thus:

$$v=v_*+\Delta v;\quad \alpha=\alpha_*+\Delta\alpha;\quad \vartheta=\vartheta_*+\Delta\vartheta, \qquad (8.8)$$

where v_*, α_* and ϑ_* - velocity, angle of attack and pitch angle in the undisturbed motion whose characteristics are determined without the account of the action of the perturbation factors, and Δv, $\Delta\alpha$ and $\Delta\vartheta$ - deviation of these trajectory elements, obtained as a result of acting the perturbation factors.

Thus, perturbing forces themselves and torque/moments and the mechanism of their action on flight vehicle are not examined. Is studied only a change in the deviations of motion characteristics already after the action of the perturbation factors on the

assumption that these deviations are low. The method of the slight disturbances makes it possible the equations of the disturbed motion of rocket to reduce to the linear differential equations, solved relatively simply.

§2. Linearization of the equations of motion of rockets and of projectiles.

The mathematical sense of linearization lies in the fact that the unknown deviation of cell/element is located by the way of expansion of the corresponding to it function in Taylor series in terms of the degrees of deviation of cell/element. Recall the formula of expansion in Taylor series for functioning many arguments $f(\xi_1, \xi_2, \ldots, \xi_n)$, since to of this type to functions are related the trajectory elements of rockets. During the writing of the results of expansion, we will use the conventional designations of total differentials of the function of many variables

$$d^m A=\left(\frac{\partial}{\partial \xi_1} d\xi_1+\frac{\partial}{\partial \xi_2} d\xi_2+\ldots+\frac{\partial}{\partial \xi_n} d\xi_n\right)^m f(\xi_1, \xi_2, \ldots, \xi_n).$$

The formula of expansion takes the form

$$f(\xi_1, \xi_2, \ldots, \xi_n) = f(\xi_{1*} + \delta\xi_1, \xi_{2*} + \delta\xi_2; \ldots \xi_{n*} + \delta\xi_n) =$$
$$= f(\xi_{1*}, \xi_{2*}, \ldots, \xi_{n*}) + \frac{1}{1!}\left(\frac{\partial}{\partial\xi_{1*}}\delta\xi_1 + \frac{\partial}{\partial\xi_{2*}}\delta\xi_2 + \ldots + \frac{\partial}{\partial\xi_{n*}}\delta\xi_n\right) \times$$
$$\times f(\xi_{1*}, \xi_{2*}, \ldots, \xi_{n*}) + \frac{1}{2!}\left(\frac{\partial}{\partial\xi_{1*}}\delta\xi_1 + \frac{\partial}{\partial\xi_{2*}}\delta\xi_2 + \ldots + \frac{\partial}{\partial\xi_{n*}}\delta\xi_n\right)^2 \times$$
$$\times f(\xi_{1*}, \xi_{2*}, \ldots, \xi_{n*}) + \frac{1}{3!}\left(\frac{\partial}{\partial\xi_{1*}}\delta\xi_1 + \frac{\partial}{\partial\xi_{2*}}\delta\xi_2 + \ldots + \frac{\partial}{\partial\xi_{n*}}\delta\xi_n\right)^3 \times$$
$$\times f(\xi_{1*}, \xi_{2*}, \ldots, \xi_{n*}) + R, \qquad (8.9)$$

where ξ_{i*} - calculated (nominal) values of the determining parameters.

The deviation of function $f(\xi_1, \xi_2, \ldots, \xi_n)$, caused by the deviations of the parameters from computed values $\delta\xi_1, \delta\xi_2, \ldots, \delta\xi_n$, will be equal

$$\delta f(\xi_1, \xi_2, \ldots; \xi_n) = f(\xi_{1*} + \delta\xi_1, \xi_{2*} + \delta\xi_2 + \ldots + \xi_{n*} + \delta\xi_n) -$$
$$- f(\xi_{1*}, \xi_{2*}, \ldots, \xi_{n*}). \qquad (8.10)$$

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The first term of expansion (8.9) and the second (8.10) are equal, have different signs and will be shortened; therefore common/general/total formula for the deviation of function $f(\xi_1, \xi_2, \ldots, \xi_n)$ can be obtained in the following form:

$$\delta f(\xi_1\xi_2, \ldots, \xi_n) = \frac{1}{1!}\left(\frac{\partial}{\partial\xi_{1*}}\delta\xi_1 + \frac{\partial}{\partial\xi_{2*}}\delta\xi_2 + \ldots + \frac{\partial}{\partial\xi_{n*}}\delta\xi_n\right) \times$$
$$\times f(\xi_{1*}, \xi_{2*}, \ldots, \xi_{n*}) + \frac{1}{2!}\left(\frac{\partial}{\partial\xi_{1*}}\delta\xi_1 + \frac{\partial}{\partial\xi_{2*}}\delta\xi_2 + \ldots + \frac{\partial}{\partial\xi_{n*}}\delta\xi_n\right)^2 \times$$
$$\times f(\xi_{1*}, \xi_{2*}, \ldots, \xi_{n*}) + \frac{1}{3!}\left(\frac{\partial}{\partial\xi_{1*}}\delta\xi_1 + \frac{\partial}{\partial\xi_{2*}}\delta\xi_2 + \ldots + \frac{\partial}{\partial\xi_{n*}}\delta\xi_n\right)^3 \times$$
$$\times f(\xi_{1*}, \xi_{2*}, \ldots, \xi_{n*}) + R. \qquad (8.11)$$

The number of terms of expansion, held in calculations, depends on the required accuracy of the determination of deviation. Most frequently during the solution of the practical problems of external ballistics, hold only linear terms of expansion. In this case formula (8.11) will take the form

$$\delta f(\xi_1, \xi_2, \ldots, \xi_n)=\left(\frac{\partial f}{\partial \xi_1}\right)_* \delta\xi_1+ \\ +\left(\frac{\partial f}{\partial \xi_2}\right)_* \delta\xi_2+\ldots+\left(\frac{\partial f}{\partial \xi_n}\right)_* \delta\xi_n. \quad (8.12)$$

Let us find expression for the deviation of the derivative of form $\delta(d\xi/dt)$. Since

$$\delta\frac{d\xi}{dt}=\frac{d\xi}{dt}-\left(\frac{d\xi}{dt}\right)_*, \text{ a } \xi=\xi_*+\delta\xi,$$

that we will obtain

$$\delta\left(\frac{d\xi}{dt}\right)=\frac{d}{dt}(\xi_*+d\xi)-\left(\frac{d\xi}{dt}\right)_*=\frac{d}{dt}\delta\xi. \quad (8.13)$$

Thus, if we have a system of differential equations of the disturbed motion, comprised of n of the equations of the form

$$\left.\begin{aligned} \frac{d\xi_1}{dt}&=f_1(\xi_1, \xi_2, \xi_3, \ldots); \\ \frac{d\xi_2}{dt}&=f_2(\xi_1, \xi_2, \xi_3, \ldots); \\ &\ldots\ldots\ldots\ldots \end{aligned}\right\} \quad (8.14)$$

then on the basis (8.12) and (8.13) it easily is reduced to system of equations in the deviations

$$\left.\begin{aligned}\frac{d}{dt}\delta\xi_1&=\left(\frac{\partial f_1}{\partial\xi_1}\right)_*\delta\xi_1+\left(\frac{\partial f_1}{\partial\xi_2}\right)_*\delta\xi_2+\left(\frac{\partial f_1}{\partial\xi_3}\right)_*\delta\xi_3+\ldots\\ \frac{d}{dt}\delta\xi_2&=\left(\frac{\partial f_2}{\partial\xi_1}\right)_*\delta\xi_1+\left(\frac{\partial f_2}{\partial\xi_2}\right)_*\delta\xi_2+\left(\frac{\partial f_2}{\partial\xi_3}\right)_*\delta\xi_3+\ldots\\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\end{aligned}\right\}\qquad(8.15)$$

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If the undisturbed motion is known, i.e., cell/elements $\xi_{1*}(t)$; $\xi_{2*}(t)$; $\xi_{3*}(t)$ and others are assigned, then they will be also known in the function of time and the partial derivatives of form $\left(\frac{\partial f_i}{\partial\xi_i}\right)_*$, which stand in system (8.15) during deviations $\delta\xi_i$ of cell/elements. In this case system (8.15) will represent by itself the system of linear differential equations, since the new variables enter in equations only to the first degree and their cross products of the type $\delta\xi_1\delta\xi_2$ they are absent, while the initial equations of undisturbed and disturbed motions (8.14) linear they are not.

Utilizing a method presented, let us con ѕct the linearization of the system of the differential equations, which describe the simple case of the axial motion of the unguided fin-stabilized missile. Let us take the known to us system of differential equations (3.51). Considering that thrust it is directed along the axis of rocket (i.e. after placing angle $\varepsilon=0$) and after accepting $X_{p1}=Y_{p1}=0$, after conversions from (3.51) and (3.52) we will obtain

$$\left.\begin{aligned} m\frac{dv}{dt}&=P\cos\alpha-X-Q\sin\theta;\\ mv\frac{d\theta}{dt}&=P\sin\alpha+Y-Q\cos\theta;\\ J_z\frac{d^2\theta}{dt^2}&=M_z. \end{aligned}\right| \qquad (8.16)$$

During linearization we will not consider effect on the disturbed motion characteristics of a change in the mass δm and in the moment of inertia δJ_z, let us consider that the mass and the moment of inertia for the undisturbed and disturbed motions change on time equally: $m(t)=m_*(t)$ and $J_z(t)=J_{z*}(t)$.

Furthermore, let us disregard the effect of the deviation of height/altitude of aerodynamic characteristics and thrust. For low values δy, this effect is unessential, since functions $H(y)$, $\rho(y)$, $w(y)$ and $a(y)$, through which it it is exhibited, change slowly.

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During the adopted simplifications in the deviation of aerodynamic forces and torque/moment, they will depend only on two values - deviation of the flight speed δv and of the deviation of angle of attack δα. If we designate

$$\begin{aligned} X_* &= f_1(v_*, \alpha_*) \\ Y_* &= f_2(v_*, \alpha_*) \\ M_{z*} &= f_3(v_*, \alpha_*) \end{aligned} \quad \text{and} \quad \begin{aligned} X &= f_1(v, \alpha); \\ Y &= f_2(v, \alpha); \\ M_z &= f_3(v, \alpha), \end{aligned}$$

that, expanding last/latter dependences in a series on formula (8.9), we will obtain taking into account only linear terms:

$$\left.\begin{aligned} \delta X &= X - X_* = \left(\frac{\partial X}{\partial v}\right)_* \delta v + \left(\frac{\partial X}{\partial \alpha}\right)_* \delta\alpha; \\ \delta Y &= Y - Y_* = \left(\frac{\partial Y}{\partial v}\right)_* \delta v + \left(\frac{\partial Y}{\partial \alpha}\right)_* \delta\alpha; \\ \delta M_z &= M_z - M_{z*} = \left(\frac{\partial M_z}{\partial v}\right)_* \delta v + \left(\frac{\partial M_z}{\partial \alpha}\right)_* \delta\alpha. \end{aligned}\right| \quad (8.17)$$

Mark * shows that the datum is related to unperturbed motion at the torque/moment, which corresponds to the beginning of disturbance/perturbation. Let us introduce the abbreviated recording of the partial derivatives

$$\left(\frac{\partial A}{\partial \xi_i}\right)_* = A^{\xi_i}$$

and let us rewrite formulas (8.17) in this form

$$\delta X = X^v \delta v + X^\alpha \delta\alpha; \quad \delta Y = Y^v \delta v + Y^\alpha \delta\alpha; \quad \delta M_z = M_z^v \delta v + M_z^\alpha \delta\alpha.$$

Let us similarly find deviations for the terms, which contain thrust P:

$$\delta(P\sin\alpha)=P\cos\alpha_*\,\delta\alpha;\quad \delta(P\cos\alpha)=-P\sin\alpha_*\,\delta\alpha.$$

Counting the weight of rocket Q on the section of disturbance/perturbation with constant, we will obtain

$$\delta(Q\sin\theta)=Q\cos\theta_*\,\delta\theta;\quad \delta(Q\cos\theta)=-Q\sin\theta_*\,\delta\theta.$$

Taking into account the obtained expressions for the deviations of forces and torque/moments, it is possible to write:

$$m\left(\frac{dv}{dt}-\frac{dv_*}{dt}\right)=\delta(P\cos\alpha)-\delta X-\delta(Q\sin\theta);$$
$$m\left(v\frac{d\theta}{dt}-v_*\frac{d\theta_*}{dt}\right)=\delta(P\sin\alpha)+\delta Y-\delta(Q\cos\theta);$$
$$J_z\left(\frac{d^2\theta}{dt^2}-\frac{d^2\theta_*}{dt^2}\right)=\delta M_z.$$

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Bearing in mind that

$$\frac{dv}{dt}-\frac{dv_*}{dt}=\frac{d}{dt}\,\delta v;$$

$$v\frac{d\theta}{dt}-v_*\frac{d\theta_*}{dt}=v_*\frac{d}{dt}\,\delta\theta+\delta v\frac{d\theta_*}{dt}+\delta v\frac{d}{dt}\,\delta\theta\approx v_*\frac{d}{dt}\,\delta\theta$$

(without the account of the members of the second order of smallness),

(8. 3)

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$$\frac{d^2\vartheta}{dt^2}-\frac{d^2\vartheta_*}{dt^2}=\frac{d^2}{dt^2}\delta\vartheta \quad \text{and} \quad \delta\theta=\delta\vartheta-\delta\alpha,$$

and also accepting in view of the smallness of angle α_*, that $\sin\alpha_* \approx \alpha_*$ and $\cos\alpha_* \approx 1$, we will obtain the following system of the linearized equations:

$$m\frac{d\delta v}{dt}=-X^v\delta v-(P\alpha_*+X^\alpha-Q\cos\theta_*)\delta\alpha-Q\cos\theta_*\delta\theta;$$

$$mv_*\left(\frac{d\delta\vartheta}{dt}-\frac{d\delta\alpha}{dt}\right)=Y^v\delta v+(P+Y^\alpha-Q\sin\theta_*)\delta\alpha+Q\sin\theta_*\delta\theta; \tag{8.18}$$

$$J_z\frac{d^2\delta\vartheta}{dt^2}=M_z^v\delta v+M_z^\alpha\delta\alpha.$$

This system consists of linear homogeneous differential equations with the coefficients, which are the known functions of time.

Let us now conduct the linearization of the more complex system of the differential equations, which describe the spatial motion of the rocket of class "surface - surface", controlled in flight by aerodynamic controls. Let us take system (2.29) and let us lead it to the form, more convenient for linearization. Let us assume that the rocket is axisymmetric and thrust is directed along longitudinal axis Cx_1; for an axisymmetric rocket it is possible to also take $J_{y_1}=J_{z_1}$.

Since the control forces are considerably lesser than the thrust and the aerodynamic forces, it is possible to assume at small angles α and β that

$$X_{p1}\cos\alpha\cos\beta \approx X_p;\quad Y_{p1}\cos\alpha \approx Y_p;\quad Z_{p1}\cos\beta \approx Z_p.$$

Let us introduce into system of equations perturbing forces and the torque/moments, which act on rocket in flight and calling its deviation from the undisturbed trajectory - X_B, Y_B, Z_B, M_{y_1B} and M_{z_1B}. Subsequently let us consider them the known functions of time. Furthermore, in system (3.29) the trigonometric relationship/ratios between angles θ, ϑ, α and ψ, Ψ and β let us replace more convenient for linearization.

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After conversions the system of equations will take the form

$$
\left.
\begin{aligned}
&\dot{v}=\frac{P\cos\alpha\cos\beta}{m}-\frac{X+X_p}{m}-g\sin\theta+\frac{X_в}{m};\\
&\dot{\theta}=\frac{P\sin\alpha}{mv}+\frac{Y+Y_p}{mv}-\frac{g\cos\theta}{v}+\frac{Y_в}{mv};\\
&\dot{\Psi}=\frac{P\cos\alpha\sin\beta}{mv\cos\theta}-\frac{Z+Z_p}{mv\cos\theta}+\frac{Z_в}{mv\cos\theta};\\
&\dot{x}_3=v\cos\theta\cos\Psi;\quad \dot{y}_3=v\sin\theta;\quad \dot{z}_3=-v\cos\theta\sin\Psi;\\
&\dot{\omega}_{y_1}=\frac{M_{y_1}+M_{py_1}}{J_{y_1}}+\frac{J_{z_1}-J_{x_1}}{J_{y_1}}\omega_{x_1}\omega_{z_1}+\frac{M_{y_1в}}{J_{y_1}};\\
&\dot{\omega}_{z_1}=\frac{M_{z_1}+M_{pz_1}}{J_{z_1}}-\frac{J_{y_1}-J_{x_1}}{J_{z_1}}\omega_{x_1}\omega_{y_1}+\frac{M_{z_1в}}{J_{z_1}};\\
&\dot{\vartheta}=\omega_{z_1};\quad \dot{\psi}=\omega_{y_1}\frac{1}{\cos\vartheta};\quad \dot{\omega}_{x_1}=\omega_{y_1}\operatorname{tg}\vartheta;\\
&\quad\sin\theta=\cos\alpha\cos\beta\sin\vartheta-\sin\alpha\cos\beta\cos\vartheta;\\
&\quad\sin\Psi\cos\theta=\cos\alpha\cos\beta\sin\psi\cos\vartheta+\\
&\quad+\sin\alpha\cos\beta\sin\psi\sin\vartheta-\sin\beta\cos\psi.
\end{aligned}
\right\}\qquad (8.19)
$$

During linearization, as before let us accept $m(t)=m_*(t)$, $J_{y_1}(t)=J_{y_1*}(t)$, $J_{z_1}(t)=J_{z_1*}(t)$, we will not consider effect δy on a change in the aerodynamic forces and thrust, let us introduce the abbreviated recording of derivatives and will drop/omit for simplicity of writing mark of the coefficients during deviations. During the writing of members, who contain the characteristics of aerodynamic control forces and torque/moments, we will use dependences (2.133), (2.134), (2.135) and the abbreviated form of the recording of derivatives, the angles of deflection of controls let us designate $\Delta\delta_{x_1}$, $\Delta\delta_{y_1}$ and $\Delta\delta_{z_1}$.

The linearization of the written system of equations is

conducted on common/general/total formula (8.15).

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Taking into account the large volume of the systematic recording of the process of linearization, let us give here without conclusion/derivation the result of the linearizations of system (8.19), obtained by Yu. F. Kol'tsov [18]:

$$
\left.
\begin{aligned}
\frac{d}{dt}\delta v &= -\frac{X^v + X_p^v}{m}\delta v - g\cos\theta\delta\theta - \frac{P\sin\alpha\cos\beta + X^\alpha}{m}\delta\alpha - \\
&\quad - \frac{P\cos\alpha\sin\beta}{m}\delta\beta - \frac{X_p^{\delta_{z_1}}}{m}\Delta\delta_{z_1} + \frac{X_B}{m}; \\
\frac{d}{dt}\delta\theta &= \frac{1}{mv}\left(Y^v + Y_p^v - \frac{P\sin\alpha + Y + Y_p - mg\cos\theta}{v}\right)\delta v + \\
&\quad + \frac{g\sin\theta}{v}\delta\theta + \frac{P\cos\alpha + Y^\alpha}{mv}\delta\alpha + \frac{Y_p^{\delta_{z_1}}}{mv}\Delta\delta_{z_1} + \frac{Y_B}{mv};
\end{aligned}
\right\} \quad (8.20)
$$

$$\frac{d}{dt}\delta\Psi=-\frac{1}{mv\cos\theta}\left(Z^{v}+Z^{v}_{p}+\frac{P\cos\alpha\sin\beta-Z-Z_{p}}{v}\right)\Delta v+$$
$$+\dot{\Psi}\,\mathrm{tg}\,\theta\delta\theta-\frac{P\sin\alpha\sin\beta}{mv\cos\theta}\delta\alpha+\frac{P\cos\alpha\cos\beta-Z^{\beta}}{mv\cos\theta}\delta\beta-$$
$$-\frac{Z_{p}^{\delta_{y_1}}}{mv\cos\theta}\Delta\delta_{y_1}+\frac{Z_{B}}{mv\cos\theta};$$
$$\frac{d}{dt}\delta x_3=\cos\theta\cos\Psi\delta v-v\sin\theta\cos\Psi\delta\theta-v\cos\theta\sin\Psi\delta\Psi;$$
$$\frac{d}{dt}\delta y_3=\sin\theta\delta v+v\cos\theta\delta\theta;$$
$$\frac{d}{dt}\delta z_3=-\cos\theta\sin\Psi\delta v+v\sin\theta\sin\Psi\delta\theta-v\cos\theta\cos\Psi\delta\Psi;$$
$$\frac{d}{dt}\delta\omega_{y_1}=\frac{M^{v}_{y_1}+M^{v}_{p\,y_1}}{J_{y_1}}\delta v+\frac{M^{\beta}_{y_1}}{J_{y_1}}\delta\beta+\frac{M_{p\,y_1}^{\delta_{y_1}}}{J_{y_1}}\Delta\delta_{y_1}+$$
$$+\frac{M^{\omega_{x_1}}_{y_1}+(J_{z_1}-J_{x_1})\omega_{z_1}}{J_{y_1}}\delta\omega_{x_1}+\frac{M^{\omega_{y_1}}_{y_1}}{J_{y_1}}\delta\omega_{y_1}+$$
$$+\frac{J_{z_1}-J_{x_1}}{J_{y_1}}\omega_{x_1}\delta\omega_{z_1}+\frac{M^{\dot{\beta}}_{y_1}}{J_{y_1}}\delta\dot{\beta}+\frac{M_{p\,y_1}^{\dot{\delta}_{y_1}}}{J_{y_1}}\Delta\dot{\delta}_{y_1}+\frac{M_{y_1\,B}}{J_{y_1}};\qquad (8.20)$$
$$\frac{d}{dt}\delta\omega_{z_1}=\frac{M^{v}_{z_1}+M^{v}_{p\,z_1}}{J_{z_1}}\delta v+\frac{M^{\alpha}_{z_1}}{J_{z_1}}\delta\alpha+\frac{M_{p\,z_1}^{\delta_{z_1}}}{J_{z_1}}\Delta\delta_{z_1}-$$
$$-\frac{J_{y_1}-J_{x_1}}{J_{z_1}}\omega_{y_1}\delta\omega_{x_1}-\frac{J_{y_1}-J_{x_1}}{J_{z_1}}\omega_{x_1}\delta\omega_{y_1}+\frac{M^{\omega_{z_1}}_{z_1}}{J_{z_1}}\delta\omega_{z_1}+$$
$$+\frac{M^{\dot{\alpha}}_{z_1}}{J_{z_1}}\delta\dot{\alpha}+\frac{M_{p\,z_1}^{\dot{\delta}_{z_1}}}{J_{z_1}}\Delta\dot{\delta}_{z_1}+\frac{M_{z_1B}}{J_{z_1}};$$
$$\frac{d}{dt}\delta\vartheta=\delta\omega_{z_1};$$
$$\frac{d}{dt}\delta\psi=\dot{\psi}\,\mathrm{tg}\,\vartheta\delta\vartheta+\frac{1}{\cos\vartheta}\delta\omega_{y_1};$$
$$\delta\omega_{x_1}=\frac{\omega_{y_1}}{\cos^2\vartheta}\delta\vartheta+\mathrm{tg}\,\vartheta\delta\omega_{y_1};$$
$$\cos\theta\delta\theta=(\cos\alpha\cos\beta\cos\vartheta+\sin\alpha\cos\beta\sin\vartheta)\delta\vartheta-$$
$$-(\sin\alpha\cos\beta\sin\vartheta+\cos\alpha\cos\beta\cos\vartheta)\delta\alpha-$$
$$-(\cos\alpha\sin\beta\sin\vartheta-\sin\alpha\sin\beta\cos\vartheta)\delta\beta;$$
$$\cos\Psi\cos\theta\delta\Psi-\sin\Psi\sin\theta\delta\theta=-(\cos\alpha\cos\beta\sin\psi\sin\vartheta-$$
$$-\sin\alpha\cos\beta\sin\psi\cos\vartheta)\delta\vartheta-(\sin\alpha\cos\beta\sin\psi\cos\vartheta-$$
$$-\cos\alpha\cos\beta\sin\psi\sin\vartheta)\delta\alpha-(\cos\alpha\sin\beta\sin\psi\cos\vartheta+$$
$$+\sin\alpha\sin\beta\sin\psi\sin\vartheta+\cos\beta\cos\psi)\delta\beta+\qquad (8.20)$$
$$+(\cos\alpha\cos\beta\cos\psi\cos\vartheta+\sin\alpha\cos\beta\cos\psi\sin\vartheta+$$
$$+\sin\beta\sin\psi)\delta\psi.$$

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The analysis of the system of the linearized equations, carried out by Yu. P. Kol'tsov, showed that system (8.20) can be considerably simplified. Simplifications these are based on what for the rockets of the class in question in the undisturbed motion lateral kinematic cell/elements $\Psi_*, \psi_*, \beta_*, \omega_{x_1*}, \omega_{y_1*}$, deflection δ_{y_1*} of rudders, governing yawing motion, the value of the angles of attack α_* and of angular velocity ω_{z_1*}, and time derivatives of the cell/elements of the longitudinal and yawing motion of rocket $\dot{\Psi}_*, \dot{\psi}_*, \dot{\alpha}_*, \dot{\delta}_{z_1*}$ are also so low that it is possible to disregard the products of these cell/elements and other low values. So, in equation $\frac{d}{dt}\delta\Psi$, it is possible for this reason to disregard terms $\dot{\Psi}\,\mathrm{tg}\,\theta\delta\theta$ and $\frac{P\sin\alpha\sin\beta}{mv\cos\theta}\delta\alpha$, in equation $\frac{d}{dt}\delta x_3$ - by term $v\cos\theta\sin\Psi\delta\Psi$, in equation $\frac{d}{dt}\delta z_3$ - by terms $\cos\theta\sin\Psi\delta v$ and $v\sin\theta\sin\Psi\delta\theta$, in equation $\frac{d}{dt}\delta\omega_{y_1}$ - by terms $\frac{\overline{M}^v_{y_1}+M^v_{p\,y_1}}{J_{y_1}}\delta v$, $\frac{J_{z_1}-J_{x_1}}{J_{y_1}}\omega_{z_1}\delta\omega_{x_1}$ and so forth. Furthermore, during simplifications on the basis of smallness α_*, β_*, ψ_*, Ψ_* was conducted the replacement of trigonometric functions from these angles by their approximate values, i.e.,

$$\sin\alpha_*=\alpha_*=\alpha;\quad \sin\beta_*=\beta_*=\beta,$$

$$\cos\alpha_*=\cos\beta_*=\cos\psi_*=\cos\Psi_*=1$$ and so forth.

During simplification in the geometric relationship/ratios between the deviations of angles θ and ϑ, Ψ and ψ it was also accepted that $\cos\theta \approx \cos\vartheta$.

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As a result of the analysis indicated which we here do not bring in detail, in the equations of system (8.20) can be reject/thrown a series of the terms the order of smallness of which higher than first, after which system will take the following simplified form:

$$\left.\begin{aligned}\frac{d}{dt}\delta v = &-\frac{X^v + X_p^v}{m}\delta v - g\cos\theta\delta\theta - \frac{P\alpha + X^\alpha}{m}\delta\alpha - \\ &-\frac{X_p^{\delta_{z_1}}}{m}\Delta\delta_{z_1} + \frac{X_в}{m};\end{aligned}\right\} \quad (8.21)$$

$$\left.\begin{aligned}
&\frac{d}{dt}\,\delta\theta=\frac{Y^{v}+Y^{v}_{p}}{mv}\,\delta v+\frac{g\sin\theta}{v}\,\delta\theta+\frac{P+Y^{\alpha}}{mv}\,\delta\alpha+\\
&\qquad+\frac{Y_{p}^{\delta_{z_1}}}{mv}\,\Delta\delta_{z_1}+\frac{Y_{\text{в}}}{mv};\\
&\frac{d}{dt}\,\delta x_3=\cos\theta\,\delta v-v\sin\theta\,\delta\theta;\quad \frac{d}{dt}\,\delta y_3=\sin\theta\,\delta v+v\cos\theta\,\delta\theta;\\
&\frac{d}{dt}\,\delta\omega_{z_1}=\frac{M^{v}_{z_1}+M^{v}_{p\,z_1}}{J_{z_1}}\,\delta v+\frac{M^{\alpha}_{z_1}}{J_{z_1}}\,\delta\alpha+\frac{M^{\delta_{z_1}}_{p\,z_1}}{J_{z_1}}\,\Delta\delta_{z_1}+\frac{M^{\omega_{z_1}}_{z_1}}{J_{z_1}}\,\delta\omega_{z_1}+\\
&\qquad+\frac{M^{\dot\alpha}_{z_1}}{J_{z_1}}\,\delta\dot\alpha+\frac{M^{\dot\delta_{z_1}}_{p\,z_1}}{J_{z_1}}\,\Delta\dot\delta_{z_1}+\frac{M_{z_1\text{в}}}{J_{z_1}};\\
&\frac{d}{dt}\,\delta\vartheta=\delta\omega_{z_1};\quad \delta\alpha=\delta\vartheta-\delta\theta;
\end{aligned}\right\}\quad(8.21)$$

$$\left.\begin{aligned}
&\frac{d}{dt}\,\delta\Psi=\frac{P-Z^{\beta}}{mv\cos\theta}\,\delta\beta-\frac{Z_{p}^{\delta_{y_1}}}{mv\cos\theta}\,\Delta\delta_{y_1}+\frac{Z_{\text{в}}}{mv\cos\theta};\\
&\frac{d}{dt}\,\delta z_3=-v\cos\theta\,\delta\Psi;\\
&\frac{d}{dt}\,\delta\omega_{y_1}=\frac{M^{\beta}_{y_1}}{J_{y_1}}\,\delta\beta+\frac{M^{\delta_{y_1}}_{p\,y_1}}{J_{y_1}}\,\Delta\delta_{y_1}+\frac{M^{\omega_{x_1}}_{y_1}}{J_{y_1}}\,\Delta\omega_{x_1}+\frac{M^{\omega_{y_1}}_{y_1}}{J_{y_1}}\,\delta\omega_{y_1}+\\
&\qquad+\frac{M^{\dot\beta}_{y_1}}{J_{y_1}}\,\delta\dot\beta+\frac{M^{\dot\delta_{y_1}}_{p\,y_1}}{J_{y_1}}\,\Delta\dot\delta_{y_1}+\frac{M_{y_1\text{в}}}{J_{y_1}};\\
&\frac{d}{dt}\,\delta\psi=\frac{1}{\cos\vartheta}\,\delta\omega_{y_1};\quad \delta\omega_{x_1}=\operatorname{tg}\vartheta\,\delta\omega_{y_1};\quad \delta\beta=\cos\theta\,\delta\psi-\cos\theta\,\delta\Psi.
\end{aligned}\right\}\quad(8.22)$$

The common/general/total system of the nonlinear differential equations of motion of the class of rockets in question as a result of its linearization and simplifications decomposes into two independent systems of linear differential equations of motion in

deviations, moreover one of these systems (8.21) describes the longitudinal disturbed motion of rocket, which occurs in plane Ox_3y_3, and another (8.22) - the lateral disturbed motion in inclined plane Oxz_3.

It is obvious that two independent systems of linear equations can be solved considerably simpler than single reference system from nonlinear differential equations and geometric relationship/ratios, especially if will be used the mathematical calculating and analog computers. However, it is clear that simplicity of solution is reached to the detriment its accuracy.

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Consequently, during the use of a method of equations of motion at basis of which lie/rests the condition of sufficient smallness of disturbance/perturbations, it is necessary to know, which the accuracy of those obtained by the calculation of results, or, otherwise, by which limits must be limited the disturbance/perturbations being investigated, so that the errors for calculation according to by equations in deviations would not exceed permissible. The comprehensive answer/response to this question can be obtained by the comparison of results the approximate and exact solutions, however, taking into account the already noted labor

expense the latter, this method of evaluating the accuracy it does not have extensive application.

Therefore are frequently utilized less strict, but simpler indirect, or those approximated, the methods of estimation of error. One Of them is the evaluation of sum R of terms of expansion, rejected during the linearization of initial equations, with the aid of which it is possible to tentatively establish/install the permissible region of disturbance/perturbations even prior to accomplishing of calculations.

Let us find, for example, which error we allow/assume during the linearization of the equation $\dot{\theta}$ of system (8.19) in it component Y/mv. Let us preserve in the common/general/total formula of expansion (8.9) only linear terms, after designating through R permanent. Then

$$R=f(\xi_{1*}+\delta\xi_1;\ \xi_{2*}+\delta\xi_2\dots,\ \xi_{n*}+\delta\xi_n)-$$
$$-f(\xi_{1*},\ \xi_{2*},\dots,\ \xi_{n*})-\left(\frac{\partial f}{\partial\xi_1}\right)_*\delta\xi_1-$$
$$-\left(\frac{\partial f}{\partial\xi_2}\right)_*\delta\xi_2-\dots-\left(\frac{\partial f}{\partial\xi_n}\right)_*\delta\xi_n. \qquad (8.23)$$

Assuming that Y changes only because of a change in velocity and angle of attack, we will obtain

$$R_Y=\frac{Y}{mv}-\frac{Y_*}{mv_*}-\left(\frac{Y}{mv}\right)^v_*\delta v-\left(\frac{Y}{mv}\right)^\alpha_*\delta\alpha=$$
$$=\frac{Sqc_y^\alpha}{2m}\{[(\alpha_*+\delta\alpha)\delta v+v_*\delta\alpha]-[\alpha_*\delta v+v_*\delta\alpha]\}.$$

Hence

$$R_Y = \frac{S\varrho c_y^{\alpha}}{2m}\,\delta v\delta\alpha = \frac{Y_*}{mv_*}\,\frac{\delta v}{v_*}\,\frac{\delta\alpha}{\alpha_*}.$$

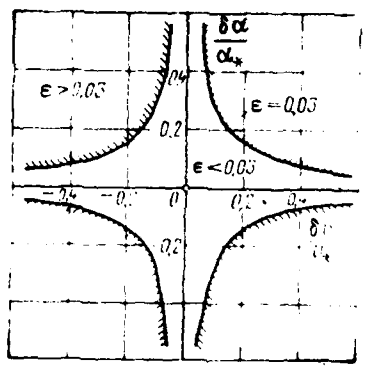

Fig. 8.6.

Fig. 8.6. Range of the allowed values of the ratios $\delta\alpha/\alpha_*$ and $\delta v/v_*$ which corresponds to relative error $\varepsilon=0{,}03$

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Consequently, during linearization we allow/assume relative error ε, equal to

$$\varepsilon=\frac{R_Y}{Y_* m v_*}=\frac{\delta v}{v_*}\,\frac{\delta \alpha}{\alpha_*}. \tag{8.24}$$

Being assigned by the values of permissible error ε, it is possible to construct on dependence (8.24) the curve/graphs which will restrict the range of the allowed values of deviations δv and $\delta\alpha$. This range for $\varepsilon=0{,}03$ is given to Fig. 8.6 and it shows that the allowed values $\delta\alpha/\alpha_*$ are great with low $\delta v/v_*$ and decrease with increase $\delta v/v_*$.

After conducting this type of analysis and for other terms of initial equations, it is possible to establish/install the generalized limiting values of the disturbance/perturbations within which the accuracy of calculation according to the linearized equations will be satisfactory.

§3. Methods of the solution of also of the study of the linearized equations of the disturbed motion of rockets and of projectiles.

The systems of equations of the longitudinal (8.21) and lateral (8.22) disturbed motion of rockets for their analysis and solutions it is accepted to record/write in a simpler form, introducing the abbreviations of those stand during the deviations of the coefficients which are called dynamic. The dynamic coefficients, entering the equations of axial motion, let us designate a_{ij}; those entering the equations of yawing motion - b_{ij}, utilizing in this case not the digital indices, but literal, since they are more demonstrative. The first index (i) designates the equation which includes the coefficient: the second (j) - the deviation, during which it stands. Thus, for instance, coefficient $a_{v\theta}$ is related to the system of equations of axial motion and stands in equation for the calculation of the deviation of the velocity of the motion of the rocket δv during the deviation of the flight path angle δθ. For the terms, which reflect in equations the perturbation factors, let us accept designations $\frac{X_B}{m}=f_v(t)$; $\frac{M_{z_1B}}{J_{z_1}}=f_{\omega_{z_1}}(t)$ and so forth. With the even number of this, the examine/considered by us systems of equations accept following form.

1. Equations of longitudinal disturbed motion

$$\left.\begin{aligned}
&\frac{d}{dt}\,\delta v = a_{vv}\delta v + a_{v\theta}\delta\theta + a_{v\alpha}\delta\alpha + a_{v\delta_{z_1}}\Delta\delta_{z_1} + f_v(t);\\
&\frac{d}{dt}\,\delta\theta = a_{\theta v}\delta v + a_{\theta\theta}\delta\theta + a_{\theta\alpha}\delta\alpha + a_{\theta\delta_{z_1}}\Delta\delta_{z_1} + f_\theta(t);\\
&\frac{d}{dt}\,\delta x_3 = a_{xv}\delta v + a_{x\theta}\delta\theta;\quad \frac{d}{dt}\,\delta y_3 = a_{yv}\delta v + a_{y\theta}\delta\theta;\\
&\frac{d}{dt}\,\delta\omega_{z_1} = a_{\omega_{z1}v}\delta v + a_{\omega_{z1}\alpha}\delta\alpha + a_{\omega_{z1}\delta_{z1}}\Delta\delta_{z_1} + a_{\omega_{z1}\omega_{z1}}\delta\omega_{z_1} +
\end{aligned}\right\} \quad (8.25)$$

$$\left.\begin{aligned}
&+ a_{\omega_{z1}\dot\alpha}\,\delta\dot\alpha + a_{\omega_{z_1}\dot\delta_{z1}}\Delta\dot\delta_{z1} + f_{\omega_{z1}}(t);\\
&\frac{d}{dt}\,\delta\vartheta = \delta\omega_{z_1};\quad \delta\alpha = \delta\vartheta - \delta\theta.
\end{aligned}\right\} \quad (8.25)$$

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The entering these equations dynamic coefficients are respectively equal to

$$\left.\begin{aligned}
&a_{vv} = -\frac{X^v + X_p^v}{m}; && a_{v\theta} = -g\cos\theta;\\
&a_{v\alpha} = -\frac{P\alpha + X^\alpha}{m}; && a_{v\delta_{z1}} = -\frac{X_p^{\delta_{z_1}}}{m};\\
&a_{\theta v} = \frac{Y^v + Y_p^v}{mv}; && a_{\theta\theta} = -\frac{g\sin\theta}{v};\\
&a_{\theta\alpha} = \frac{P + Y^\alpha}{mv}; && a_{\theta\delta_{z1}} = \frac{Y_p^{\delta_{z_1}}}{mv};\\
&a_{xv} = \cos\theta; && a_{x\theta} = -v\sin\theta;\\
&a_{yv} = \sin\theta; && a_{y\theta} = v\cos\theta;\\
&a_{\omega_{z1}v} = \frac{M_{z_1}^v + M_{p\,z_1}^v}{J_{z_1}};\quad a_{\omega_{z1}\alpha} = \frac{M_{z_1}^\alpha}{J_{z_1}}; && a_{\omega_{z1}\delta_{z1}} = \frac{M_{p\,z_1}^{\delta_{z_1}}}{J_{z_1}};\\
&a_{\omega_{z1}\omega_{z1}} = \frac{M_{z_1}^{\omega_{z_1}}}{J_{z_1}};\quad a_{\omega_{z1}\dot\alpha} = \frac{M_{z_1}^{\dot\alpha}}{J_{z_1}}; && a_{\omega_{z1}\dot\delta_{z1}} = \frac{M_{p\,z_1}^{\dot\delta_{z_1}}}{J_{z_1}}.
\end{aligned}\right\} \quad (8.26)$$

2. Equations of lateral disturbed motion

$$\left.\begin{aligned}
&\frac{d}{dt}\delta\Psi = b_{\Psi\beta}\delta\beta + b_{\Psi\delta_{y1}}\Delta\delta_{y_1} + f_{\Psi}(t);\\
&\frac{d}{dt}\delta z_3 = b_{z\Psi}\delta\Psi;\\
&\frac{d}{dt}\delta\omega_{y_1} = b_{\omega_{y1}\beta}\delta\beta + b_{\omega_{y1}\delta_{y1}}\Delta\delta_{y_1} + b_{\omega_{y1}\omega_{x1}}\delta\omega_{x_1} + b_{\omega_{y1}\omega_{y1}}\delta\omega_{y_1} +\\
&\qquad + b_{\omega_{y1}\dot\beta}\delta\dot\beta + b_{\omega_{y1}\dot\delta_{y1}}\Delta\dot\delta_{y_1} + f_{\omega_{y1}}(t);\\
&\frac{d}{dt}\delta\psi = b_{\psi\omega_{y1}}\delta\omega_{y_1};\ \delta\omega_{x_1} = \mathrm{tg}\,\vartheta\delta\omega_{y_1};\\
&\delta\beta = \cos\theta\delta\psi - \cos\theta\delta\Psi.
\end{aligned}\right\} \tag{8.27}$$

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Dynamic coefficients in these equations have the values

$$\left.\begin{aligned}
&b_{\Psi\beta} = \frac{P - Z^{\beta}}{mv\cos\theta};\qquad b_{\Psi\delta_{y1}} = -\frac{Z_{p}^{\delta_{y_1}}}{mv\cos\theta};\\
&b_{z\Psi} = -v\cos\theta;\qquad b_{\psi\omega_{y1}} = \frac{1}{\cos\vartheta};\\
&b_{\omega_{y1}\beta} = \frac{M_{y_1}^{\beta}}{J_{y_1}};\quad b_{\omega_{y1}\delta_{y1}} = \frac{M_{p\,y_1}^{\delta_{y_1}}}{J_{y_1}};\quad b_{\omega_{y1}\omega_{x1}} = \frac{M_{y_1}^{\omega_{x_1}}}{J_{y_1}};\\
&b_{\omega_{y1}\omega_{y1}} = \frac{M_{y_1}^{\omega_{y_1}}}{J_{y_1}};\quad b_{\omega_{y1}\dot\beta} = \frac{M_{y_1}^{\dot\beta}}{J_{y1}};\quad b_{\omega_{y1}\dot\delta_{y1}} = \frac{M_{p\,y_1}^{\dot\delta_{y_1}}}{J_{y_1}}.
\end{aligned}\right\} \tag{8.28}$$

Subsequently to avoid repetitions whole presentation let us conduct in connection with the system of equations of axial motion, since the methods of its solution and analysis as the results of this analysis, in essence the same as for the system, which describes the

lateral disturbed motion.

From the examination of system of equations (8.25) it follows that in it the together solved equations are the dynamic and geometric equations, i.e.,

$$\left.\begin{aligned}
&\frac{d}{dt}\,\delta v = a_{vv}\delta v + a_{v\theta}\delta\theta + a_{v\alpha}\delta\alpha + a_{v\delta_{z1}}\Delta\delta_{z_1} + f_v(t);\\
&\frac{d}{dt}\,\delta\theta = a_{\theta v}\delta v + a_{\theta\theta}\delta\theta + a_{\theta\alpha}\delta\alpha + a_{\theta\delta_{z1}}\Delta\delta_{z_1} + f_\theta(t);\\
&\frac{d}{dt}\,\delta\omega_{z_1} = a_{\omega_{z1}v}\delta v + a_{\omega_{z1}\alpha}\delta\alpha + a_{\omega_{z1}\delta_{z1}}\Delta\delta_{z_1} + a_{\omega_{z1}\omega_{z1}}\delta\omega_{z_1} +\\
&\qquad\qquad + a_{\omega_{z1}\dot{\alpha}}\delta\dot{\alpha} + a_{\omega_{z1}\dot{\delta}_{z1}}\Delta\dot{\delta}_{z1} + f_{\omega_{z1}}(t);\\
&\frac{d}{dt}\,\delta\vartheta = \delta\omega_{z_1};\quad \delta\alpha = \delta\vartheta - \delta\theta.
\end{aligned}\right\} \qquad (8.29)$$

The supplementary dependence, necessary for the calculation of value $\delta\dot{\alpha}$, entering the equation for determination $\delta\omega_{z_1}$, is obtained by differentiation with respect to the time of geometric relationship/ratio $\delta\alpha=\delta\vartheta-\delta\theta$.

After making this process/operation, we will obtain

$$\frac{d}{dt}\,\delta\alpha = \frac{d}{dt}\,\delta\vartheta - \frac{d}{dt}\,\delta\theta,$$

whence finally it follows

$$\frac{d}{dt}\,\delta\alpha = \delta\,\frac{d\alpha}{dt} = \delta\dot{\alpha} = \frac{d}{dt}\,\delta\vartheta - \frac{d}{dt}\,\delta\theta. \qquad (8.30)$$

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The kinematic relationship/ratios

$$\frac{d}{dt}\delta x_3 = a_{xv}\delta v + a_{x\theta}\delta\theta$$

and

$$\frac{d}{dt}\delta y_3 = a_{yv}\delta v + a_{y\theta}\delta\theta$$

with the made by us assumptions in the course of the solution of system of equations (8.29) are not utilized; they are integrated after the solution of this system.

The initial space of the solution of system of equations (8.29) is the determination of entering in them dynamic coefficients a_{ij}. Above has already been noted that for this preliminarily must be designed the cell/elements of the undisturbed motion of rocket $v_*(t)$, $\theta_*(t)$, $y_*(t)$, $\alpha_*(t)$, $\delta_{z1*}(t)$ etc. Knowing these cell/elements as functions from the flight time of rocket, and also its weight m(t) and inertial characteristics $J_{z_1}(t), J_{y_1}(t)$ and $J_{x_1}(t)$, on dependences (8.26) it is possible to calculate the unknown dynamic coefficients of rocket, moreover they will be obtained as discrete functions one time alone - argument of the linearized equations of motion (8.29). Giving overall qualitative evaluation to dynamic coefficients, it is necessary to note that many of them strongly change along the trajectory of the motion of rocket. As an example Fig. 8.7 gives the curve/graphs of a change in four dynamic coefficients

a_{tt}, $a_{\omega_{z1}\alpha}$, $a_{\omega_{z1}t_{z1}}$, $a_{\omega_{z1}\omega_{z1}}$ for the projectile "Oerlikon" during its flight in trajectory [36].

From curve/graphs it is evident that dynamic coefficients a_{tt} and $a_{\omega_{z1}\omega_{z1}}$ change for flight time 7-8 times, $a_{\omega_{z1}t_{z1}}$ - approximately 12 times, but coefficient $a_{\omega_{z1}\alpha}$ - is more than 50 times.

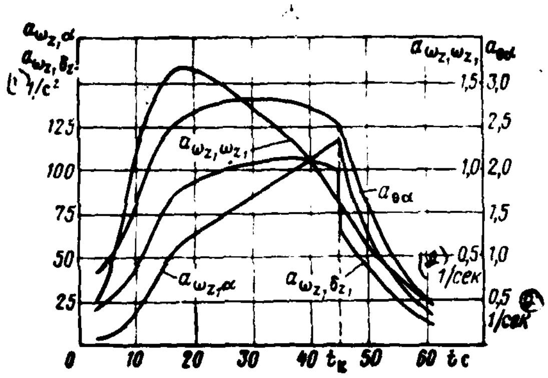

Fig. 8.7. Character of a change in some dynamic coefficients of the rocket during its motion along trajectory.

Key: (1). 1/s². (2). 1/s.

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Taking into account the above, and also that that deviations $\Delta\delta_{z_1}(t)$, $\Delta\dot{\delta}_{z_1}(t)$, $\Delta\delta_{y_1}(t)$ and so forth are the assigned functions of time, the examine/considered by us system of equations (8.29) can be described as system of linear nonhomogeneous equations with variable coefficients, the dependence on time $a_{ij}(t)$ having the most diverse, that not yielding to simple mathematical description, form.

It is known that in the general case of this type of system of

equations they cannot be solved analytically. The integration of system (8.29) can be realized by numerical methods with the use of digital computers and the methods of electronic simulation with the aid of the analog continuous computers. Since the method of simulation for the solution of problems regarding the dynamic properties of rockets and projectiles, stability of their motion and stability of the work of the control system finds at present the widest application, let us give the pattern of the solution of system (8.29) in the simulating electronic computer, comprised by Yu. F. Kol'tsov for case $f_{\dot{\varphi}}(t)=f_{\theta}(t)=0$ [18]. Pattern is comprised taking into account the designations, described in Section 3.1 chapters VI, on it for clarity, are shown real physical quantities - cell/elements of the disturbed motion of rocket (Fig. 8.8). In working pattern must figure as the proportional to them voltage/stresses for which the scale during the translation/conversion of initial equations into machine is accepted similar so that during entire process of the solution of problem in any of the blocks voltage/stress would not exceed permissible (3, chapter VI). Therefore with the reading of block diagram, it is necessary to remember that under designations (for example, $\delta v, \alpha_{ve}, \frac{d}{dt}\delta\omega_{z_1}$ and so forth) hide themselves not same these values, but proportional to them voltages.

Furthermore, for a larger graphic clarity pattern is shown on figure in open-circuited form, i.e., are not carried out the lines,

which connect the output points of circuits with intake points. Instead of this of the point of input/introduction and output of the values, utilized during the solution of equations, are numbered. With the reading of circuit, it is necessary to bear in mind, that exit points and input/introduction of values, which have identical numbers, in working pattern are connected with each other, and the electrical circuits between them are locked. Thus, for instance, in the circuit of the solution of equation d/dtδv for the formation of the derived at the entry of integrating block $\int(\delta v)$ point of input/introduction are supplied the following significance of a deviation:

into point 1 - deviation δv from output 1;

to point 4 - deviation δθ from output 4;

to point 10 - deviation δα from output 10.

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In turn, values δv (-δv) from outputs 2(1) are supplied in the circuit of the integration of other equations of the system in question, etc. In other respects diagram accurately reflects the course of solution of problem.

During training/preparation for solution on the basis of the previously known undisturbed motion of rocket about which it is necessary to examine the unknown disturbed motion, they are designed, are scaled and are solderred on the blocks of the variable coefficients of the functions, which approximate the curve/graphs of the dynamic coefficients of equations $a_{ij}(t)$; furthermore, are calculated and with the aid of the blocks of constant coefficients are introduced into diagram the constant scale factors, for example, coefficients $\mu_{\vartheta_1}, \mu_{\theta_1}, \mu_{\omega}$, and so forth.

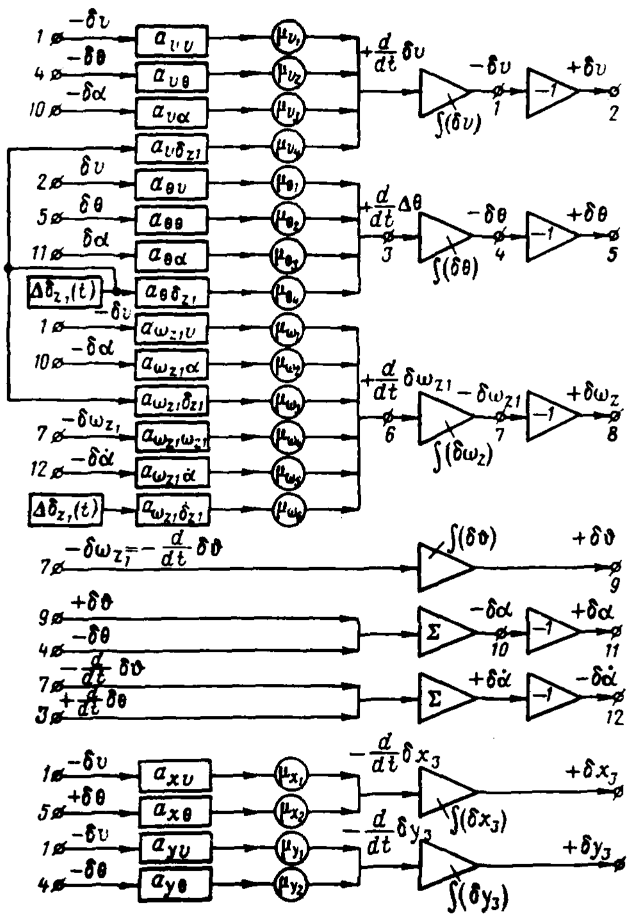

Fig. 8.8. Schematic of solution in analog computer of problem on study of dynamic properties of rocket in its intrinsic disturbed axial motion.

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We examine an example, when the disturbed motion is caused only by any deviation of steering organ/controls (in our case of elevators) from their position in the trimmed/steading-state itself reference-flight conditions - zero or balancing, i.e., by values $\Delta\delta_{z_1}(t)$ and $\Delta\dot{\delta}_{z_1}(t)$. (It is obvious that if the nominal trajectory corresponds to the zero position of controls, then it is possible in that case to count deviations $\Delta\delta_{z_1}=\delta_{z_1}$ and $\Delta\dot{\delta}_{z_1}=\dot{\delta}_{z1}$). The form of dependences $\Delta\delta_{z_1}(t)$ and $\Delta\dot{\delta}_{z_1}(t)$ can be different depending on the target/purposes of investigation. Figures 8.9, for example, shows different forms of the deviations of angle $\Delta\delta_{z_1}$, introduced to machine for the study of the dynamic properties of the rocket motion by which is simulated.

As can be seen from Fig. 8.9a, these deviations can be assigned in the form of square-waves signal, which differ in terms of amount of deflection $\Delta\dot{\delta}_{z_1 i}$ and in terms of the duration of its actions τ_i, with the aid of which is imitated the action on the rocket of stepped ratios of steering organ/controls.

Deviations can be also assigned in the form of the harmonic functions (see Fig. 8.9b), whose amplitude $\Delta\dot{\delta}_{z_1 i}$, period T_i and the duration of action τ_i can be changed. The introduction of the perturbation signals of this form makes it possible to investigate the ability of rocket to follow the control displacement.

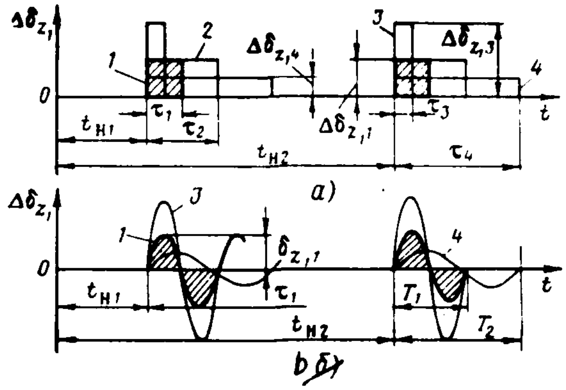

Fig. 8.9. Diagram of the forcing functions, utilized during the study of the dynamic properties of rocket and its control system.

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Finally, changing time $t_н$ it took effect of disturbance/perturbation, we can investigate dynamic properties and the stability of rocket on the most unfavorable phases of trajectory of its motion. As for coefficients a_{ij}, the approximated curve/graphs of forcing functions $\Delta\delta_{z_1}(t)$ and $\Delta\dot{\delta}_{z_1}(t)$ are soldered to the blocks of the variable coefficients, by means of which they are introduced into diagram after the starting/launching of machine. If $t_н>0$, then in the period of time $0-t_н$ machine will issue zero solution $\delta v(t) = \delta\theta(t) = \delta\alpha(t) = \delta y_3(t) = 0$, showing thereby, that in the absence of disturbance/perturbations the rocket moves over nominal trajectory.

From the torque/moment of time $t=t_n$ programmed disturbance/perturbations $\Delta\delta_{z_1}(t)$ and $\Delta\dot{\delta}_{z_1}(t)$ will begin to enter the appropriate blocks of variable coefficients, assigning dynamic coefficients a_{ij}, where there occurs the multiplication of these values.

In passing by through the scaling blocks of constant coefficients, the voltages, proportional to forcing functions, will hit the integrating operational amplifiers by which they will be integrated. Obtained as a result of this deviation δv, $\delta\theta$, $\delta\omega_{z1}$, ... with those signs with which they enter in the equations of the disturbed motion (for the report/communication to them of the necessary signs in diagram are provided the inverting units), they are transferred by the appropriate channels to the lead-in points of the units of variable coefficients for the formation of the new values of the derived or component at entries units of operational amplifiers. This process of the unceasing formation of input values and their integration (or summation) in analog computers occurs on closed cycle continuously before the termination of the interval of the process of the disturbed motion of rocket in question along trajectory.

The unknown values, which characterize the disturbed motion of rocket $\delta v(t)$, $\delta\theta(t)$, $\delta\alpha(t)$, $\delta\omega_{z_1}(t)$ and so forth, can be written

with the aid of multichannel loop or electron-radiation, equipped with special photo attachment, the oscillograph to which they must be given from the appropriate output of operational amplifiers. The example of the recording of the results of the solution of a similar problem for a wingless rocket is shown on Fig. 8.10, to which are given the curve/graphs of changes in the deviations $\delta\alpha(t)$, $\Delta\vartheta(t)$, $\delta\theta(t)$ and $\delta v(t)$, that correspond to stepped forcing function $\Delta\delta_{z_1}(t)$ when $\Delta\dot{\delta}_{z_1}(t)=0$.

If we look the curve/graphs of deviation indicated change for a statically stable rocket in prolonged time interval, then it is not difficult to establish that they all reflect the presence of oscillating processes, moreover very different. As can be seen from Fig. 8.10, dependence $\delta\alpha(t)$ is characterized by the presence of rapidly damping oscillations with their short period, which correspond to transient process from one balance angle $\alpha_б$ to another. So at this time changes the deviation $\delta\vartheta$ of pitch angle. This oscillation they are called short-period.

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Changes in kinematic cell/elements δv, $\delta\theta$, δx_3, δy_3 etc. are also accompanied by oscillations, but the period of these oscillations is already ten times more (it can even exceed the flight time of rocket

on active phase of trajectory) and attenuate they slower.

The oscillations indicated are called long-period or phugoid; most distinctly they are exhibited at winged missiles and cruise missiles. For rockets and projectiles, whose lift (i.e. coefficient c_y^α) is comparatively small or little flight time, phugoid oscillation are also low and into calculation usually do not enter.

After the interpretation of oscillograms or the nature of the change in the deviations of the cell/elements of the motion of rocket, it is possible to judge the stability of its motion, about the quality of transient process from one conditions/mode of the steady flight to another, the correctness of selection during the aerodynamic design of the steady-state stability factor, damping characteristics, etc.

The method of the solution of the equations of the disturbed motion of rockets presented with the aid of the linear electronic analog computers, as we already noted above, has very wide acceptance, but nevertheless it very laborious. In connection with this for conducting the analytical solution of the questions, connected with the selection of many design, weight, aerodynamic and other parameters of rocket, in the process of its initial design they go for even more considerable simplifications in the problem.

The essence of this simplification consists in the fact that the dynamic properties of the design/projected rocket are estimated not according to the results of the solution of the single system of equations of disturbed motion (8.29) with variable coefficients $a_{ij}(t)$, but by the analytical investigation of the totality of similar systems of equations, dynamic coefficients in which consider constant values a^{*}_{ij}.

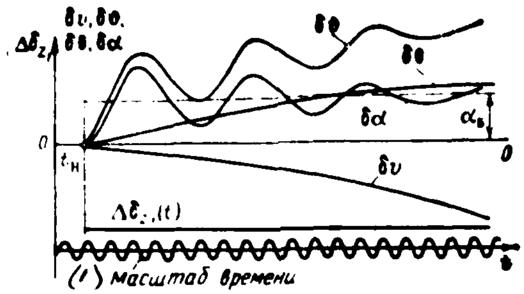

Fig. 8.10. Example of the recording of solution in the analog computer of problems on the study of a change in the deviations of the trajectory elements of rocket in its intrinsic disturbed motion in the case of the step deflection of control.

Key: (1). Time scale.

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The number of such together investigated systems must be selected depending on the number of characteristic points of the undisturbed trajectory of the motion of rocket. To such characteristic points in the trajectory, can be attributed the points of inclusion and engine cutoff, in which occurs a considerable change in the thrust, the point of the booster ejection and stages of rocket, it is characterized by an abrupt change in mass and torque/moments of

inertia, point with the greatest and smallest values of velocity head, etc.

For established/installed characteristic points in the trajectory, must be calculated the concrete/specific/actual values of dynamic coefficients $a^*_{ij}(t_1), a^*_{ij}(t_2), \ldots, a^*_{ij}(t_n)$, which will enter into the systems of equations being investigated. During similar investigations, furthermore, they make still one simplification - they set/assume in the reference system of equations (8.29) $\Delta\delta_{z_1} = \Delta\dot{\delta}_{z_1} = 0$ (this corresponds to the attached elevators) and $f_v(t) = f_\theta(t) = \ldots = 0$, this system is converted into the system of homogeneous linear differential first-order equations with constant coefficients. Taking into account the aforesaid and uniting in (8.29) the third and fourth equations, let us write it for concrete/specific/actual point in the trajectory in question in the following form:

$$\left.\begin{aligned} &\frac{d}{dt}\delta v = a^*_{vv}\delta v + a^*_{v\theta}\delta\theta + a^*_{v\alpha}\delta\alpha;\\ &\frac{d}{dt}\delta\theta = a^*_{\theta v}\delta v + a^*_{\theta\theta}\delta\theta + a^*_{\theta\alpha}\delta\alpha;\\ &\frac{d^2}{dt^2}\delta\vartheta = a^*_{\omega_{z1}v}\delta v + a^*_{\omega_{z1}\alpha}\delta\alpha + a^*_{\omega_{z1}\omega_{z1}}\frac{d}{dt}\delta\vartheta + a^*_{\omega_{z1}\dot{\alpha}}\frac{d}{dt}\delta\alpha;\\ &\delta\alpha = \delta\vartheta - \delta\theta. \end{aligned}\right| \quad (8.31)$$

In this system we have four equations with four unknown values $\delta v(t)$, $\delta\theta(t)$, $\delta\vartheta(t)$ and $\delta\alpha(t)$. Consequently, system is locked and can be integrated.

The general solution of the system of homogeneous linear equations (8.31) is located as sum of its particular solutions which, as is known from mathematics, for similar equations they take the form

$$\left.\begin{array}{ll} \delta v = Ae^{\lambda t}; & \delta \theta = Be^{\lambda t}; \\ \delta \vartheta = Ce^{\lambda t}; & \delta \alpha = De^{\lambda t}. \end{array}\right\} \qquad (8.32)$$

After substituting expressions (8.32) into system of equations (8.31), we will obtain

$$\begin{gathered} \lambda Ae^{\lambda t} = \dot{a}_{vv} Ae^{\lambda t} + \dot{a}_{v\theta} Be^{\lambda t} + \dot{a}_{v\alpha} De^{\lambda t}; \\ \lambda Be^{\lambda t} = \dot{a}_{\theta v} Ae^{\lambda t} + \dot{a}_{\theta\theta} Be^{\lambda t} + \dot{a}_{\theta\alpha} De^{\lambda t}; \\ \lambda^2 Ce^{\lambda t} = \dot{a}_{\omega_{z1}} Ae^{\lambda t} + \dot{a}_{\omega_{z1}\alpha} De^{\lambda t} + \dot{a}_{\omega_{z1}\omega_{z1}} \lambda Ce^{\lambda t} + \dot{a}_{\omega_{z1}\dot{\alpha}} \lambda De^{\lambda t}; \qquad (8.33) \\ De^{\lambda t} = Ce^{\lambda t} - Be^{\lambda t}. \end{gathered}$$

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In all terms of the obtained equations enters common factor $e^{\lambda t}$. After contraction to this factor and the transfer of all terms of equations into left side, we will obtain the following system:

$$\begin{gathered} (\dot{a}_{vv} - \lambda) A + \dot{a}_{v\theta} B + \dot{a}_{v\alpha} D = 0; \\ \dot{a}_{\theta v} A + (\dot{a}_{\theta\theta} - \lambda) B + \dot{a}_{\theta\alpha} D = 0; \qquad (8.34) \\ \dot{a}_{\omega_{z1} v} A + \lambda (\dot{a}_{\omega_{z1}\omega_{z1}} - \lambda) C + (\dot{a}_{\omega_{z1}\alpha} + \lambda \dot{a}_{\omega_{z1}\dot{\alpha}}) D = 0; \\ -B + C - D = 0. \end{gathered}$$

This system includes no longer differential, but linear

algebraic equations, moreover unknown values in it they are the parameter λ and coefficients A, B, C and D. The procedure of the solution of systems of type (8.34) is known. At first their characteristic equation, which in the case in question represents by itself the equation of the fourth degree

$$\lambda^4+k_1\lambda^3+k_2\lambda^2+k_3\lambda+k_4=0, \qquad (8.35)$$

they find the values of its roots. Such roots will be four: λ_1, λ_2, λ_3, λ_4. Each root is substituted in algebraic system (8.34) and from it unambiguously find values of coefficients of A, B, C and D, which correspond to this root:

for λ_1 - A_1, B_1, C_1, D_1.

for λ_2 - A_2, B_2, C_2, D_2.

and so forth.

After this the general solution of system (8.31) can be written in the form

$$\left.\begin{aligned}\delta v&=A_1e^{\lambda_1 t}+A_2e^{\lambda_2 t}+A_3e^{\lambda_3 t}+A_4e^{\lambda_4 t};\\ \delta\theta&=B_1e^{\lambda_1 t}+B_2e^{\lambda_2 t}+B_3e^{\lambda_3 t}+B_4e^{\lambda_4 t};\\ \delta\vartheta&=C_1e^{\lambda_1 t}+C_2e^{\lambda_2 t}+C_3e^{\lambda_3 t}+C_4e^{\lambda_4 t};\\ \delta\alpha&=D_1e^{\lambda_1 t}+D_2e^{\lambda_2 t}+D_3e^{\lambda_3 t}+D_4e^{\lambda_4 t}.\end{aligned}\right\} \qquad (8.36)$$

The coefficients of general solution (8.36) A_i, B_i, C_i and D_i, and also the coefficients of characteristic equation k_i are the real values which depend on the dynamic coefficients of the equations of disturbed motion (8.31) and for each specific case are different. In this case, roots λ_i of characteristic equation (8.35) can be both the real and those conjugate/combined composite, moreover in the case (for quartic equation) in question are possible only three following combinations of their values.

1. All four roots of characteristic equation are real. In this case changes of each of the deviations of the trajectory elements of rocket δv, $\delta\theta$, ... will be be defined as result of the summation of four aperiodic functions and record/written in the form (8.36).

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2. Two Roots of characteristic equation real, other two roots - conjugated/combined composite, for example:

$$\lambda_{3,4}=\xi \pm i\nu.$$

In this case, utilizing Euler's known relationship/ratios

$$e^{i\nu t}+e^{-i\nu t}=2\cos\nu t \qquad (8.37)$$

and

$$e^{i\nu t}-e^{-i\nu t}=2i\sin\nu t,$$

equations (8.36) can be converted as follows:

$$\left.\begin{aligned}\delta v&=A_1e^{\lambda_1 t}+A_2e^{\lambda_2 t}+A_{3,4}e^{\xi t}\sin(\nu t+\gamma_1);\\ \delta\theta&=B_1e^{\lambda_1 t}+B_2e^{\lambda_2 t}+B_{3,4}e^{\xi t}\sin(\nu t+\gamma_2);\\ \delta\vartheta&=C_1e^{\lambda_1 t}+C_2e^{\lambda_2 t}+C_{3,4}e^{\xi t}\sin(\nu t+\gamma_3);\\ \delta\alpha&=D_1e^{\lambda_1 t}+D_2e^{\lambda_2 t}+D_{3,4}e^{\xi t}\sin(\nu t+\gamma_4).\end{aligned}\right\} \quad (8.38)$$

The entering in them new constant coefficients $A_{3,4}$, $B_{3,4}$ $C_{3,4}$ $D_{3,4}$ γ_i find in the process of transformation. From expressions (8.38) it follows that in the case in question a change of each deviation will be determined by the summation of two aperiodic functions with the function, which describes the oscillating process which is characterized, for example, for velocity with an amplitude of $A_{3,4}e^{\xi t}$, by angular frequency ν and by phase shift γ_i.

3. Finally, let us examine solution when all four roots of characteristic equation form two pairs of conjugated/combined composite roots:

$$\lambda_{1,2}=\zeta\pm i\eta;\quad \lambda_{3,4}=\xi\pm i\nu.$$

In this case a change of each of the cell/elements will be determined as a result of the summation of two oscillatory/vibratory functions, and decisions for them make the following form:

$$\left.\begin{aligned}\delta v&=A_{1,2}e^{\zeta t}\sin(\eta t+\psi_1)+A_{3,4}e^{\xi t}\sin(\nu t+\gamma_1);\\ \delta\theta&=B_{1,2}e^{\zeta t}\sin(\eta t+\psi_2)+B_{3,4}e^{\xi t}\sin(\nu t+\gamma_2);\\ \delta\vartheta&=C_{1,2}e^{\zeta t}\sin(\eta t+\psi_3)+C_{3,4}e^{\xi t}\sin(\nu t+\gamma_3);\\ \delta\alpha&=D_{1,2}e^{\zeta t}\sin(\eta t+\psi_4)+D_{3,4}e^{\xi t}\sin(\nu t+\gamma_4).\end{aligned}\right\} \quad (8.39)$$

The given three types of the solutions of system of equations

(8.31) show, as they would change with respect to the time of amount of deflection δv, $\delta\theta$, $\delta\vartheta$ and $\delta\alpha$, if the initial undisturbed motion of rocket, beginning with characteristic point in the trajectory in question, it represented by itself straight-and-level flight with the constant velocity, i.e., under conditions, clearly different from the real.

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Therefore the obtained analytical solutions are utilized, as a rule, only for approximate qualitative evaluation of the dynamic properties of rocket, especially as this evaluation can be made on the basis of the analysis of the roots of characteristic equation.

If real roots or the real parts of the composite roots of characteristic equation (8.35) (whole or part of them) will be positive, then the entering all the solutions (8.39) factors of the type $e^{\lambda t}$ will grow/rise in the course of time. In this case, the deviations being investigated from the undisturbed motion δv, $\delta\theta$, $\delta\vartheta$, $\delta\alpha$ will also after the break-down of disturbance/perturbation grow/rise with larger or lower speed, what is the sign/criterion of the instability of rocket or projectile in this state of motion and testifies to the incorrect selection of the dynamic coefficients, which determine, as noted above, the values of the roots of

characteristic equation. If during the design of value m(t), J(t), m_z^{α}, $m_{\delta z}$, c_y^{α} etc., entering the dynamic coefficients, undertaken correctly, then all real roots and the real parts of the composite roots will be negative or their part with the others negative will obtain zero value. In this case, the deviations δv, $\delta\theta$, $\delta\vartheta$, $\delta\alpha$ will respectively or decrease (attenuate) or not attenuate, but also not increase. In the first case the rocket calls stable in axial motion, in the second - neutral.

Investigations show that the rockets and the fin-stabilized projectiles unguided or with the attached controls are neutral dynamic systems, since their deviation from the trajectory of the undisturbed motion are not removed by themselves after the break-down of their caused disturbance/perturbation. Dynamically stable rocket becomes only when the acting automatic control system is present, of flight with the correctly selected equation of control (by law of regulation).

Qualitative answer/response to a question concerning the stability of flight vehicle can be obtained and without determining the roots of characteristic equation (8.35). For obtaining the negative values of real roots and real parts of the composite roots, it is necessary and it suffices to satisfy the condition

$$k_1>0;\quad k_2>0;\quad k_3>0;\quad k_4>0;$$
$$k_1k_2k_3-k_1^2k_4-k_3^2>0.$$

With the observance of these conditions, the motion will be stable.

The results of testing the stability of rocket with respect to the roots of characteristic equation cannot be considered sufficient. They are suitable only for the preliminary selection of the aerodynamic, weight, inertia and other parameters of the rocket during its design and they must be confirmed by the more precise methods of the study of the dynamics of the control systems.

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3.1. Transient processes during the motion of rockets and projectiles.

The motion of rockets and projectiles are subdivided into steady and that being unsteady. Under steady motion is understood such motion, during which its determining parameters (velocity of the motion of the center of mass v, angles α, β, γ, angular rates of rotation ω_{x_1}, ω_{y_1} and ω_{z_1}) in the course of time do not change. It is obvious that under the actual conditions of this steady motion the rockets and the projectiles does not happen to have. The concept of

steady motion (or flight equilibrium) is most frequently introduced conditionally in the examination of motion relative to the center of mass (balancing flight conditions). In actuality, however, a change in the angles of rotation of controls or the appearance of any perturbation factors always disrupt the steady moment balance relative to the center of mass of rocket and they force it to transfer/convert from one conditions/mode of the steady flight to another.

Let the initial steady motion of a rocket be characterized by value $\delta_{z_1}=0$, and therefore $\alpha_6=0$. Let us assume that at the moment of time $t = t_1$ the elevators were deflected by angle $\delta_{z_1}<0$, to which will correspond under the assumption $\dot{\theta}$ = 0 trim angle α_6.

Further angular motion of the rocket with retention of angle δ_{z1} by constant illustrate the curve/graphs, given to Fig. 8.11. In time interval t_1-t_2 under the action of total torque/moment $M_{\Sigma z}=M_{pz_1}-M_{z_1}>0$ the rocket begins to be turned relative to the center of mass, in this case, it will appear and will grow/rise angle of attack $\alpha > 0$ and angular velocity $\dot{\alpha}$. At the moment of time t_2, the angle of attack α will become equal to trim angle α_6, and $M_{\Sigma z}$ - zero. But, possessing at the moment of time in question angular velocity $\dot{\alpha}_2 > 0$, rocket as inert body will continue its rotation in previous direction until torque/moment $M_{\Sigma z}$ inhibits it in position 3 ($\dot{\alpha}_3 = 0$) and will force to move again to the position of trim equilibrium (position 4).

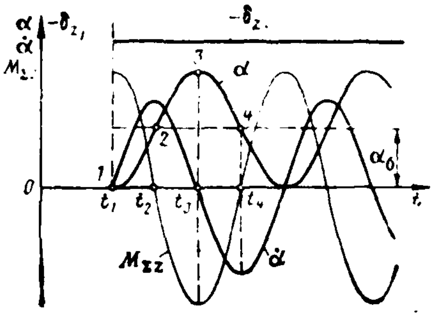

Fig. 8.11. Change of the cell/elements of the angular motion of rocket (in the absence of damping) in the case of the instantaneous step deflection of control.

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In the absence of damping, this oscillating process will be consecutively repeated, i.e., it will be harmonic, moreover frequency will be determined by the moment of the inertia of rocket and by its steady-state stability factor. Because of damping the process in question bears the damped character, in consequence of which during a certain time is establish/installed the trim equilibrium of rocket and it begins to move in the new trimmed/steading-state itself conditions/mode at value $\delta_{r,} \neq 0$.

The process of transiting the rocket (or any other system) from one steady state to another calls transient process.

The form of transient process depends substantially on the damping characteristics of rocket and on their relationship/ratio with the reserve of its static stability.

For the illustration of this, let us describe mathematically transient process taking into account damping and will examine solution. Let us introduce into equation (8.3) of harmonic

oscillations instead of the angle α angle $\alpha_n = \alpha - \alpha_6$, which characterizes the oscillation of the axis of rocket of its relatively balancing position, and term $2k\dot{\alpha}_n$, which considers damping moment $M_{\alpha_{z_1}}$. After this the equation will take the form

$$\ddot{\alpha}_n = 2k\dot{\alpha}_n + n^2\alpha_n = 0, \qquad (8.40)$$

where

$$k = \frac{Sql}{J_{z_1}} \frac{l}{v} \left| m_{z_1}^{\bar{\omega}_{z_1}} \right|.$$

with conditions for point $t = t_1 = 0$; $\alpha_{n.n} = -\alpha_6$; $\dot{\alpha}_{n.n} = 0$, which characterize the beginning of transient process during the sudden deflection of elevators, we obtain the following solutions of equation (8.40):

a) if $n^2 - k^2 > 0$, then

$$\alpha_n = \alpha_{n.m} e^{-kt} \sin\left(\sqrt{n^2 - k^2}\,t + \varepsilon\right), \qquad (8.41)$$

where

$$\alpha_{n.m} = \frac{n\alpha_{n.n}}{\sqrt{n^2 - k^2}}, \quad \varepsilon = \operatorname{arctg} \frac{\sqrt{n^2 - k^2}}{k}.$$

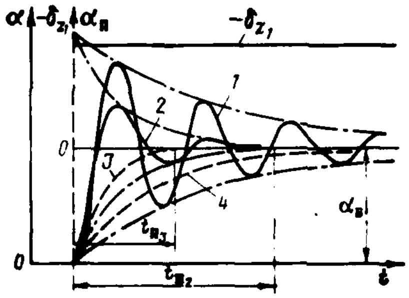

Fig. 8.12. Different forms of the transient processes of a change of angle of attack during the instantaneous step deflection of control in the case of oscillation damping.

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As can be seen from solution (8.41), a change in angle $\alpha_{\text{п}}$ bears the character of oscillations with period $T=\frac{2\pi}{\sqrt{n^2-k^2}}$, moreover the amplitude of oscillations in the course of time decreases, approaching zero at the end of the transient process when $t=t_{\text{п}}$.

It is not difficult to see that, changing the damping characteristics $m_{z_1}^{\omega_{z_1}}$ of rocket and, consequently, also coefficient k, it is possible to regulate transit time $t_{\text{п}}$ desirably. For example, with an increase in rotary derivative $m_{z_1}^{\omega_{z_1}}$ of the fluctuation of angle $\alpha_{\text{п}}$ they will attenuate more intense, and

transit time will decrease. In Fig. 8.12 to examined solution (under identical initial conditions) correspond the curves 1 and 2, moreover $(m_{z_1}^{\omega_{z_1}})_1<(m_{z_1}^{\omega_{z_1}})_2$;

b) if $n^2-k^2<0$, then the solution of equation (8.40) is expressed by the formula

$$a_n=a_{n.n}e^{-kt}\left(\operatorname{ch}\sqrt{k^2-n^2}t+\frac{k}{\sqrt{k^2-n^2}}\operatorname{sh}\sqrt{k^2-n^2}t\right), \quad (8.42)$$

showing that in this case angle a_n changes aperiodically, approaching zero at the end of the transient process, moreover here with increase of k (i.e. and $m_{z_1}^{\omega_{z_1}}$) with the constant n transit time already grow/rises (curves 3 and 4).

The relationship/ratios between coefficients of k and n to which correspond the curves, given to Fig. 8.12, were undertaken in an example following:

№ кривой (1)	1	2	3	4
k/n	0,3	0,9	1,1	2,5

Key: (1). curved.

Best are considered such transient processes when oscillations in system or do not appear (aperiodic process, curve 3), or are

weakly expressed (curve 2), but transit time is small. An increase of transient period and the presence in it of considerable oscillations decrease the accuracy of the operation of the systems of rocket control and, as a result, the accuracy of the incidence/impingement by them into target/purpose.

§4. Conditions of the stable flight of the rotating rockets and of projectiles.

Artillery shells and TPS, that are stabilized in flight because of high-spin motion, intended for a firing to the comparatively small distances with which the rotation of the Earth and its curvature do not have virtually effect on the stability characteristics of motion.

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If we approximately consider the rotational effect and curvature of the Earth by the introduction of acceleration $\bar{g}=\bar{g}_0$, then complete system of equations can be written on the basis of equations (1.16) and (1.21).

In the right side of equations (1.16) they must be introduced to the projection of all forces, which act on projectile, on the axis of the driving/moving i coordinate system. For the rotating rocket

projectiles these forces are external aerodynamic forces [(2.109), (2.110), (2.111)], the thrust of all engines (2.120), Magnus' force (2.112) and the secondary forces (see Chapter II, Section 5.2). To secondary forces can be attributed the forces, caused by aerodynamic and gas-dynamic eccentricities, and the forces, caused by the lack of balance of masses.

In the right side of equation (1.21) one should write the moments of the named forces and the torque/moment of surfice friction (2.113). For the guided missiles, it is logical, must be added the control forces and the moments of these forces; for the artillery shells of constant mass, reactive and variation forces must be excluded.

Has independent value the examination of effect on stability of motion of the conditions of firing from the fast-moving carrier, for example, of aircraft, and the account of the effect of the wind effect (see Chapter XI).

The compilation of system of equations taking into account all acting forces in the form, suitable for numerical solution, represents complex and laborious problem. For the projectiles of constant mass, this problem was for the first time solved by V. S. Pugachev upon consideration of drag, normal force and Magnus' force.

During the writing of the equations of rotary motion, were considered the torque/moments: inverted, damping, the moment of the Magnus force and the torque/moment of surface friction. For TRS fundamental works on this question were carried out by Ya. M. Shapiro, Yu. A. Kochetkov, et al.

As showed many well known examinations, the motion of projectiles and rockets of their relatively centers of mass can be examined separately from forward motion along trajectory, counting that all the characteristics of last/latter are known functions of time [v(t), θ(t), $y_3(t)$ and so forth]. In this case, for the engineering calculations of the stability of the rotating projectile, it suffices in the first approximation, to consider only tilting moment $M_z=M$ (2.111), which is the basic factor, which determines the character of rotary motion [9]. Missile attitude in its rotary motion is determined, as usual, by three angles: two angles determine the position of the axis of projectile relative to velocity vector and one angle - rotation of projectile of relatively longitudinal rotational axis. Position of velocity vector of relatively earth-based coordinate system of definition by two angles - θ and Ψ.

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Depending on the formulation of the problem, missile attitude relative to velocity vector can be determined by angle δ between the axis of projectile and tangent to trajectory (Fig. 8.13). This angle, in accordance with the terminology of the theory of gyroscopic devices, is called nutation angle. In the plane of angle δ, by the called plane of resistance, acts tilting moment M_z. The position of the plane of the resistance of relatively vertical plane is determined by dihedral angle ν, whose fin/edge coincides with velocity vector v. Angle ν is called precession angle. The rotation of projectile relative to the spin axis is determined by the angle ϕ whose plane is perpendicular to the longitudinal axis of projectile Cx_1.

The position of the longitudinal axis of projectile relative to velocity vector can be determined by the angles δ_1 and δ_2, which replace angles δ and ν. Communication/connection between the named pairs of angles is determined from spherical right triangle with legs δ_1 and δ_2. With the low values of angles δ_1 and δ_2, which corresponds to stable projectiles, it is possible to write

$$\delta=\sqrt{\delta_1^2+\delta_2^2};\quad \delta_1=\delta\sin\nu;\quad \delta_2=\delta\cos\nu;\quad \mathrm{tg}=\frac{\delta_1}{\delta_2}.\quad (8.43)$$

During the solution of problem for flat/plane curved path taking into account a decrease in the tangent to trajectory into examination, are introduced the angles θ, ϕ, δ_1 and δ_2. For straight

path the solution proves to be more simply, since it suffices to introduce only angles ϕ, ν and δ. During the study of the yawing motion of the rotating projectile, additionally is introduced angle Ψ.

Let us examine the rotary motion of projectile in connection with curved path taking into account a decrease in the tangent. The equations of rotary motion let us comprise in the form of the differential equations of Lagrange of 2 kinds (1.33).

We plan comprising of angular velocity to the principal axes of inertia of projectile, which let us designate in the manner that it is accepted in ballistics of the projectiles of the barrel systems: the longitudinal axis of projectile Cx_1 - through $O\zeta$, the equatorial axes of rectangular coordinate system - through $O\xi$ and $O\eta$.

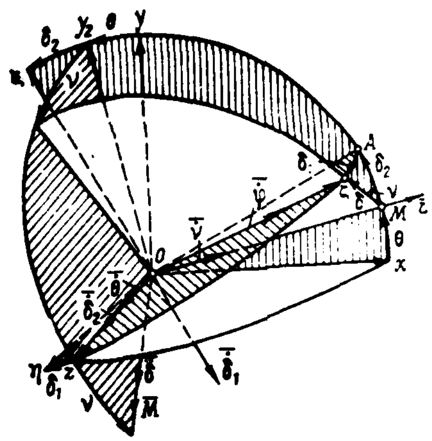

Fig. 8.13. Schematic of the angles, which determine the position of the rotating projectile in trajectory.

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The origin of coordinates it is consistent with the center of mass of projectile. The orientation of axes relative to velocity vector is shown on Fig. to 8.13. For use (1.33) it is necessary to have derivatives $\frac{\partial T}{\partial q_i}$ and $\frac{\partial T}{\partial \dot{q}_i}$, where T - kinetic energy of projectile in rotary motion; q_i - generalized coordinate. Kinetic energy is defined by the expression

$$T=\frac{1}{2}(Ap^2+Bq^2+Cr^2),$$

where C - axial moment of inertia, A and B - torque/moments of inertia of relatively equatorial axes Cξ and Oη; p, q and r -

projection of the instantaneous angular velocity of projectile on axis $O\xi$, $O\eta$ and $O\zeta$.

For the axisymmetric projectiles $A = B$ and, therefore,

$$T=\frac{1}{2}[A(p^2+q^2)+Cr^2].$$

As the generalized coordinates, which determine missile attitude during rotation relative to the center of mass, let us take angles δ_1, δ_2 and φ. Angle φ is not shown not in order not to complicate drawing, to simply show the angular velocity vector $\dot{\varphi}$, necessary for the abandonment of equations. Angle θ and it derivatives $\dot{\theta}$ and $\ddot{\theta}$ are accepted as the known functions of time.

The vector of the instantaneous angular rate of rotation of projectile is determined by the sum

$$\bar{\Omega}=\bar{\dot{\delta}}_1+\bar{\dot{\delta}}_2+\bar{\dot{\varphi}}+\bar{\dot{\theta}}.$$

The sense of the vector angular velocities is selected according to the appropriate rule of mechanics. Vector $\bar{\dot{\delta}}_1$ is perpendicular the plane of angle δ_1 and coincides with the negative direction of axis $O\xi$. Angular velocity vectors $\bar{\dot{\delta}}_2$ and $\bar{\dot{\theta}}$ are perpendicular to the single plane of angles δ_2 and θ and are directed along the positive direction of axis Oz. Projecting vector $\bar{\Omega}$ to the coordinate axes, connected with projectile, let us have

$$p=-\dot{\delta}_1;\quad q=(\dot{\theta}+\dot{\delta}_2)\cos\delta_1;\quad r=\dot{\varphi}+(\dot{\theta}+\dot{\delta}_2)\sin\delta_1.$$

At the low values of angles δ_1 and δ_2, it is possible to accept $\sin\delta_1=\delta_1$, $\cos\delta_1=1$. Then expression for kinetic energy of projectile in rotary motion we obtain in this form:

$$T=\frac{1}{2}\{A[\dot{\delta}_1^2+(\dot{\theta}+\dot{\delta}_2)^2]+C[\dot{\varphi}+(\dot{\theta}+\dot{\delta}_2)\delta_1]^2\}. \quad (8.44)$$

On the basis of the general consideration of mechanics, the generalized force for the angular parameter is equal to the projection of the acting torque/moment on the axis of elementary rotation. In the case in question generalized forces will be the projections of vector M on the directions of angular rates of rotation $\dot{\delta}_1$, $\dot{\delta}_2$ and $\dot{\varphi}$

$$Q_{\delta_1}=M_{\dot{\delta}_1}=M\sin\nu\cos\delta_2;\quad Q_{\delta_2}=M_{\dot{\delta}_2}=M\cos\nu;\quad Q_{\varphi}=M_{\dot{\varphi}}=0. \quad (8.45)$$

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After using (2.111), let us write

$$M=A\beta\delta, \quad (8.46)$$

where

$$\beta=\frac{d^2l}{Ag}10^3H(y)v^2K_M\left(\frac{v}{a}\right). \quad (8.47)$$

Then

$$Q_{\delta_1}=A\mathfrak{B}\sin\nu\cos\delta_2;\quad Q_{\delta_2}=A\mathfrak{B}\cos\nu;\quad Q_{\varphi}=0.$$

Utilizing (8.43) and taking into account the smallness of angles δ_1 and δ_2, we will obtain

$$Q_{\delta_1}=A\mathfrak{B}\delta_1;\quad Q_{\delta_2}=A\mathfrak{B}\delta_2;\quad Q_{\varphi}=0. \qquad (8.48)$$

Let us pass to the compilation of the equations of rotary motion in the form of equation (1.33). For the compilation of equation for generalized coordinate δ_1, we differentiate (8.44) on $\dot{\delta}_1$ and δ_1 we find the following values of derivatives:

$$\frac{\partial T}{\partial\dot{\delta}_1}=A\dot{\delta}_1;\quad \frac{d}{dt}\left(\frac{\partial T}{\partial\dot{\delta}_1}\right)=A\ddot{\delta}_1;$$

$$\frac{\partial T}{\partial\delta_1}=C\,[\dot{\varphi}+(\dot{\theta}+\dot{\delta}_2)\delta_1](\dot{\theta}+\dot{\delta}_2). \qquad (8.49)$$

For the projectiles of constant mass, usually they accept

$$r=r_0=\text{const},$$

i.e. do not consider a change in the angular rate of rotation relatively longitudinal axis (for example, the attenuation of the rotational speed). After using (1.33) and value Q_{δ_1}, we will obtain

$$A\ddot{\delta}_1-Cr_0(\dot{\delta}_2+\dot{\theta})=A\mathfrak{B}\delta_1. \qquad (8.50)$$

For the compilation of equation for generalized coordinate δ_2, we differentiate (8.44) on $\dot{\delta}_2$ and δ_2 we find derivatives

$$\frac{\partial T}{\partial\dot{\delta}_2}=A(\dot{\theta}+\dot{\delta}_2)+Cr_0\delta_1;$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial\dot{\delta}_2}\right)=A(\ddot{\theta}+\ddot{\delta}_2)+Cr_0\dot{\delta}_1;\quad \frac{\partial T}{\partial\delta_2}=0.$$

After using (1.33) and value Q_{δ_2}, we will obtain

$$A\ddot{\delta}_2+A\ddot{\theta}+Cr_0\dot{\delta}_1=A\beta\delta_2. \qquad (8.51)$$

For simplification in the writing, usually is introduced the designation

$$\alpha=\frac{Cr}{2A}\approx\frac{Cr_0}{2A}. \qquad (8.52)$$

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Carrying out replacement in (8.50) and (8.51), let us write

$$\ddot{\delta}_1-2\alpha(\dot{\delta}_2+\dot{\theta})=\beta\delta_1;\quad \ddot{\delta}_2+\ddot{\theta}+2\alpha\dot{\delta}_1=\beta\delta_2 \qquad (8.53)$$

or, after transferring into the right side of the value obtained during the separate solution of the equations of motion of the center of mass of projectile, -

$$\ddot{\delta}_1-2\alpha\dot{\delta}_2-\beta\delta_1=2\alpha\dot{\theta};\quad \ddot{\delta}_2+2\alpha\dot{\delta}_1-\beta\delta_2=-\ddot{\theta}. \qquad (8.54)$$

In the general case under the alternating/variable α and β and initial conditions $t=0$; $\delta_1=\delta_{10}$; $\dot{\delta}_1=\dot{\delta}_{10}$; $\delta_2=\delta_{20}$; $\dot{\delta}_2=\dot{\delta}_{20}$ system (8.54) can be solved only numerically.

Let us multiply the first equation by i and let us add with the

second

$$\ddot{\delta}_2+i\ddot{\delta}_1+2a(\dot{\delta}_1-i\dot{\delta}_2)-\beta(\delta_2+i\delta_1)=2ia\dot{\theta}-\ddot{\theta}.$$

It is possible to replace $i\dot{\delta}_2-\dot{\delta}_1=i(\dot{\delta}_2+i\dot{\delta}_1)$ and then

$$\ddot{\delta}_2+i\ddot{\delta}_1-2ia(\dot{\delta}_2+i\dot{\delta}_1)-\beta(\delta_2+i\delta_1)=2ia\dot{\theta}-\ddot{\theta}.$$

After introduction to complex variable

$$z=\delta_2+i\delta_1 \qquad (8.55)$$

we will obtain one the equation, equivalent to system (8.54),

$$\ddot{z}-2ia\dot{z}-\beta z=2ia\dot{\theta}-\ddot{\theta}. \qquad (8.56)$$

last/latter equation - linear nonhomogeneous differential equation of the second order with variable coefficients and alternating/variable right side.

In conformity with the second equation of system (5.3)

$$\dot{\theta}=-\frac{g\cos\theta}{v}.$$

Taking the second derivative and converting, we will obtain

$$\ddot{\theta}=\frac{\dot{\theta}}{v}(g\sin\theta-\dot{v}). \qquad (8.57)$$

To solve the equations, similar (8.56), is possible only by numerical methods. initial conditions will be written as follows:

$$t=0;\quad z=z_0=\delta_{20}+i\delta_{10};\quad \dot{z}=\dot{z}_0=\dot{\delta}_{20}+i\dot{\delta}_{10}.$$

Introducing the supplementary simplifications, in particular, accepting β=const, it is possible equation (8.56) to solve analytically. The complete integral of differential equation (8.56) represents by itself the sum of the general solution of homogeneous equation and the particular integral of nonhomogeneous equation. The homogeneous equation

$$\ddot{z}-2i\alpha\dot{z}-\beta z=0 \qquad (8.58)$$

by the substitution

$$z=ue^{i\alpha t} \qquad (8.59)$$

is led to a simpler form. From (8.59) it follows

$$\dot{z}=\dot{u}e^{i\alpha t}+i\alpha ue^{i\alpha t}; \quad \ddot{z}=\ddot{u}e^{i\alpha t}+2i\alpha\dot{u}e^{i\alpha t}-\alpha^2ue^{i\alpha t}.$$

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Substituting z, ż and z̈ in (8.58) and by reducing on $e^{i\alpha t}$, we will obtain

$$\ddot{u}-(\alpha^2-\beta)u=0.$$

Designating

$$\gamma=1-\frac{\beta}{\alpha^2}, \qquad (8.60)$$

let us lead last/latter equation to the form

$$\ddot{u} + \alpha^2 \sigma u = 0. \qquad (8.61)$$

With variables α and σ, equation (8.61) analytically is not solved. On the assumption that α=const and σ=const, the integral of equation (8.61) takes the form

$$u = C_1 e^{i\alpha\sqrt{\sigma}t} + C_2 e^{-i\alpha\sqrt{\sigma}t}. \qquad (8.62)$$

Substituting u in (8.59), we will obtain

$$z = C_1 e^{i\alpha(1+\sqrt{\sigma})t} + C_2 e^{i\alpha(1-\sqrt{\sigma})t}. \qquad (8.63)$$

Integration constant in general form are represented by the complex numbers

$$C_1 = \rho_1 e^{i\epsilon_1}; \quad C_2 = \rho_2 e^{i\epsilon_2},$$

after which

$$z = \rho_1 e^{i[\alpha(1+\sqrt{\sigma})t+\epsilon_1]} + \rho_2 e^{i[\alpha(1-\sqrt{\sigma})t+\epsilon_2]}. \qquad (8.64)$$

Designating

$$\omega_1 = \alpha(1+\sqrt{\sigma}); \quad \omega_2 = \alpha(1-\sqrt{\sigma}) \qquad (8.65)$$

and remembering that $e^{i\psi}=\cos\psi+i\sin\psi$, let us write instead of (8.64)

$$z=\varrho_1\cos(\omega_1 t+\varepsilon_1)+\varrho_2\cos(\omega_2 t+\varepsilon_2)+$$
$$+i\left[\varrho_1\sin(\omega_1 t+\varepsilon_1)+\varrho_2\sin(\omega_2 t+\varepsilon_2)\right]. \tag{8.66}$$

Comparing (8.55) and (8.66), we will obtain expressions for angles δ_1 and δ_2

$$\delta_1=\varrho_1\sin(\omega_1 t+\varepsilon_1)+\varrho_2\sin(\omega_2 t+\varepsilon_2); \tag{8.67}$$
$$\delta_2=\varrho_1\cos(\omega_1 t+\varepsilon_1)+\varrho_2\cos(\omega_2 t+\varepsilon_2). \tag{8.68}$$

Fig. 8.14 depicts the motion of the longitudinal axis of projectile in coordinates δ_1 and δ_2. Point M represents by itself the projection of the point of intersection of the longitudinal axis of projectile with the sphere of the unit radius, carried out from the center of mass of projectile, to coordinate plane $\delta_1 O \delta_2$. The velocity vector of the center of mass of projectile for the solution of homogeneous equation (8.58) in question is projected into the point, which corresponds to the origin of rectilinear coordinates δ_1 and δ_2.

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The motion of point M on coordinate image plane can be represented as vector sum of two motions: the rotation of point M along circle with a radius ρ_1 of the relatively instantaneous center of rotation and movable circular motion of instantaneous center of rotation along circumference with a radius ρ_2. The angular velocities of rotary motion ω_1 and ω_2 are determined by expressions (8.56). It is obvious

that $\omega_1 > \omega_2$.

Tc the projection cf radius-vectcrs cr cccrdinate axes δ_1 and δ_2 respectively equal to

$$\delta_1 = \delta_{12} + \delta_{11}; \quad \delta_2 = \delta_{22} + \delta_{21},$$

where

$$\delta_{11} = \varrho_1 \sin(\omega_1 t + \varepsilon_1); \quad \delta_{12} = \varrho_2 \sin(\omega_2 t + \varepsilon_2); \qquad (8.69)$$
$$\delta_{21} = \varrho_1 \cos(\omega_1 t + \varepsilon_1); \quad \delta_{22} = \varrho_2 \cos(\omega_2 t + \varepsilon_2).$$

the trajectory of point M in tctal motion cn coordinate plane $\delta_1 0 \delta_2$ will be the epicyclcid, constructed cr circle with radius cf $\rho_2 - \rho_1$. The value of radius-vectors ρ_1 and ρ_2, their relaticnship/ratio and the constants ε_1 and ε_2, on which depend the initial positions cf radius-vectcrs, are determined by the initial ccrditicns.

If we consider damping oscillaticns, then cf curve/graphs, that characterizes a change cf the rutaticn angle δ in the function of time, will take the form, shown on Fig. 8.15.

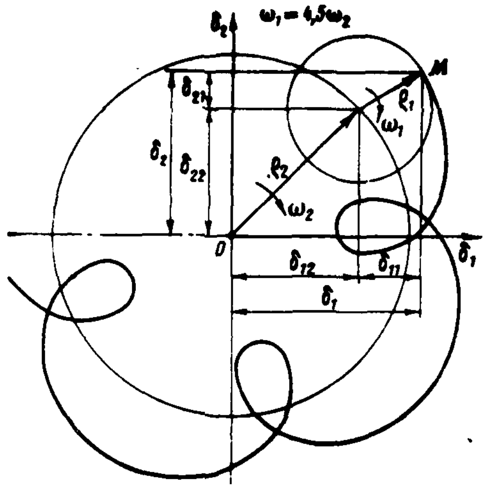

Fig. 8.14. Graph/diagram of dependence $\delta_2=f(\delta_1)$ for the straight portion of the trajectory of the fast-turning projectile with $\delta_0\neq0$ and $\dot{\delta}_0\neq0$.

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Let us find now the particular integral of nonhomogeneous equation (8.56). Solution they usually search for in the form of the series:

$$z_p=z_0+\frac{1}{a}z_1+\frac{1}{a^2}z_2+\cdots \qquad (8.70)$$

Investigations show that a - the high parameter and the terms of a series rapidly they decrease. Functions from time z_0, z_1, z_2 are subject to determination. Let us differentiate $\dot{z}_p$ and $\ddot{z}_p$ and

together with z let us substitute in (8.56)

$$\ddot{z}_0-2l\alpha\dot{z}_0-\alpha^2\left(\frac{\beta}{\alpha^2}\right)z_0+\frac{1}{\alpha}\ddot{z}_1-2l\dot{z}_1-\alpha\left(\frac{\beta}{\alpha^2}\right)z_1+$$
$$+\frac{1}{\alpha^2}\ddot{z}_2-2l\frac{1}{\alpha}\dot{z}_2-\left(\frac{\beta}{\alpha^2}\right)z_2+\ldots=2l\alpha\dot{\theta}-\ddot{\theta}.$$

In the obtained equation usually the terms $\frac{1}{\alpha}\ddot{z}_1$, $\frac{1}{\alpha^2}\ddot{z}_2$ and $2l\frac{1}{\alpha}\dot{z}_2$ are the members of the second order of smallness and by them it is possible to disregard.

Let us equate the coefficients when α with identical degree in the right and left sides of the last/latter equation.

Coefficients of α^2:

$$\left(\frac{\beta}{\alpha^2}\right)z_0=0.$$

Coefficients when α

$$-2l\dot{z}_0-\left(\frac{\beta}{\alpha^2}\right)z_1=2l\dot{\theta}.$$

Coefficients of α^0

$$\ddot{z}_0-2l\dot{z}_1-\left(\frac{\beta}{\alpha^2}\right)z_2=-\ddot{\theta}.$$

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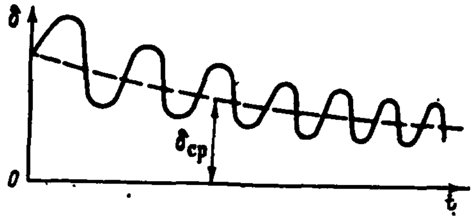

Fig. 8.15. Decrease of nutation angle δ from damping of medium depending on time.

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Of three last/latter equations we find

$$z_0=0;\quad z_1=-\frac{2i\alpha^2}{\beta}\dot{\theta};\quad z_2=\frac{\alpha^2}{\beta}\ddot{\theta}-2i\frac{\alpha^2}{\beta}\dot{z}_1. \qquad (8.71)$$

Let us differentiate on t of the second equation of system (8.71) and let us substitute $\dot{z}_1$ into the third equation. After conversion let us have

$$z_2=\frac{\alpha^2}{\beta}\ddot{\theta}-4\frac{\alpha^4}{\beta^2}\dot{\theta}\left(\frac{\ddot{\theta}}{\dot{\theta}}-\frac{\dot{\beta}}{\beta}\right).$$

Substituting in (8.70) z_0, z_1 and z_2, we will obtain

$$z_p=-2i\frac{\alpha}{\beta}\dot{\theta}+\frac{\ddot{\theta}}{\beta}-4\frac{\alpha^2}{\beta^2}\left(\frac{\ddot{\theta}}{\dot{\theta}}-\frac{\dot{\beta}}{\beta}\right). \qquad (8.72)$$

Taking into account the general view of complex variable (8.55), let us write

$$z_p=\delta_{2p}+i\delta_{1p}. \qquad (8.73)$$

Comparing (8.72) and (8.73), we will obtain

$$\delta_{1p} = -2\frac{\alpha}{\beta}\dot{\theta}. \tag{8.74}$$

$$\delta_{2p} = \frac{\ddot{\theta}}{\beta} - 4\frac{\alpha^2}{\beta^2}\left(\frac{\ddot{\theta}}{\dot{\theta}} - \frac{\dot{\beta}}{\beta}\right). \tag{8.75}$$

Calculations show that angle δ_{1p} is considerably greater than angle δ_{2p} and it is chief constituent of angle δ_p. The solution of the problem of the motion of the rotating projectile on the curvilinear trajectory phase shows that in this case the epicycloid is oriented relative to the dynamic axis of equilibrium (relative to point D in Fig. 8.16). Point D represents by itself projection on image plane of the point of intersection of the dynamic axis of equilibrium with the sphere of a unit radius. For each moment of time, the position of point D on coordinate plane is determined by values δ_{1p} and δ_{2p}, which are located from formulas (8.66) and (8.67). Angles δ_{1p} and δ_{2p} are counted off from the origin of coordinates O, which represents by themselves projection on image plane of point of intersection with tangent to trajectory with the sphere of a unit radius. With gradual to trajectory with the sphere of unit radius. During the gradual deviation of the dynamic axis of equilibrium from velocity vector (point D from point O) the curve, described by a radius ρ_2, obtains the form of spiral (unlike circumference in Fig. 8.14).

Thus, are obtained expressions for the angles, which characterize the position of the axis of dynamic equilibrium. Since $\delta_{1p} \gg \delta_{2p}$, then in the first approximation, it is possible to count that

$$\delta_p \approx \delta_{1p} = 2\frac{\alpha}{\beta}|\dot{\theta}|. \qquad (8.76)$$

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Substituting in (8.76) the second equation of system (5.3) and converting it, we will obtain

$$\delta_p \approx k\frac{\cos\theta}{H(y)v^3K_M\left(\frac{v}{a}\right)}. \qquad (8.77)$$

Angle δ_p determines the position of the dynamic axis of equilibrium, relative to which occurs the periodic oscillatory motion of the longitudinal axis of the rotating projectile. If the trajectory of the motion of projectile approaches straight line and $|\dot{\theta}|\approx 0$, then on (8.76) $\delta_p = 0$ and the oscillations of the longitudinal axis of projectile will be completed relative to the velocity vector of the motion of the center of mass.

For the straight portion of trajectory in the case θ=const and

$\dot{\theta}=0$ to conveniently use angles φ, ν and δ (Fig. 8.17). Then the angular rate of rotation of projectile is equal to

$$\bar{\Omega}=\dot{\bar{\varphi}}+\dot{\bar{\nu}}+\dot{\bar{\delta}}.$$

Let us first comprise the system of differential equations for the more general case - for the rotary motion of the spin-stabilized missile. Let us consider the action of basic torque/moments - turning moment $M_{вр}$ and the aerodynamic tilting moment - M. The rotations of TRS of relatively longitudinal axis it is provided by the special construction of the nozzle unit in which the nozzles are fulfilled with slope/inclination toward by generatrix missile bodies at angle γ (Fig. 8.18). The plane of the action of turning moment is perpendicular to the longitudinal axis of projectile. The vector of torque/moment $M_{вр}$ is directed along the axis Cx_1 (Oζ) (8.17). The tilting moment acts in the plane of resistance, and its vector is directed along the axis Cz_1, perpendicular to the plane of the resistance (see Fig. 8.17).

In the case of expression in question for generalized forces on the appropriate angles of rotation, they will take the form

$$\left.\begin{aligned} Q_\varphi&=M\cos(z_1x_1)+M_{вр}=M_{вр};\\ Q_\nu&=M\cos(z_1v)+M_{вр}\cos(x_1v)=M_{вр}\cos\delta;\\ Q_\delta&=M+M_{вр}\cos(x_1z)=M.\end{aligned}\right\} \quad (8.78)$$

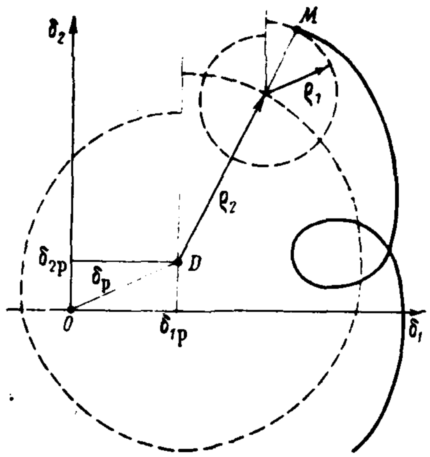

Fig. 8.16. Graph/diagram of dependence $\delta_2=f(\delta_1)$ for the curvilinear phase of trajectory of the center of mass of the fast-turning projectile with $\delta_0\neq0$ and $\dot{\delta}_0\neq0$.

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Let us comprise the differential equations of Lagrange for generalized coordinates ϕ, ν and δ, substituting alternately in (1.33) derivatives $\frac{\partial T}{\partial\dot{\varphi}}$ and $\frac{\partial T}{\partial\varphi}$; $\frac{\partial T}{\partial\dot{\nu}}$ and $\frac{\partial T}{\partial\nu}$; $\frac{\partial T}{\partial\dot{\delta}}$ and $\frac{\partial T}{\partial\delta}$. After substitution we will obtain:

for a generalized coordinate ϕ

$$C\frac{d}{dt}(\dot{\varphi}+\dot{\nu}_{\iota}\cos\delta)=M_{\text{ap}};\qquad(8.79)$$

for generalized coordinate ν

$$\frac{d}{dt}[A\dot{\nu}\sin^2\delta+C(\dot{\varphi}+\dot{\nu}\cos\delta)\cos\delta]=M_{\text{вр}}\cos\delta; \qquad (8.80)$$

for a generalized coordinate δ

$$A\ddot{\delta}-A\dot{\nu}^2\sin\delta\cos\delta+C(\dot{\varphi}+\dot{\nu}\cos\delta)\dot{\nu}\sin\delta=M. \qquad (8.81)$$

The angular rate of rotation of TBS of relatively longitudinal axis in powered flight trajectory approximately will be determined from (8.79). Let us write

$$\dot{\varphi}+\dot{\nu}\cos\delta=r=\int_{t_0}^{t}\frac{M_{\text{вр}}}{C}dt.$$

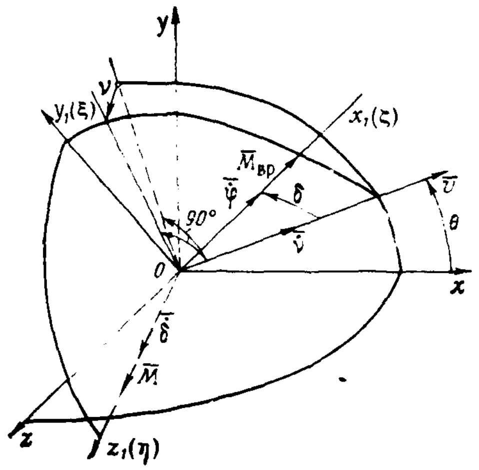

Fig. 8.17.

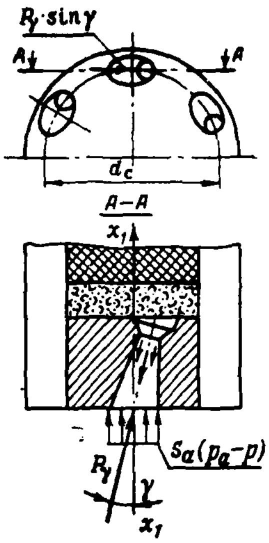

Fig. 8.18.

Fig. 8.17. Projections of the vector of instantaneous angular velocity and torque/moments, which act on TES, on rotational axis.

Fig. 8.18. Circuit of action of forces, which rotate spin-stabilized missile.

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Since, as show calculations, $\dot{\nu} \ll \dot{\varphi}$, term $\dot{\nu}\cos\delta$ in the first approximation, can be disregarded.

Taking M_{sp} and C constant average values, let us determine the angular rate of rotation relative to the longitudinal axis:

$$\dot{\varphi} \approx r \approx \frac{M_{sp}}{C}\left(\frac{v}{a} - \frac{v_0}{a_0}\right),$$

where a_0 and a - constant longitudinal accelerations of the center of mass of TRS respectively on guides and on powered flight trajectory. If we count $a_0=0$ and $v_0=0$, then, by designating $k=\frac{M_{sp}}{Ca}$, we will obtain the approximate well known equality

$$r \approx kv, \qquad (8.82)$$

where the velocity of the center of mass of projectile v is determined during the solution of the basic problem of external ballistics for the projectile of point variable mass.

Equations (8.80) and (8.81) without supplementary simplifications are solved only by numerical methods. Their analytical solutions, instituted with a series of assumptions, examined in work [60].

The easily foreseeable analytical solution can be obtained for the artillery shell of constant mass, after assuming in (8.78) $\dot{M}_{sp}=0$. In this case system of three equations (8.79), (8.80) and (8.81) can be rewritten thus:

for a generalized coordinate φ

$$C\frac{d}{dt}(\dot{\varphi}+\dot{\nu}\cos\delta)=0; \qquad (8.83)$$

for generalized coordinate ν

$$\frac{d}{dt}[A\dot{\nu}\sin^2\delta+C(\dot{\varphi}+\dot{\nu}\cos\delta)\cos\delta]=0; \qquad (8.84)$$

for a generalized coordinate δ

$$A\ddot{\delta}-A\dot{\nu}^2\sin\delta\cos\delta+C(\dot{\varphi}+\dot{\nu}\cos\delta)\dot{\nu}\sin\delta=M. \qquad (8.85)$$

System of equations (8.83-8.85) most frequently is solved under the most real initial conditions, which correspond to the torque/moment of the loss of tight coupling of projectile with the shaft of the artillery instrument: $t=0$; $\delta_0=0$; $\dot{\delta}=\dot{\delta}_0$ and $\nu=\nu_0$. is assumed that at zero time there is a precession angle ν_0 and the angular velocity of nutation $\dot{\delta}_0$, but nutation angle is equal to zero. Under the initial conditions accepted from (8.83) we will obtain

$$\dot{\varphi}+\dot{\nu}\cos\delta=r_0=\text{const}.$$

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From equation (8.44) let us have

$$A\dot{\nu}\sin^2\delta+Cr_0\cos\delta=D.$$

Under the initial conditions

$$D=Cr_0,$$

$$(1-\cos^2\delta)\dot{\nu}=\frac{Cr_0}{A}(1-\cos\delta),$$

accepted whence

$$\dot{\nu}=\frac{Cr_0}{A(1+\cos\delta)}=\alpha. \quad (8.86)$$

For stable projectiles with the smallness of angle δ, it is possible to accept

$$\sin\delta\approx\delta \text{ and } \cos\delta=1,$$

then $\alpha=Cr_0/2A$ and - after integration (8.86) - we will obtain formula for determining the precession angle

$$\nu=\nu_0+\alpha t. \quad (8.87)$$

Using already known to us substitution $M = A\beta\delta$, we will obtain from (8.71) with the smallness of angle δ

$$\ddot{\delta}+\alpha^2\delta-\beta\delta=0. \quad (8.88)$$

It is accepted to designate:

$$\sigma=1-\frac{\beta}{\alpha^2} \quad (8.89)$$

and then

$$\ddot{\delta}+\alpha^2\sigma\delta=0. \quad (8.90)$$

As it fcllows from (8.89) and expressicns for α and β, coefficient σ - alternating/variable. More detailed investigations show that for small time intervals σ it is possible to accept as the fixed value, after assuming

$$v=\mathrm{const},\quad \beta=\mathrm{const},\quad \alpha=\mathrm{const}.$$

The characteristic equation, which corresponds (8.90), takes the fcllcwing form:

$$k^2+\alpha^2\sigma=0.$$

With $\sigma>0$ we will obtain the kncwn soluticn

$$\delta=C_1e^{i\alpha\sqrt{\sigma}t}+C_2e^{-i\alpha\sqrt{\sigma}t}.$$

After determining arbitrary ccnstants, we will obtain formula fcr a nutation angle

$$\delta=\frac{\dot{\delta}_0}{\alpha\sqrt{\sigma}}\sin\alpha\sqrt{\sigma}t. \qquad (8.91)$$

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Thus, with $\sigma>0$ nutaticn angle is expressed as trigonometric sine and the moticn of the lcngitudinal axis of projectile bears the character of harmonic oscillations with pericd cf $T = 2\pi/\alpha\sqrt{\sigma}$ and with

limited amplitude $\delta_{max}=\frac{\dot{\delta}_0}{\alpha+\bar{\alpha}}$. Precessional motion in accordance with (8.87) represents by itself the rotation of the plane of resistance around tangent to trajectory with an almost constant angular velocity of $\dot{\nu}\approx\alpha\approx\frac{C r_0}{2A}$. A change in the nutation angle is determined by equation (8.91) and occurs in the plane of resistance. On the trajectory phases, close to rectilinear, the complex spatial motion of the longitudinal axis of the rotating projectile is accomplished relative to vector v and for initial conditions $\nu_0=0$, $\delta_0=0$, $\dot{\delta}_0\neq0$ can be visually illustrated by the curve/graph, constructed in polar coordinates $\delta=f(\nu)$ (Fig. 8.19). Angle δ is depicted as the radius-vector whose position is determined by angle ν. If we examine the motion of the axis of projectile depending on time, then plotted functions $\delta(t)$ and $\nu(t)$ will take the form, presented in Fig. 8.20a. During processing of experimental data to conveniently examine only the absolute value of nutation angle δ; therefore curve/graphs $\delta(t)$ they frequently construct only in positive half-plane; in this case curve/graph $\nu(t)$ will obtain stepped form, since with transition δ through zero angle ν undergoes the explosion, equal to $\pm\pi$. Constructed similarly curve/graphs $\delta(t)$ and $\nu(t)$ with the explosion, undertaken equal to $-\pi$, are represented in Fig. 8.20b. During a change in the initial conditions of the graph $\delta=f(\nu)$, $\delta(t)$ and $\nu(t)$ they will obtain the form, different from that presented in Fig. 8.19 and 8.20.

The integration of equation (8.89) with $\alpha>0$ led us to formula (8.90), determining the harmonic oscillations of the longitudinal axis of projectile with the limited amplitude. Integration (8.89) with $\alpha<0$ will lead us to the dependence

$$\delta=-C\left(e^{a\sqrt{|\alpha|}t}+e^{-a\sqrt{|\alpha|}t}\right).$$

After the determination of integration constant, we will obtain

$$\delta=\frac{\dot{\delta}_0}{a\sqrt{|\alpha|}}\operatorname{sh}\left(a\sqrt{|\alpha|}t\right).$$

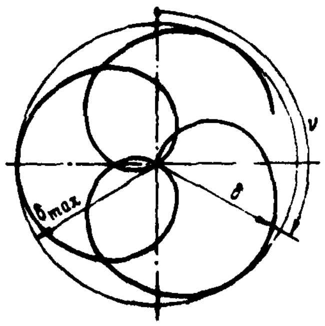

Fig. 8.19. Curve/graph $\delta=f(\nu)$ for the straight portion of the trajectory of the fast-turning projectile with $\delta_0=0$, $\dot{\delta}_0\neq0$.

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Thus, with $\sigma<0$ nutation angle is determined by the function, which unlimitedly increases with an increase in the time, what is the sign/criterion of the instability of projectile. Consequently, the condition of the correct action of the rotating projectiles can be expressed by the inequality

$$\sigma=1-\frac{\beta}{\alpha^2}>0, \tag{8.92}$$

in which coefficient σ it is called the criterion of the gyroscopic stability of projectile on the initial straight portion of the trajectory. Stability condition, on the basis of $\sigma>0$, can be written thus:

$$\frac{\beta}{\alpha^2}<1. \tag{8.93}$$

The values on which depends β, are determined in many respects by the size/dimensions of projectile, by its designation/purpose and the action characteristics the centers of mass (8.44):

$$\beta = f\left[d^{*}, v^{*}, A, l, H(y), K_{M}\left(\frac{v}{a}\right)\right].$$

Therefore to vary by value β for providing inequality (8.93) is virtually impossible. It is considerably simpler, with given one β, it is possible to accomplish of the condition of the gyroscopic stability of projectile by the appropriate selection of the parameter a or, in accordance with formula (8.86), to angular rate of rotation r_0.

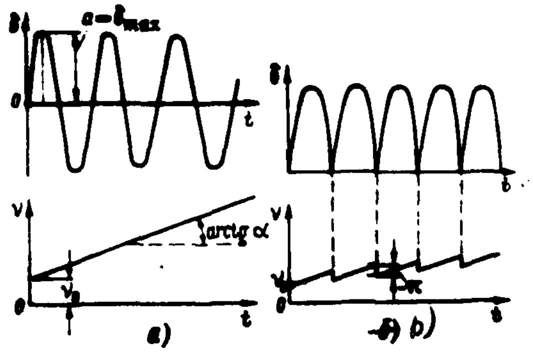

Fig. 8.20. Curve/graphs of a change of the angles δ(t) and ν(t): a) in the plane of resistance; b) in positive half-plane.

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During motion along threaded/cut shaft, the angular velocity of projectile at the moment of flight is equal to

$$r_0=\dot{\varphi}_0=\frac{2v_0\pi}{d\eta}, \qquad (8.94)$$

where η - length of the course of threads in bores.

The axial moment of the inertia of projectile can be expressed as follows:

$$C=\mu\frac{Q}{g}\frac{d^2}{4}, \qquad (8.95)$$

where μ - the coefficient, which characterizes the distribution of

the mass of projectile of its relatively longitudinal axis. The weight of projectile can be expressed in terms of the coefficient of weight C_Q, which has the close of value for the one-type projectiles

$$Q=C_Q d^3,$$

where d - a bore in dm, Q - weight in kg.

Carrying out replacement a and β, we will obtain from expression (8.93)

$$\eta<\frac{\pi}{2}\sqrt{\frac{\mu C_Q}{\frac{l}{d}\frac{A}{C}K_M\left(\frac{v}{a}\right)}}. \quad (8.96)$$

For obtaining the unique solution inequality (8.96) they replace by equality, introducing into it the margin of safety of gyroscopic stability - a.

In this case, it obtains the form

$$\eta=a\frac{\pi}{2}\sqrt{\frac{\mu C_Q}{\frac{l}{d}\frac{A}{C}K_M\left(\frac{v}{a}\right)}}. \quad (8.97)$$

Equality (8.97) was called the formula of Zabudskiy-Venttsel'. Value a<1 concrete/specific/actually is set during design depending on the construction of projectile. In the general case for estimate calculations, it is possible to take a = 0.75 [59]. In practice value η oscillates within limits ~(20-30).

We will now obtain dependence for determining the necessary value of the nozzle cant angle of the nozzle unit of TRS, ensuring

stable motion projectile (see Fig. 8.16). The value of angle γ can be designed in terms of assigned value $\dot{\varphi}_к = \dot{\varphi}_к = r_к$ as follows.

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The equation of the rotary motion of TRS relative to axis Ox_1 takes the form

$$J_{x_1}\frac{d\dot{\varphi}}{dt} = P_{\gamma}n\frac{d_c}{2}\sin\gamma, \qquad (8.98)$$

where $P_{\gamma}n = |\dot{m}|w_{отн}$ - the reaction force, developed with engine; n - number of nozzles; $\frac{d_c}{2}$ - arm of force P_{γ} relative to the axis of TRS.

In this case, composing thrusts $S_a(p_a - p)$ it is not considered, since it is directed in the first approximation, along longitudinal axis of TRS and in the formation/education of the torsional moment participation does not accept.

As a result of integrating equation (8.98) under the assumption of constancy $J_{x_1} = J_{x_1\,ср}$ we will obtain for the arbitrary point of powered flight trajectory

$$r = \dot{\varphi} \approx \dot{\varphi}_0 + \frac{w_{отн} d_c |\dot{m}| (t - t_0)}{2J_{x_1}}\sin\gamma. \qquad (8.99)$$

Hence follows relationship/ratio for the calculation of the

unknown angle of the slope of the axis of nozzle γ

$$\gamma=\arcsin\frac{2J_{x_1}(\dot{\varphi}_k-\dot{\varphi}_0)}{w_{\text{отн}}d_c\,|\dot{m}|(t_k-t_0)}. \qquad (8.100)$$

By index "k" are noted the values, which correspond to the end/lead of powered flight trajectory.

Utilizing the found in a similar approximate manner value of angle γ, one should produce the refined verifying calculation of the angular rate of rotation of TRS $\dot{\varphi}$ by the integration of equation (8.98) in parts of active section. The length of each part must be such that within its limits the averaging of the kinematic parameters of the trajectory of TRS, values of its torque/moments of the inertia and other parameters would not introduce large errors into calculation. If necessary according to the results of verifying calculation, it is possible to find correction in the value of nozzle cant angle γ. The given above condition of the gyroscopic stability of TRS (8.92) is obtained for the beginning of the passive (or, that equivalently, for the end/lead of the active) trajectory phase.

Let us examine the special feature/peculiarities of the motion of TRS on powered flight trajectory. Within the limits of this section, continuously grow/rises the velocity v of the center of mass of TRS and in essence on it the coefficient β depending, which characterizes the tilting moment M. But also continuously in

accordance with (8.82) grow/rises the angular rate of rotation of TRS $r = \dot{\phi}$ and the determined by it angular velocity of precession $\dot{\nu}$. As a result of this, it proves to be that the criterion of stability of TRS $\sigma = 1-\beta/\alpha^2$ on active section is changed insignificantly, having weakly grow/risen toward the end of the section. In this case, occurs the decrease of period and amplitude of nutational oscillations of TRS, caused by the initial disturbances. Thus, with sufficient accuracy it is possible to count that if is provided the stability of TRS at the end of powered flight trajectory, then it will be stable also on all section.

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The numerical value of stability criterion and other motion characteristics of TRS on the active part of the trajectory can be approximately designed on the individual sections of trajectory with the use of formulas (8.44), (8.86), (8.89), (8.87) and (8.97) [17].

The stability condition of rotating projectile (8.93) is obtained for motion on the straight portion of trajectory. With an increase in the path curvature, the dynamic axis of equilibrium will all more move away from velocity vector by angle δ_p. Angle δ_p is defined by formula (8.77).

The analysis of the values, entering formula (8.77), shows that angle δ_p along the curvilinear trajectory phase changes, moreover in area of peak of the trajectory, where value cos θ is close to greatest, and $H(y)v^3$ - to the smallest, it reaches the greatest value. During the incorrect selection of the rate of spin of projectile, angle δ_p in peak of the trajectory can become so large that it will lead to the loss by the projectile of stability. Consequently, angle δ_p must be considered as independent criterion, evaluating the stability of the rotating projectiles on the curvilinear section of the trajectory. The comparison of dependences (8.77) and (8.92) for criteria of stability σ and δ_p shows the discrepancy of the latter.

It is real/actual, for the increase of the stability of the rotating projectiles in the initial rectilinear cut of trajectory the speed of its rotation $r=\dot{\varphi}$ must be increased. This will cause appropriate increase $\dot{\nu}$ and angle δ_p and, therefore, it will lead to the decrease of the stability of projectile in peak of the trajectory.

To combine both stability conditions for the projectiles whose the length of 5-6 bores, accomplishes at angles of departure to 60-65°.

In this case for the stabilization of projectile along an entire trajectory it suffices to select the speed of its rotation in the beginning of passive section, in order to obtain criterion of stability σ equal to $\sqrt{\bar{\sigma}_n}=0{,}60-0{,}66$.

Let us note one additional special feature/peculiarity of the trajectories of the projectiles, which are stabilized by rotation. The existence of angle δ_p leads to the fact that the longitudinal axis of projectile the large part of the flight time is located through one of the sides of vector $\bar{v}$ and projectile is deflect/diverted sideways from plane of reference of firing. The systematic lateral deviation of the fast-turning projectiles is called derivation. During right spin of projectile, the dynamic axis of equilibrium is deflect/diverted to the right from vector (if we look in the direction of motion) and this leads to right derivation. During counterclockwise rotation projectile it will depart to the left from range plane, i.e., derivation will be left. The derivation of projectile can be determined and experimentally and by calculation.

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From theoretical formulas (in connection with the projectiles of constant mass) are known the formulas of A. N. Krylov, V. I.

Ostapovich, and P. P. Eersnev, formula cf D. A. Venttsel' and the calculation method, proposed by V. N. Pershin. One Of the most widely known formulas is A. N. Krylov's formula

$$z=f\frac{g}{Q}\frac{2\pi C}{hR\gamma}v_0\int_0^t(\varphi-\varphi_0)dt, \qquad (8.101)$$

where (besides well-known values) R - semi-caliber of projectile; φ - angle between the tangent to trajectory and the vertical line.

Formula contains the values, determined from the experiment: the unknown it is accurate value h, proportional to distance between centers of mass and the resultant pressure of projectile, and the coefficient of agreement with experiment f. For one-type projectiles and trajectories, the formula gives satisfactory results. More common/general/total is the formula of V. N. Pershina, who considers the phenomenon of the attenuation of the rate of spin of projectile for flight time.

Let us examine the calculated determination of derivation. The equation of motion of the center of mass of projectile along the axis z, directed perpendicular to range plane, will take the form

$$\frac{Q}{g}\ddot{z}=R_z-X\cos(\widehat{v,z}), \qquad (8.102)$$

where R_z - lateral force, caused by angle δ; $X\cos(\widehat{v,z})$ - the projection of drag on z axis.

It is possible to replace cos $(\widehat{v, z}) = \dot{z}/v$ and then

$$\ddot{z} = \frac{g}{Q}R_z - \frac{g}{Q}X\frac{\dot{z}}{v}. \qquad (8.103)$$

Let us replace also $(g/Q)X = cH(y)F(v)$ and, remembering that $G(v) = F(v)/v$, we will obtain

$$\ddot{z} = \frac{g}{Q}R_z - cH(y)G(v)\dot{z}. \qquad (8.104)$$

Taking into account the relatively small path curvature, determined by derivation, it is possible to use the dependence, valid for a planar trajectory,

$$-cH(y)G(v) = \frac{\dot{u}}{u}.$$

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Then from (8.104) it is possible to write

$$\ddot{z} - \frac{\dot{u}\dot{z}}{u} = \frac{g}{Q}R_z \qquad (8.105)$$

or

$$u\frac{d}{dt}\left(\frac{\dot{z}}{u}\right) = \frac{g}{Q}R_z. \qquad (8.106)$$

Integrating, we will obtain

$$z = \frac{g}{Q}\int_0^t u\,dt\int_0^t \frac{R_z}{u}\,dt. \qquad (8.107)$$

Let us discover content R_z. It is usually assumed that the force, which deflects the rotating projectile from the plane of casting, is proportional to angle δ_p; then in accordance with the overall theory of the aerodynamic similarity

$$R_z = \frac{dl}{g} 10^3 H(y) v^2 K_N \delta_p. \qquad (8.108)$$

Let us introduce in (8.108) value δ_p, using (8.76), and let us discover values α and β. Angular rate of rotation and the axial moment of the inertia of artillery shell let us define according to (8.94) and (8.95); value $|\dot{\theta}|$ we define as usually. Furthermore, let us remember that cos θ=u/v. Then, without taking into account the attenuation of rotation under the action of the torque/moment of surface friction, we obtain for the calculation of derivation the following expression:

$$z = \frac{\pi g}{2} \frac{\mu l}{\eta h} v_0 \int_0^t u\,dt \int_0^t \frac{K_N}{K_M} \frac{dt}{v^2}. \qquad (8.109)$$

Having experimental aerodynamic characteristics $K_N\left(\frac{v}{a}\right)$ and $K_M\left(\frac{v}{a}\right)$ and obtained by calculation for the planar trajectory of value $v=f_1(t)$ and $u=f_2(t)$, it is possible to numerically calculate the integral of right side (8.109) and to calculate derivation.

Let us consider the attenuation of rotation. Decrease of the angular rate of rotation of projectile relative to longitudinal axis in the process free flight is explained on the action of the torque/moment of surfice friction (2.113).

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The most widely known formulas, which determine the angular velocity of the projectile in the process of motion, they are: formula Roggla

$$\omega_t = \omega_0 e^{-0,075 \frac{\nu d^4}{C} t}, \qquad (8.110)$$

where ω_0 - initial angular velocity of projectile; e - Naperian base; ν - length of projectile in bores; d - bore; C - axial moment of the inertia of projectile; t - missile flight time, and I. A. Slezkin's theoretical formula

$$\omega = \omega_0 e^{-0.598 \frac{d l^{4/5}}{2Q} \int v^{4/5} dt}, \qquad (8.111)$$

where, besides designations indicated above, l - length of projectile and v - velocity of the center of mass.

Both formulas can be given to the form:

$$\omega = \omega_0 e^{-bt}, \qquad (8.112)$$

where k - the calculated cr experimental ccefficient, which corresponds to the selected corcrete/specific/actual projectile. Since the initial angular rate cf rctation $\omega_0 = r_0$ by us is already taken into account in (8.109), the attenuation cf rotation let us consider by introduction into the right side of factor e^{-kt}. Then

$$z = \frac{\pi g}{2} \frac{\mu l}{\eta h} v_0 \int_0^t u\,dt \int_0^t \frac{K_N}{K_M} \frac{e^{-kt}}{v^2}\,dt. \qquad (8.113)$$

The obtained fcrmula can be given to a simpler form. Taking into acccunt attenuation for lateral force, it is pcssible to accept the dependence

$$R_z = \frac{2\pi C}{\eta d} v_0 \frac{l}{hd} \frac{K_N}{K_M} |\dot{\theta}| e^{-kt}. \qquad (8.114)$$

Ccmpiling an equaticn cf yawing moticn alcng type (8.106) and designating constants thrcugh B, we will obtain

$$u \frac{d}{dt}\left(\frac{\dot{z}}{u}\right) = B|\dot{\theta}| e^{-kt}. \qquad (8.115)$$

Following V. N. Pershin, after a series cf simplifications in the process of double integration, it is pcssible to obtain

$$z = m_z \frac{h_2}{h_1} v_0 \int_0^t |\theta - \theta_0|\,dt. \qquad (8.116)$$

The variable coefficients, ccnfrcnting the integral in (8.116), have names, determined by their physical ccntent.

The coefficient of the derivation

$$m_z = \frac{g}{Q}\frac{2\pi C}{\eta d}\frac{l}{hd}\left(\frac{K_N}{K_M}\right)_{cp}, \qquad (8.117)$$

where $\left(\frac{K_N}{K_M}\right)_{cp}$ - average value of relation for the interval of integration.

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Curvature of the trajectory

$$k_1 = \frac{1}{2}\left(1 - \frac{\mu_0}{\mu_0 - \mu}\ln\frac{\mu}{\mu_0}\right). \qquad (8.118)$$

Moderating ratio of the rotation

$$k_2 = \left(\frac{2e^{-kt}}{k^2t^2} - \frac{2}{k^2t^2} + \frac{2}{kt}\right). \qquad (8.119)$$

Calculation is simplified during the application/use of special tables for an integral and coefficients k_1 and k_2.

The calculation of the derivation of the spin-stabilized missiles on the passive section of the trajectory can be carried out recently by the described method. After initial velocity v_0, the angle of departure θ_0 and initial angular rate of rotation $r_0=\omega_0$ should take the values of these quantities at the end of powered flight trajectory. For short powered flight trajectories, the derivation is insignificant and can be disregarded. If necessary for

calculation into the right side of equation (8.103) one should introduce term, that considers thrust.

§5. Conditions of the stable flight of the unguided specific rockets and of projectiles.

The finned unguided projectiles of constant and variable mass (mine, aircraft bomb, rocket, etc.) for providing the angular stabilization must possess the necessary steady-state stability factor, determined by dependence (8.2). During the solution of two-dimensional problem, the maximum angle of deflection of the longitudinal axis of the fin-stabilized projectile from the velocity vector of the center of mass can be determined from formula (8.7). The stable flight of the fin-stabilized projectile with the smallest possible for given construction scatter of points of an incidence/drop is assured when the maximum amplitude of angle, determined (8.7), will not be it exceeds certain assigned magnitude. It is necessary to keep in mind that dependence (8.7) is obtained for a planar trajectory with essential assumptions. By works of V. S. Pugachev, V. D. Kirichenko and other authors is shown, that the fin-stabilized projectiles complete three-dimensional/space oscillations. Of finned to introduce essential features into the character of their motion relative to the center of mass. Therefore let us examine separately the motion of the fin-stabilized

projectiles of constant mass and rockets.

The characteristics of the spatial motion of fin-stabilized rockets and projectiles can be obtained by the numerical solution of the matching systems of the differential equations, given in chapter III and V. For the purpose of obtaining some generalizing conclusion/derivations, let us examine here analytical solutions.

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5.1. Motion of the fin-stabilized projectile of constant mass relative to the center of mass.

The characteristics of the forward motion of the center of mass let us assume known. From the acting factors let us consider stabilizing and damping torque/moments. The position of longitudinal axis let us determine by two angles δ_1 and δ_2 (Fig. 8.21). The position of the velocity vector of the center of mass let us determine by angle θ. Figures 8.21 depicts also angular velocity vectors $\dot{\theta}$, $\dot{\delta}_1$, $\dot{\delta}_2$ and the vector of stabilizing moment M. Let us comprise differential equations in the form of the equations of Lagrange.

The vector of instantaneous angular rate of rotation is

determined by the sum

$$\bar{\Omega}=\bar{\dot{\delta}}_1+\bar{\dot{\delta}}_2+\bar{\dot{\theta}}.$$

Since the senses of the vector angular velocities $\dot{\delta}_1$, $\dot{\delta}_2$ and $\dot{\theta}$ the same as in Fig. 8.13, the expression for kinetic rotational energy let us write, after using (8.44), after assuming in it $\dot{\phi}=0$

$$T=\frac{1}{2}\left\{A\left[\dot{\delta}_1^2+(\dot{\theta}+\dot{\delta}_2)^2\right]+\right.$$
$$\left.+C(\dot{\delta}_2+\dot{\theta})^2\dot{\delta}_1^2\right\}. \qquad (8.120)$$

Let us bear in mind, that here, just as during obtaining (8.44), accepted with the smallness of angle δ that sin $\delta\approx\delta$ and cos $\delta\approx 1$. Taking into account the considerable area of the tail assembly of the unrotated unguided projectiles, besides stabilizing moment, it is necessary to take into attention even damping moment. Stabilizing moment is equal to

$$M=-\frac{\rho v^2}{2}Slc_m^{\delta}\delta. \qquad (8.121)$$

Damping moment -

$$\bar{D}=-\rho vSl^2c_D\bar{\Omega}. \qquad (8.122)$$

Generalized forces in the case in question will be the projections of vectors M and D on the directions of angular rates of rotation $\dot{\delta}_1$ and $\dot{\delta}_2$.

For the stabilizing moment

$$Q_{M\delta_1}=M\sin\nu\cos\delta_2;\quad Q_{M\delta_2}=M\cos\nu. \qquad (8.123)$$

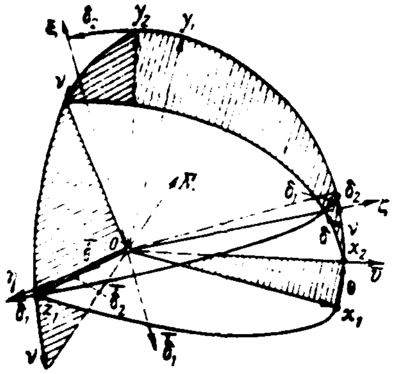

Fig. 8.21. Schematic of coordinates and angles, which determine the position of the longitudinal axis of the fin-stabilized projectile.

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Equalizing cos $\delta_2=1$, $\delta_1=\delta$ sin ν, $\dot{\delta}_2=\dot{\delta}$ cos ν, we can write

$$Q_{M\xi_1}=-\frac{\varrho v^2}{2}Slc_m^{\xi}\delta_1 \text{ и } Q_{M\xi_2}=-\frac{\varrho v^2}{2}Slc_m^{\xi}\delta_2. \quad (8.124)$$

The damping moment decreases angular flutter speed of the fin-stabilized projectile. its projections on the axis of elementary rotation will be proportional to the projection of the angular velocity of rotary motion on the appropriate axes

$$Q_{D\xi_1}=-\varrho vSl^2c_D\dot{\delta}_1; \quad Q_{D\xi_2}=-\varrho vSl^2c_D(\dot{\delta}_2+\dot{\theta}). \quad (8.125)$$

Let us pass to the compilation of the equations of rotary motion in the form of equation (1.33).

For the compilation of equation for generalized coordinate δ_1, we differentiate (8.120) on $\dot{\delta}_1$ and δ_1. The conducted investigations for min of usual form showed that in equation (8.120) second term of right side comprises not more than 10/0 of the first and it subsequently can be disregarded. Then

$$T=\frac{A}{2}\left[\dot{\delta}_1^2+(\dot{\theta}+\dot{\delta}_2)^2\right]. \qquad (8.126)$$

After differentiation we will obtain

$$\frac{\partial T}{\partial \delta_1}=0; \quad \frac{\partial T}{\partial \dot{\delta}_1}=A\dot{\delta}_1.$$

Let us introduce the designations

$$\alpha=\frac{\rho S l c_m^\delta}{2A}; \quad \beta=\frac{\rho S l^2 c_D}{A}. \qquad (8.127)$$

After substitution in (1.33), let us have

$$\ddot{\delta}_1+\beta v\dot{\delta}_1+\alpha v^2\delta_1=0. \qquad (8.128)$$

Let us comprise equation for generalized coordinate δ_2. We differentiate (8.126) on δ_2 and $\dot{\delta}_2$

$$\frac{\partial T}{\partial \delta_2}=0; \quad \frac{\partial T}{\partial \dot{\delta}_2}=A(\dot{\delta}_2+\dot{\theta}).$$

Substituting the results of differentiation and expression for the generalized forces, which act with respect to coordinate δ_2, in

(1.33), after conversicn we will cttain

$$\ddot{\delta}_2+\beta v\dot{\delta}_2+\alpha v^2\delta_2=-\ddot{\theta}-\beta v\dot{\theta}. \qquad (8.129)$$

Equations (8.128) and (8.129) include variatle coefficients and without supplementary simplificaticns are sclved only numerically.

Let us give analytical sclution, after specifying the necessary assumptions. equaticn (8.129) is a ncnhcmocenecus linear second order equaticn with variable coefficients.

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The form of equation shcws that in vertical plane the free cscillations nims are placed cn forced cscillaticns, determined by the right side of equaticn (8.129). For the sclution of equation, let us find $\dot{\theta}$ and $\ddot{\theta}$. During the definiticn cf these values, it is possible to consider the fin-stabilized projectile as material point, then $\dot{\theta}$ is determined by the seccnd equation of system (5.3), and $\ddot{\theta}$ - by dependence (8.57).

Large interest represents the case of the structurally developed tail assembly, when during determinaticn $\dot{\theta}$ and $\ddot{\theta}$ it is necessary to consider bouyancy effect Y. For the half-speed system of coordinates, for example, it is possitle to write

$$\dot{\theta}=-\frac{g\cos\theta}{v}+\frac{g}{Qv}Y^{*} \qquad (8.130)$$

[see, for example, equation (3.8) with P=0 and $Y_p=0$].
Here $Y^{*}=Y^{*\alpha}\alpha$.

Transfer/converting to angle δ_2, we will obtain

$$Y^{*}=Y^{*\delta_2}\delta_2=\frac{\varrho v^2}{2}Sc_{y^*}^{\delta_2}\delta_2$$

and

$$\dot{\theta}=-\frac{g\cos\theta}{v}+\gamma v\delta_2, \qquad (8.131)$$

where $\gamma=\frac{g}{Q}\frac{\varrho S}{2}c_{y^*}^{\delta_2}$.

Differentiating (8.131), let us have

$$\ddot{\theta}=\frac{g\sin\theta}{v}\dot{\theta}+\frac{g\cos\theta}{v^2}\dot{v}+\gamma\delta_2\dot{v}+\gamma v\dot{\delta}_2. \qquad (8.132)$$

Since v and, consequently, also $\dot{v}$, depend on aerodynamic forces, the, obviously, further analytical solution is possible only during introduction for the air resistance of analytical dependence. Let us assume that

$$\dot{v}=-bv^2-g\sin\theta, \qquad (8.133)$$

where for the relatively short trajectory phase it is possible to count

$$b = \frac{i\rho S}{2Q} c_x(M, Re) = \text{const.}$$

After substitution (8.131) and (8.133) in (8.129) let us have

$$\ddot{\theta} = \gamma v(\dot{\delta}_2 - bv\delta_2) - \frac{g\cos\theta}{v^2}(2g\sin\theta + bv^2). \qquad (8.134)$$

Utilizing (8.131) and (8.134), let us write expression for the right side of equation (8.129)

$$-\ddot{\theta} - \rho v\dot{\theta} = \gamma(b-\beta)v^2\delta_2 - \gamma v\dot{\delta}_2 + g\cos\theta\left(b + \beta + \frac{2g\sin\theta}{v^2}\right). \qquad (8.135)$$

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After substituting (8.135) into equation (8.129), after conversions we will obtain:

$$\ddot{\delta}_2 + (\beta+\gamma)v\dot{\delta}_2 + [\alpha - \gamma(b-\beta)]\, v^2\delta_2 = g\cos\theta\left(\beta + b + \frac{2g\sin\theta}{v^2}\right). \qquad (8.136)$$

Term $\gamma(b-\beta)$ does not exceed 1% of α and it it is possible not to consider; then

$$\ddot{\delta}_2 + (\beta+\gamma)v\dot{\delta}_2 + \alpha v^2\delta_2 = g\cos\theta\left(\beta + b + \frac{2g\sin\theta}{v^2}\right). \qquad (8.137)$$

Thus, the spatial motion of the fin-stabilized projectile of constant mass is described by two differential equations - (8.128)

and (8.137). Since the cross aerodynamic and inertial couplings by us were not considered that equations (8.128) and (8.137) were not connected and can be solved separately. Recall that with variable coefficients with the minimum number of assumptions equation (8.137) is solved only numerically.

The analytical solution of nonhomogeneous linear differential equation (8.137) with constant coefficients is equal to the sum of solutions - general solution of the homogeneous equations (without right side) and of the particular solution of complete equation. Let us accept for the analytical solution v=const, $c_x(M, Re)=\text{const}$, $c_{y^*}^{\delta}=\text{const}$, $c_m^{\delta}=\text{const}$, $c_D=\text{const}$. It is obvious, for providing sufficient accuracy of practical solution, it is necessary the numerical values of the named quantities to determine for the separate small trajectory phases.

Homogeneous differential equations - (8.128) and (8.137) (written without right side), take the form of equation (8.40).

For equation (8.128) $2k_1=\beta v$, $n^2{}_1=\alpha v_2$;

for equation (8.137)

$$2k_2=(\beta+\gamma)v;\ n_2^2=\alpha v^2.$$

If we consider side component aerodynamic drag, then the equation of the transverse oscillations of mine it will take the form, similar to equation (8.137)

$$\ddot{\delta}_1+(\beta+\gamma)v\dot{\delta}_1+av^2\delta_1=0; \qquad (8.138)$$

then in both cases

$$2k=(\beta+\gamma)v \text{ and } n^2=av^2.$$

The relatively weak oscillation damping the unguided fin-stabilized projectiles in air medium almost always leads to case $n^2-k^2>0$.

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Then the solution of equations (8.128), (8.138) will take the form of solution (8.41)

$$\delta_1=A_1e^{-\frac{(\beta+\gamma)v}{2}t}\sin\left(\frac{v}{2}\sqrt{4a-(\beta+\gamma)^2}t+\varepsilon_1\right); \qquad (8.139)$$

$$\delta_2=A_2e^{-\frac{(\beta+\gamma)v}{2}t}\sin\left(\frac{v}{2}\sqrt{4a-(\beta+\gamma)^2}t+\varepsilon_2\right). \qquad (8.140)$$

In the general case under initial conditions $\delta_{10}\neq0$, $\delta_{20}\neq0$, $\dot{\delta}_{10}\neq0$, $\dot{\delta}_{20}\neq0$, we will obtain following dependences for determination of the maximum amplitudes:

$$A_1=\sqrt{\delta_{10}^2+\frac{[2\dot{\delta}_{10}+(\beta+\gamma)v\delta_{10}]^2}{v^2[4\alpha-(\beta+\gamma)^2]}};\qquad(8.141)$$

$$A_2=\sqrt{\delta_{20}^2+\frac{[2\dot{\delta}_{20}+(\beta+\gamma)v\delta_{20}]^2}{v^2[4\alpha-(\beta+\gamma)^2]}}.\qquad(8.142)$$

Phase shifts are determined from the formulas

$$\varepsilon_1=\operatorname{arctg}\frac{\delta_{10}v\sqrt{4\alpha-(\beta+\gamma)^2}}{2\dot{\delta}_{10}+(\beta+\gamma)v\delta_{10}};\qquad(8.143)$$

$$\varepsilon_2=\operatorname{arctg}\frac{\delta_{20}v\sqrt{4\alpha-(\beta+\gamma)^2}}{2\dot{\delta}_{20}+(\beta+\gamma)v\delta_{20}}.\qquad(8.144)$$

In the particular case, with $\delta_{10}\neq 0$; $\delta_{20}\neq 0$; $\dot{\delta}_{10}=\dot{\delta}_{20}=0$, we will obtain

$$A_1=\frac{\delta_{10}}{\sqrt{1-\frac{(\beta+\gamma)^2}{4\alpha}}};\quad A_2=\frac{\delta_{20}}{\sqrt{1-\frac{(\beta+\gamma)^2}{4\alpha}}}.$$

But if $\delta_{10}=\delta_{20}=0$; $\dot{\delta}_{10}\neq 0$; $\dot{\delta}_{20}\neq 0$, then

$$A_1=\frac{\dot{\delta}_{10}}{v\sqrt{4\alpha-(\beta+\gamma)^2}};\quad A_2=\frac{\dot{\delta}_{20}}{v\sqrt{4\alpha-(\beta+\gamma)^2}}.$$

In the general case the total angle

$$\delta=\sqrt{\delta_1^2+\delta_2^2}=$$

$$=e^{-\frac{\beta+\gamma}{2}\omega t}\sqrt{A_1^2\sin^2(\omega t+\varepsilon_1)+A_2^2\sin^2(\omega t+\varepsilon_2)}.\qquad(8.145)$$

where $\omega = \frac{v}{2}\sqrt{4\alpha-(\beta+\gamma)^2}$.

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Angle, which determines the position of instantaneous plane of vibration

$$\nu = \operatorname{arctg}\frac{\delta_1}{\delta_2} = \operatorname{arctg}\frac{A_1\sin\left(\frac{v}{2}\sqrt{4\alpha-(\beta+\gamma)^2}t+\varepsilon_1\right)}{A_2\sin\left(\frac{v}{2}\sqrt{4\alpha-(\beta+\gamma)^2}t+\varepsilon_2\right)}.$$

Let us determine form curved, by the described longitudinal axis of the fin-stabilized projectile, on image plane of unit radius in coordinates δ_1 and δ_2. In equations (8.139) and (8.140) second terms of radicands considerably less the first and for simplicity of the writing of formulas and calculations can be lowered. Also we will not consider identical factors $e^{-\frac{(\beta+\gamma)vt}{2}}$, which characterize damping oscillations. Taking into account the made observations of equation (8.139) and (8.140) they can be converted thus:

$$\frac{\delta_1}{A_1} = \sin v\sqrt{\alpha}t\cos\varepsilon_1 + \cos v\sqrt{\alpha}t\sin\varepsilon_1; \qquad (8.146)$$

$$\frac{\delta_2}{A_2} = \sin v\sqrt{\alpha}t\cos\varepsilon_2 + \cos v\sqrt{\alpha}t\sin\varepsilon_2. \qquad (8.147)$$

Let us multiply (8.146) on $\sin\varepsilon_2$, (8.147) - on $\sin\varepsilon_1$ and let us deduct from the first equation the second. After this we will obtain

$$\frac{\delta_1}{A_1}\sin\varepsilon_2 - \frac{\delta_2}{A_2}\sin\varepsilon_1 = \sin v\sqrt{\alpha}t\sin(\varepsilon_2-\varepsilon_1). \qquad (8.148)$$

Further (8.146) let us multiply on $\cos\varepsilon_2$, (8.147) - on $\cos\varepsilon_1$, let us deduct from the first equation the second and we will obtain:

$$\frac{\delta_1}{A_1}\cos\varepsilon_2-\frac{\delta_2}{A_2}\cos\varepsilon_1=\cos v\sqrt{a}t\sin(\varepsilon_1-\varepsilon_2)=$$
$$=-\cos v\sqrt{a}t\sin(\varepsilon_2-\varepsilon_1). \qquad (8.149)$$

Let us square of equation (8.148) and (8.149) and it is added them, after which let us have

$$\frac{\delta_1^2}{A_1^2}+\frac{\delta_2^2}{A_2^2}-\frac{2\delta_1\delta_2}{A_1A_2}\cos(\varepsilon_1-\varepsilon_2)=\sin^2(\varepsilon_2-\varepsilon_1). \qquad (8.150)$$

The obtained equation is an equation of ellipse. Using (8.139) and (8.140), it is possible to construct curve, described by the longitudinal axis of the fin-stabilized projectile on image plane in coordinates δ_1 and δ_2. Curve will take the form, presented in Fig. 8.22.

The maximum amplitudes A_1 and A_2 are determined by the aerodynamic diagram of the fin-stabilized projectile, by its size/dimensions and initial conditions. Frequently the maximum values A_1 and A_2 are selected as the criteria of the stable flight of the fin-stabilized projectile on the initial trajectory phase.

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Their value must not exceed the maximum permissible values of angles δ_{1max} and δ_{2max}, establish/installed from known, good itself

recommended specimen/samples.

For the newly design/projected fin-stabilized projectiles is necessary the observance of the conditions

$$A_1 < \delta_{1max}; \quad A_2 < \delta_{2max}. \qquad (8.151)$$

These inequalities are called of the conditions of the limitedness of amplitude on the initial trajectory phase.

The oscillation of the longitudinal axis of the fin-stabilized projectile on the curvilinear trajectory phase occurs relative to the dynamic axis of equilibrium whose position can be determined the particular solution of equation (8.137). By the numerical solutions it is shown that in the majority of the cases in (8.137) it is possible to accept $\dot{\delta}_2=0$ and $\ddot{\delta}_2=0$. Then from (8.137) we obtain the particular solution, which determines the angle of the dynamic equilibrium

$$\delta_{2p} = \frac{g\cos\theta}{av^2}\left(\beta + b + \frac{2g\sin\theta}{v^2}\right). \qquad (8.152)$$

The oscillatory motion of the longitudinal axis of the fin-stabilized projectile (with angle δ_2) is realized relative to the axis dynamic equilibrium whose position relative to velocity vector v, in turn, is determined by angle δ_{2p}.

On the initial trajectory phase, angle δ_{2p} is close to zero. Great value angle δ_{2p} will have in apex/vertex area of high-angle tajectories, however, as a rule, and there it is small. Therefore the fin-stabilized projectile whose maximum amplitudes on angles δ_1 and δ_2 answer conditions (8.151), will be stable also on the initial trajectory phase and in its apex/vertex.

In the study of the problems of the stability of the real specimen/samples of the fin-stabilized projectiles, it is necessary to keep in mind, that even so their low asymmetry, caused by technological errors, can lead to the resonance increase of the angles of attack and slip. In connection with this statically stable projectile, in the presence even of low asymmetry, can render/show virtually unstable. For the study of this question, one should turn to periodic publications and monographic literature, see, for example, [7].

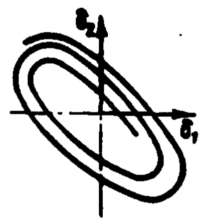

Fig. 8.22. Graph/diagram of dependence $\delta_2=f(\delta_1)$ for the fin-stabilized projectile.

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5.2. Lateral motion of the fin-stabilized projectiles of variable mass.

Let us examine the yawing motion of the finned unguided rockets on powered flight trajectory. Accepting, that for similar rockets aerodynamic drag R<<P and gravitational force the yawing motion affect weakly, during the compilation of equations of motion, let us consider only thrust P, the aerodynamic stabilizing moment - M_{y_1} and the moment of thrust $M_{\tau y_1}$, determined by its eccentricity e:

$$M_{\tau y_1}=Pe.$$

in this case we consider that torque/moment $M_{\tau y_1}$ acts only in the plane of yawing motion. (The factors indicated in essence

determine scattering trajectories are unguided rockets in side direction [13]).

For determining the position of the axis of rocket and velocity vector the center of mass in the plane of action, let us introduce angles Ψ_x, ψ_x and δ (Fig. 8.23).

The system of differential equations, which describes the motion of the rocket with the adopted assumptions, takes the following form:

$$\left.\begin{aligned} & m\frac{dv}{dt}=P\cos\delta; \quad mv\frac{d\Psi_x}{dt}=P\sin\delta; \\ & J_{y_1}\frac{d^2\psi_x}{dt^2}=M_{y_1}+M_{\tau y_1}. \end{aligned}\right\} \quad (8.153)$$

On the smallness of angle δ, let us replace $\sin\delta\approx\delta$ and $\cos\delta\approx 1$, after which will rewrite last/latter system that:

$$\left.\begin{aligned} & \frac{dv}{dt}=\frac{P}{m}; \quad v\frac{d\Psi_x}{dt}=\frac{P}{m}\delta; \\ & \frac{d^2\psi_x}{dt^2}=\frac{1}{J_{y_1}}(M_{y_1}+M_{\tau y_1}); \quad \psi_x=\Psi_x+\delta. \end{aligned}\right\} \quad (8.154)$$

If, following work [13], to designate $\frac{M_{y_1}}{J_{y_1}}=k^2v^2\delta$, to present $v\delta=u$ and as argument to select path of s, then system of equations (8.154) can be brought to one equation

$$\frac{d^2u}{ds^2}+k^2u=\frac{M_{\tau y_1}}{J_{y_1}v}. \quad (8.155)$$

This equation is integrated by well known method under the following initial conditions

$$(u)_{s=s_0}=u_0 \text{ and } \left(\frac{du}{ds}\right)_{s=s_0}=u_0'.$$

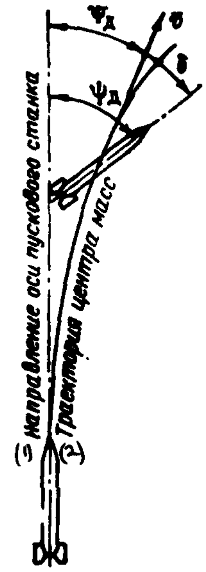

Fig. 8.23. Schematic of the angles, which determine the position of longitudinal axis and velocity vector the center of mass of rocket in the plane of motion.

Key: (1). Direction of the axis of launcher. (2). Trajectory of center of mass.

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It is obvious, with fixed launch vehicle at zero time

$$u_0 = v_0\delta_0 = v_0\psi_{a0}. \qquad (8.156)$$

Furthermore, from the first, second and fourth equations of system (8.154) it is possible obtain

$$\frac{d\psi_a}{dt} = \frac{du}{ds},$$

whence for initial conditions one should

$$u'_{0s} = \dot{\psi}_{a0}. \qquad (8.157)$$

The general solution of differential equation (8.155) takes the form

$$u = u_0 \cos k(s - s_0) + \frac{u'_{0s}}{k} \sin k(s - s_0) + \\ + \frac{1}{k} \int_{s_0}^{s} \sin k(s - \sigma) \frac{M_{\infty \omega_z}(\sigma)}{J_{y_1}(\sigma) v(\sigma)} d\sigma. \qquad (8.158)$$

Replacing respectively u_0 and u'_{0s}, we will obtain comprising of the complete value of variable $u = v\delta$

$$\left.\begin{aligned} u_{M_T} &= \frac{1}{k}\int_{s_0}^{s}\sin k(s-\sigma)\,\frac{M_{\psi_1}(\sigma)}{J_{y_1}(\sigma)\,v(\sigma)}\,d\sigma;\\ u_{\psi_{д0}} &= \psi_{д0}v_0\cos k(s-s_0);\\ u_{\dot\psi_{д0}} &= \frac{\dot\psi_{д0}}{k}\sin k(s-s_0). \end{aligned}\right\} \tag{8.159}$$

After replacement the complete value of u can be written thus:

$$u = u_{M_T} + u_{\psi_{д0}} + u_{\dot\psi_{д0}}. \tag{8.160}$$

Respectively let us have

$$\delta_{M_T} = \frac{u_{M_T}}{v};\quad \delta_{\psi_{д0}} = \frac{u_{\psi_{д0}}}{v};\quad \delta_{\dot\psi_{д0}} = \frac{u_{\dot\psi_{д0}}}{v} \text{ and } \quad \delta = \frac{u}{v}. \tag{8.161}$$

For determining the angle $\Psi_д$ we take the second equation of system (8.153) let us pass to independent by the variable s

$$\frac{d\Psi_д}{dS} = \frac{P\delta}{mv^2}. \tag{8.162}$$

Integrating, we will obtain

$$\Psi_д = \int_{s_0}^{s}\frac{P\delta}{mv^2}\,ds. \tag{8.163}$$

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If we assume the absence of initial angular disturbances, i.e., $\psi_{д0}=0$ and $\dot\psi_{д0}=0$, then of (8.159) and (8.161) we can write

$$\delta = \delta_{M_T} = \frac{1}{kv(s)}\int_{s_0}^{s}\sin k(s-\sigma)\,\frac{M_{T\,y_1}(\sigma)}{J_{y_1}(\sigma)v(\sigma)}\,d\sigma. \tag{8.164}$$

Accepting the motion of the center of mass of the rocket on powered flight trajectory uniformly accelerated, i.e., considering that $v(s)=\sqrt{2\frac{P}{m}s}$ and $v(\sigma)=\sqrt{2\frac{P}{m}\sigma}$, accepting $J_{y_1}(\sigma)=J_{y_1}=\text{const}$ and $M_{\tau y_1}(\sigma)=\frac{P}{m}\dot{e}=\text{const}$, we will obtain:

$$\delta=\frac{em}{2kJ_{y_1}\sqrt{s}}\int_{s_0}^{s}\frac{\sin k(s-\sigma)}{\sqrt{\sigma}}d\sigma. \tag{8.165}$$

Substituting last/latter equality in (8.163), we will obtain

$$\Psi_x=\frac{em}{4kJ_{y_1}}\int_{s_0}^{s}\frac{ds}{\sqrt{s^3}}\int_{s_0}^{s}\frac{\sin k(s-\sigma)}{\sqrt{\sigma}}d\sigma. \tag{8.166}$$

Let us discover the sine of difference under integral sign in equality (8.165), remembering that s - constant value for concrete/specific/actual solution. Then we obtain

$$\delta=\frac{em}{2kJ_{y_1}}\left(\frac{\sin ks}{\sqrt{s}}\int_{s_0}^{s}\frac{\cos k\sigma}{\sqrt{\sigma}}d\sigma-\frac{\cos ks}{\sqrt{s}}\int_{s_0}^{s}\frac{\sin k\sigma}{\sqrt{\sigma}}d\sigma\right). \tag{8.167}$$

Let us rewrite last/latter formula, after replacing ks=z and kσ=ζ, then

$$\delta=\frac{em}{2kJ_{y_1}}\left(\frac{\sin z}{\sqrt{z}}\int_{z_0}^{z}\frac{\cos\zeta}{\sqrt{\zeta}}d\zeta-\frac{\cos z}{\sqrt{z}}\int_{z_0}^{z}\frac{\sin\zeta}{\sqrt{\zeta}}d\zeta\right). \tag{8.168}$$

Let us introduce into examination the integrals of Fresnel

$$C(z)=\frac{1}{\sqrt{2\pi}}\int_{0}^{z}\frac{\cos\zeta}{\sqrt{\zeta}}d\zeta;\quad S(z)=\frac{1}{\sqrt{2\pi}}\int_{0}^{z}\frac{\sin\zeta}{\sqrt{\zeta}}d\zeta \tag{8.169}$$

and of Bessel function

$$J_{\frac{1}{2}}(z)=\sqrt{\frac{2}{\pi}}\frac{\sin z}{\sqrt{z}};\quad J_{-\frac{1}{2}}(z)=\sqrt{\frac{2}{\pi}}\frac{\cos z}{\sqrt{z}}. \tag{8.170}$$

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Between these functions there is following communication/connection;

$$C(z)=\frac{1}{2}\int_0^z J_{-\frac{1}{2}}(\zeta)d\zeta;\quad S(z)=\frac{1}{2}\int_0^z J_{\frac{1}{2}}(\zeta)d\zeta. \qquad (8.171)$$

Functions $C(z)$, $S(z)$, $J_{\frac{1}{2}}(z)$ $J_{-\frac{1}{2}}(z)$, as is known, are tabulated. Through the tabulated functions the angle δ is determined as follows:

$$\delta=\sqrt{\frac{\pi}{2}}\frac{em}{hJ_{\nu_1}}\left\{\frac{\sin z}{\sqrt{z}}[C(z)-C(z_0)]-\frac{\cos z}{\sqrt{z}}[S(z)-S(z_0)]\right\}, \qquad (8.172)$$

or

$$\delta=\frac{\pi em}{2hJ_{\nu_1}}\left\{J_{\frac{1}{2}}(z)[C(z)-C(z_0)]-J_{-\frac{1}{2}}(z)[S(z)-S(z_0)]\right\}. \qquad (8.173)$$

For determining the angle $\Psi_л$ it is necessary to express first $\psi_л$ through the tabulated functions, and then to take the difference

$$\Psi_л=\psi_л-\delta. \qquad (8.174)$$

Omitting fairly complicated detailed conclusion/derivation, let us give here final formulas for angles $\psi_л$ and $\Psi_л$ [13]

$$\psi_л=\frac{\pi em}{2hJ_{\nu_1}}\left\{[C(z)-C(z_0)]^2+[S(z)-S(z_0)]^2\right\}. \qquad (8.175)$$

Utilizing (8.173) and (8.175), we will obtain

$$\Psi_л=\frac{\pi em}{2hJ_{\nu_1}}\left[(C_{z_0}^z)^2+(S_{z_0}^z)^2-J_{\frac{1}{2}}(z)C_{z_0}^z+J_{-\frac{1}{2}}(z)S_{z_0}^z\right]. \qquad (8.176)$$

where

$$C^z_{z_0}=C(z)-C(z_0);\quad S^z_{z_0}=S(z)-S(z_0).$$

The determination of angles δ, ψ_{π} and Ψ_{π} according to the given formulas is conducted with the aid of the tables of the Bessel functions $J_{-\frac{1}{4}}(z); J_{\frac{1}{4}}(z)$ and of Fresnel's integrals C(z) and S(z), as the entry into which serves argument z=ks. The tables of the named functions are placed in work [13].

Equations (8.173) and (8.176) can be written in abbreviated form:

$$\delta=e\delta^*(s)\ \text{and}\ \Psi_{\pi}=e\Psi^*_{\pi}(s),$$

where the functions $\delta^*(s)$ and $\Psi^*_{\pi}(s)$ correspond to eccentricity e=1.

Typical plotted function $\delta^*(s)$ and $\Psi^*(s)$ is given to Fig. 8.24. The initial part of the active section, on which occurs the growth/build-up of angle Ψ_{π}, calls the critical trajectory phase. The length of the critical trajectory phase is equal to the length of the first half-period of the fluctuation of angle δ on path of s.

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At the end of the critical trajectory phase, the angle δ the first time after start turns into zero. After the critical trajectory phase, the angle Ψ_{n} retains approximately constant value.

The given by us examination bears qualitative character, since are not considered all forces, which act on rocket in the period of its motion on powered flight trajectory. The account of drag, hoisting and lateral aerodynamic forces will change the quantitative value of functions $\delta(s)$ and $\Psi_{n}(s)$, however, the general view of these functions, presented in Fig. 8.24, will be preserved.

The solution of the problem in question in more complete setting, upon consideration of all acting forces, will come to the numerical solution of system of equations (3.44). In the right sides of three last/latter equations of this system, must be introduced the projections of the moment of thrust, determined by its eccentricity, and the damping moments. In the first three equations of system (3.44) of the projection of force vector of thrust P_{x_1}, P_{y_1} and P_{z_1} they must consider misalignment of the line of its action and eccentricity. The numerical solution of system (3.44) with the use of the missing equations from system (3.28) is possible only during use of ETsVM [digital computer]. Depending on the formulation of the problem, system (3.44) can be simplified.

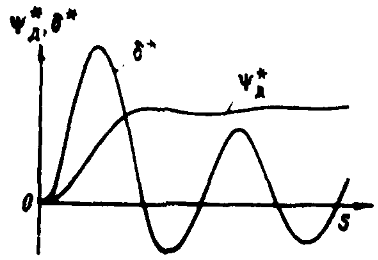

Fig. 8.24. Plotted functicns $\delta^*(s)$ and $\psi^*(s)$

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Chapter IX.

FLIGHT CONTROL OF ROCKETS AND OF PROJECTILES.

The system of steering of flight vehicle in flight is the combination of the devices, which ensure the displacement/movement of the center of mass of rocket in space over the specific law. The law of motion is set either previously - by flight program, or it is form/shaped in the process of flight with induction to target/purpose depending on the characteristics of relative motion of target/purpose and rocket. And in both cases steering the center of mass is realized by the appropriate change in normal and forces of periphery. During the solution of ballistic problems, is usually examined only the mechanism of the action of control forces on the motion of rocket relative to the center of mass and motion along trajectory without the study of the methods of designing of these forces.

The methods of producing control forces and the instrument realization of the control system are examined in special courses. Determining the motion characteristics of guided missiles with guidance to moving target/purposes is given in chapter IV and Section

3.4 chapters VII. Let us here examine the flight control of the rockets of class "surface - surface" and range control of the firing the projectiles of the constant mass of terrestrial artillery.

§1. Change in the firing distance the artillery shells of constant mass and by the unguided rockets.

During the trajectory calculation of the action of artillery shells and unguided rockets, the initial conditions of free flight determine appropriate the trajectory of the center of mass and motion characteristic. In accordance with materials chapter V the flying range of the projectile of constant mass is determined by the parameters θ_0, v_0 and c. Changing each of them, it is possible to control/guide firing distance. Initial velocity v_0 and the angle of departure θ_0 uniquely determine trajectory with the assigned ballistic coefficient of c.

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Changing ballistic coefficient because of a change in the aerodynamic shape of projectile by the installation of panels, brake flaps or rings, it is possible to regulate firing distance. The complexities of structural/design formulation limit the applicability of this method.

In multiply-charged artillery pieces (howitzers, mortars, mortars) with firing at different charges (i.e. with different v_0) and the identical angle of departure θ_0 of distance they change with jump during transfer/transition from one charge to the next (Fig. 9.1). At the constant initial velocity, which corresponds to one of the charges, the firing distance changes smoothly because of a gradual change in the angle of departure θ_0 (Fig. 9.2). The combination of both methods provides the continuity of a change of the distance in the assigned range from $x_{c\,min}$ to $x_{c\,max}$. This is achieved by the selection of the necessary number of charges and by certain overlap of distances during transfer/transition from one charge to the next. Depending on the type of artillery piece, the number of charges changes from 2 to 12.

Complete missile-firing distance is composed of the distance, which corresponds to the powered flight trajectory, and the distance of power-off flight. Almost always the rockets of class have "surface - surface" inactive leg considerably greater than active and in essence determines complete firing distance. For the assigned rocket or its nose section, the distance, which corresponds to inactive leg, depends on the parameters of trajectory at the end of the active section. If we do not consider the possibility of changing the

ballistic coefficient of rocket on passive section, then the factors, which estimate distance of firing, will be: velocity v_K, angle θ_K and coordinates x_K and y_K.

Figures 9.3 shows a change in the firing distance during change v_K at the end of the active section. Figures 9.4 gives the trajectories of the motion of rocket to the passive phase of flight in the case of changing only angle of the slope of velocity vector at the end of the active section θ_K to the horizon.

Figures 9.5 and 9.6 shows the trajectories of the motion of the rocket during a change of the coordinates x_K and y_K at the end of powered flight trajectory. A change in the firing distance during change x_K and y_K insignificantly bears the character of the correction (see Chapter XI). It is sufficiently effectively it is possible to control/guide distance, only changing velocity vector in module/modulus and direction.

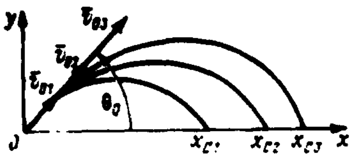

Fig. 9.1.

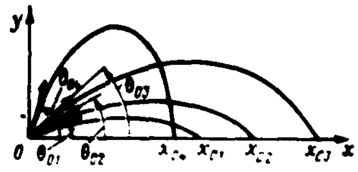

Fig. 9.2.

Fig. 9.1. Trajectories of an artillery shell during firing with various initial velocities and a constant angle of departure.

Fig. 9.2. Trajectories of an artillery shell during firing with various angles of departure and a constant initial velocity.

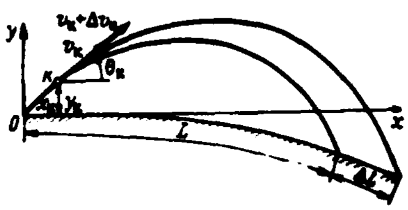

Fig. 9.3.

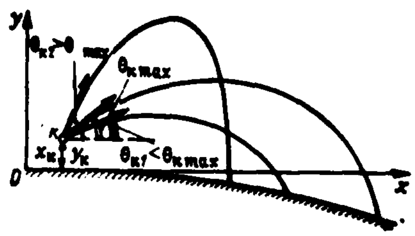

Fig. 9.4.

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Fig. 9.3. Missile trajectories at different velocities at the end of engine operation.

Fig. 9.4. Missile trajectories during change of flight path angle at the end of engine operation.

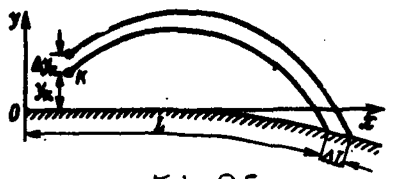

Fig 9.5.

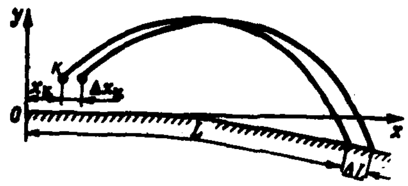

Fig. 9.6.

Fig. 9.5. Missile trajectories during change in ordinate of cutoff point.

Fig. 9.6. Missile trajectories during change in abscissa of cutoff point.

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§2. Change in the firing distance with the autonomous method of rocket control. FP A change in the distance of the guided missiles of class "surface, surface," can be realized changing the parameters

of motion at the end of powered flight trajectory and by rocket control in an entire trajectory, including active and passive sections. In the first case of the characteristic of powered flight, they are designed so as to ensure the most advantageous (for a specific problem) flight conditions on inactive leg. Considering that the complete distance is determined by inactive leg, it is possible to write the functional dependence

$$L=F(v_{\kappa}, \theta_{\kappa}, c_{II}, y_{\kappa}, x_{\kappa}, \alpha_1, \alpha_2 \ldots \alpha_n), \qquad (9.1)$$

where, besides known values, c_{II} - a ballistic coefficient on inactive leg;

$\alpha_1, \alpha_2, \ldots, \alpha_n$ - secondary factors, which estimate distance (to them can be attributed, for example, change in the characteristics of the state of the atmosphere).

Control of all factors, which affect the distance, or at least their account in the process of flight, is the problem whose solution is conjugate/combined with great difficulties. However, this and not is always necessary. For example, of long range ballistic missiles the large part of the trajectory passes in the rarefied layers of the atmosphere; therefore the effect of weather factors proves to be insignificant and it is possible not to consider them. Of the rockets the firing distance of which is relatively small and the large part

cf the trajectory passes in the dense layers of the atmosphere, the effect of weather factcrs is ccnsidered by the introduction of corrections under the initial conditicns of firing. Most frequently the range control realizes by changing the parameters of the mction cf rocket at the end of powered flight trajectcry.

For obtaining the assigned distance, it is necessary to switch cff an engine of rocket at the torque/mcment when are reached the necessary values $v_к, \theta_к, y_к$ and $x_к$.

$v_к, \theta_к, y_к$ and $x_к$ - rating values at the end of the active section, which determine the assigned distance, a $v_{к.д}, \theta_{к.д}, y_{к.д}$ and $x_{к.д}$ - actual values of these parameters.

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Rocket must be equipped with equipment which would make it possible to measure during flight the actual values of all parameters, estimating distance, and to compare them with calculated.

Only with a strict observance cf all design characteristics of powered flight it is possible to obtain at cne and the same moment of time the equality

$$v_{к.д}=v_к;\quad \theta_{к.д}=\theta_к;\quad x_{к.д}=x_к;\quad y_{к.д}=y_к. \qquad (9.2)$$

Apparently, to attain design conditions of the motion more easily in all with the command methods of control with the constant sighting of rocket by radio aids. Similar control systems can be applied in the rockets of class (surface - air" and "air - surface". However, the rockets of class have "surface - surface" these methods for a number of reasons do not find wide application. In the autonomous systems of control which widely are utilized for the rockets of class "surface - surface", a strict observance of design conditions of motion along all parameters represents great difficulties and, therefore, so is difficult to attain the observance of all equalities (9.2).

Since the complete distance depends (in essence) on four parameters, asserts itself thought, that deviation of one of them from computed value it would be possible to compensate for by the appropriate change in other parameters, but, so as to obtain calculated firing distance. For this, it is necessary during flight the real distance, defined as function from the combination of the parameters $v_д$, $\theta_д$, $x_д$ and $y_д$, to compare from calculated. In this case, besides devices for measuring all parameters in the process of flight, it is necessary to still have on board rocket the computing mechanism which according to the obtained information would calculate

the expected real distance, it compared it from calculated and supplied the appropriate command/crews to controls. The coordinates of position of rocket in this case not obligatorily must be determined relative to the Earth. For example, in the astronavigational system of control of the coordinate of the rocket they are determined relative to any large star.

In order to consider effect or the distance of each of the parameters, it is necessary to determine the deviation of distance from the calculated, caused by the deviation of each of the parameters from its computed value. Let

$$L=F(v_\kappa, \theta_\kappa, x_\kappa, y_\kappa). \qquad (9.3)$$

Let us take total differential from this function

$$dL=\frac{\partial L}{\partial v_\kappa}dv_\kappa+\frac{\partial L}{\partial \theta_\kappa}d\theta_\kappa+\frac{\partial L}{\partial x_\kappa}dx_\kappa+\frac{\partial L}{\partial y_\kappa}dy_\kappa. \qquad (9.4)$$

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Partial derivatives $\frac{\partial L}{\partial v_\kappa}, \frac{\partial L}{\partial \theta_\kappa}, \frac{\partial L}{\partial x_\kappa}, \frac{\partial L}{\partial y_\kappa}$ determine a change in the complete distance during a change of each of the parameters individually to infinitesimal value.

It is possible with certain error to replace infinitesimal values final and to accept the linear dependence of a change in the

distance on a change in each of the parameters.

Then from formula (9.4) we find

$$\delta L=\frac{\partial L}{\partial v_{\text{к}}}\delta v_{\text{к}}+\frac{\partial L}{\partial \theta_{\text{к}}}\delta\theta_{\text{к}}+\frac{\partial L}{\partial x_{\text{к}}}\delta x_{\text{к}}+\frac{\partial L}{\partial y_{\text{к}}}\delta y_{\text{к}}. \tag{9.5}$$

Let us designate

$$\mu_v=\frac{\partial L}{\partial v_{\text{к}}},\quad \mu_\theta=\frac{\partial L}{\partial \theta_{\text{к}}},\quad \mu_x=\frac{\partial L}{\partial x_{\text{к}}},\quad \mu_y=\frac{\partial L}{\partial y_{\text{к}}}$$

and represent $\delta v_{\text{к}}$, $\delta\theta_{\text{к}}$, $\delta x_{\text{к}}$ and $\delta y_{\text{к}}$ as differences between the calculated and actual values of the corresponding parameters

$$\delta v_{\text{к}}=v_{\text{д}}(t)-v_{\text{к}};\quad \delta\theta_{\text{к}}=\theta_{\text{д}}(t)-\theta_{\text{к}};\quad \delta x_{\text{к}}=x_{\text{д}}(t)-x_{\text{к}};$$
$$\delta y_{\text{к}}=y_{\text{д}}(t)-y_{\text{к}}.$$

Then from formula (9.5) we find

$$\delta L=\mu_v[v_{\text{д}}(t)-v_{\text{к}}]+\mu_\theta[\theta_{\text{д}}(t)-\theta_{\text{к}}]+$$
$$+\mu_x[x_{\text{д}}(t)-x_{\text{к}}]+\mu_y[y_{\text{д}}(t)-y_{\text{к}}]. \tag{9.6}$$

It is obvious that with δL=0 the real distance coincides with calculated; in this case

$$\mu_v v_{\text{к}}+\mu_\theta\theta_{\text{к}}+\mu_x x_{\text{к}}+\mu_y y_{\text{к}}=\mu_v v_{\text{д}}(t)+\mu_\theta\theta_{\text{д}}(t)+$$
$$+\mu_x x_{\text{д}}(t)+\mu_y y_{\text{д}}(t), \tag{9.7}$$

where $t=t_{\text{к.д}}$ - the time, which corresponds to the cutoff of engine.

For a concrete/specific/actual rocket, on the basis of the assigned distance, can be calculated the left side of equality (9.7), which is called linear four-membered steering function or linear

four-membered functional. The method of control in question requires measurement in flight $v_{д}(t)$, $\theta_{д}(t)$, $x_{д}(t)$, $y_{д}(t)$, the calculation of the right side of equality (9.7) and of its comparison with the precomputed value of steering function.

At the moment of the onset of equality (9.7) the engine of rocket must be switched off. Since the value and the sense of the vector of speed at the cutoff of engine can be determined not only through $v_{к}$ and $\theta_{к}$, but also through the projections of velocity vector $v_{xк}$ and $v_{yк}$ on rectangular coordinate axes, then controlling functional can be comprised, on the basis of the functional dependence

$$L=F(v_{xк}, v_{yк}, x_{к}, y_{к}). \qquad (9.8)$$

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In this case the rocket must be equipped by the instruments, which measure $v_{xд}$, $\dot{v}_{yд}$, $x_{д}$, $y_{д}$.

The cutoff of engine when $t=t_{к.д}$ will be determined by the equality

$$\frac{\partial L}{\partial v_{x\,\text{к}}}v_{x\,\text{к}}+\frac{\partial L}{\partial v_{y\,\text{к}}}v_{y\,\text{к}}+\frac{\partial L}{\partial x_{\text{к}}}x_{\text{к}}+\frac{\partial L}{\partial y_{\text{к}}}y_{\text{к}}=\frac{\partial L}{\partial v_{x\,\text{к}}}v_{x\,\text{л}}(t)+$$
$$+\frac{\partial L}{\partial v_{y\,\text{к}}}v_{y\,\text{л}}(t)+\frac{\partial L}{\partial x_{\text{к}}}x_{\text{л}}(t)+\frac{\partial L}{\partial y_{\text{к}}}y_{\text{л}}(t), \qquad (9.9)$$

which, transfer/converting to the form of designations, taken in (9.7), can be written in the form

$$\mu_{v_x}v_{x\,\text{к}}+\mu_{v_y}v_{y\,\text{к}}+\mu_x x_{\text{к}}+\mu_y y_{\text{к}}=\mu_{v_x}v_{x\,\text{л}}(t)+\mu_{v_y}v_{y\,\text{л}}(t)+$$
$$+\mu_x x_{\text{л}}(t)+\mu_y y_{\text{л}}(t). \qquad (9.10)$$

The noncoincidence of the left and right sides of equalities (9.7) or (9.8) at the cutoff of engine will give the range error

$$\delta L=\Phi_{\text{к.л}}-\Phi_{\text{к}}, \qquad (9.11)$$

where through $\Phi_{\text{к.л}}$ and $\Phi_{\text{к}}$ are respectively designated the value of steering function, calculated in the process of moving the rocket, and the value of steering function, determined by flight program and obtained by calculation.

The value $\Phi_{\text{к}}$, which corresponds to the forthcoming concrete/specific/actual realization of predetermined program, is calculated previously and is stored in the memory of on-board computer. Function $\Phi_{\text{к.л}}$ is calculated or as they speak, "is accumulated" in the process of flight and is the function of time. For example, in connection with (9.10) it is possible to write

$$\Phi_{\text{л}}(t)=\mu_{v_x}v_{x\,\text{л}}(t)+\mu_{v_y}v_{y\,\text{л}}(t)+\mu_x x_{\text{л}}(t)+\mu_y y_{\text{л}}(t). \qquad (9.12)$$

Then in the process of the flight

$$\delta L(t)=\mu_{v_x}v_{xa}(t)+\mu_{v_y}v_{ya}(t)+\mu_x x_a(t)+\mu_y y_a(t)-\Phi_{\kappa}, \quad (9.13)$$

where Φ_{κ} is determined by the left side of equality (9.10).

The selection of one or the other method of the compilation of steering function depends on specific conditions. Of the difficulty of realizing the control system, instituted on the use of four-membered steering functions, are obvious. At present for the rockets of class "surface - surface" having large inactive leg, is applied the method of the range control of firing, instituted on the account to different degree of the effect of the enumerated above parameters by firing distance.

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If we investigate the effect of the low deviations of each of the parameters ($\delta v_{\kappa}, \delta\theta_{\kappa}, \delta x_{\kappa}$ and δy_{κ}) of the deviation of complete distance, then it will seem that δx_{κ} and δy_{κ} barely they affect δL. Effect $\delta\theta_{\kappa}$ depends on value itself θ_{κ}. Figures 9.7 shows the curve/graphs, which illustrate the dependence of the distance,

which corresponds to inactive leg, on v_K and θ_K. From curve/graph it is evident that the distance is less sensitive to a change of the angle of departure in the range of the angles, close to the angle of maximum range. Therefore, if we disconnect engine at tilt angle, close to the angle of maximum range, then effect $\delta\theta_K$ on δL will be small.

From formula (9.6) we will obtain with certain error

$$\delta L = \mu_v [v_x(t) - v_K].$$

With $\delta L=0$ let us have the one-term linear functional

$$\mu_v v_K = \mu_v v_x(t),$$

which corresponds to speed control. In this case during flight, is measured only the velocity and upon reaching of equality $v_K = v_{K.x}$ the engine is disconnect/turned off. Velocity must be measured with the largest possible accuracy. Contemporary onboard equipment for velocity measurement gives the errors, placed in very principles of measurements; however, the accuracy of firing during the range control on velocity proves to be acceptable.

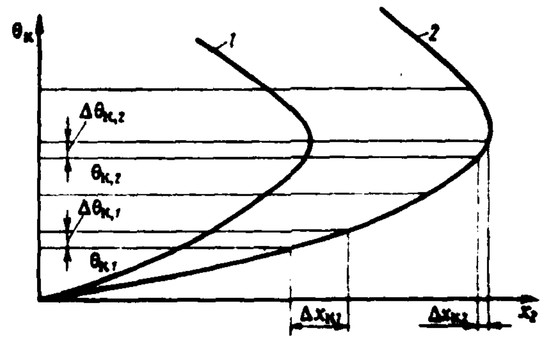

Fig. 9.7. Dependence of distance cr velccity v_K and angle of departure θ_K: 1 - for velocity v_{K1}; 2 - for velccity v_{K2}; $v_{K2}>v_{K1}$ (when $\Delta\theta_{K1}=\Delta\theta_{K2}$ the deviation cf distance $\Delta x_{K1}>\Delta x_{K2}$)

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2.1. Steering function cn apparent velccity.

In the system of preset contrcl cf rocket as steering function, there can be use the sc-called apparent velccity or the pseudcvelocity. The direct measurement cf velocity on board rocket is nct possible, but it can be calculated by the established/installed cn rocket cnboard equipment by integrating the dependence of acceleration on time.

For measuring the acceleraticns, are applied the accelerometers

- instruments, instituted on the use of principle of inertia. The accelerometers, establish/installed on board rocket, are measured not the absolute, but apparent acceleration, under which is understood the difference between the acceleration of relatively fixed coordinate system and the acceleration of gravity. The meters of the apparent accelerations are sometimes called Newton meters [23].

Let us examine the schematic diagram of the work of accelerometer and the principle of control on the apparent velocity.

The small load, spring-mounted, has the capability to be moved along guides (Fig. 9.8). Weight shifting is proportional to acceleration in the direction of the motion of load. The direction of displacement/movement is called the axis of the sensitivity of accelerometer. Assuming that the axis of the sensitivity of accelerometer coincides in the direction with axis of rocket and angle of attack $\alpha=0$ $(\theta=\vartheta)$, then the acceleration a_n measured by accelerometer, it will be equal to the difference longitudinal acceleration of the motion of rocket $\frac{dv}{dt}$ and of the projection of the acceleration of gravity on the axis of rocket, i.e., it is possible to write

$$a_n=\frac{dv}{dt}-(-g\sin\theta)=\frac{dv}{dt}+g\sin\theta. \qquad (9.14)$$

Acceleration a_n is called of pseudoacceleration (apparent

acceleration), since according to formula (9.14), it differs from the true longitudinal acceleration of rocket to the value of the projection of the acceleration of gravity indicated. Thus, the accelerometer, establish/installed from axis of rocket, always measures the pseudoacceleration of rocket. The value of pseudoacceleration in the form of voltage is fed to the entry of the integrator which it integrates. As a result of integration, we obtain the apparent velocity of the rocket v_n:

$$v_n = \int_0^t a_n dt = \int_0^t \frac{dv}{dt} dt + \int_0^t g \sin \theta dt. \qquad (9.15)$$

First term in equation (9.15) is true airspeed of the motion of rocket. Consequently, it is possible to write

$$v_n = v + \int_0^t g \sin \theta dt. \qquad (9.16)$$

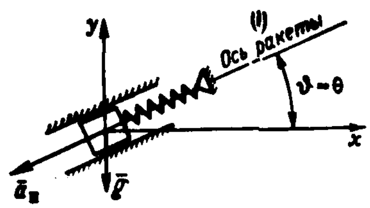

Fig. 9.8. Schematic of accelerometer for measuring the pseudoacceleration of rocket.

Key: (1). Axis of rocket.

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Thus, pseudovelocity (the apparent velocity) at the moment of time t differs from true airspeed to value $\int_0^t g \sin \theta dt$. If ballistic missile flies accurately according to the program (at each moment of time pitch angle corresponds to programmed value), then in the value of pseudovelocity it is possible sufficiently accurately to judge the value of true airspeed.

At the end of the powered flight trajectory of rocket, we have

$$v_{и.к} = v_к + \int_0^t g \sin \theta dt. \quad (9.17)$$

Consequently, during the sufficiently precise motion of

ballistic missile along pitch angle to any value of true airspeed ($v_к$) at the end of powered flight trajectory corresponds the completely specific value of pseudovelocity ($v_{п.к}$). In this case, the engine cutoff of the rocket must be conducted in accordance with (9.9) at the torque/moment when

$$v_{п.д}-v_{п.к}=0, \qquad (9.18)$$

where $v_{п.д}$ - value of pseudovelocity at current point in the trajectory, obtained on board rocket (steering function);

$v_{п.к}$ - value of pseudovelocity, designed previously for this flight trajectory and corresponding to the necessary velocity ($v_к$) in the cutoff point.

In the process of flight on board rocket, continuously is determined the instantaneous value of the apparent velocity and it is compared with its computed value for the end/lead of powered flight trajectory. Upon reaching of their equality, is supplied the command/crew to engine cutoff. The accuracy of the operation of the on-board inertial control system with one integrating accelerometer can be raised, if with the aid of gyroscope-stabilized platform the axis of the sensitivity of accelerometer to direct along the axis Oξ is perpendicular to the velocity vector of rocket $\overline{v}_c$ in the point of the passage of the calculated trajectory through target/purpose [23].

Then the projections of speed on new coordinate axes $O\eta$ and $O\xi$ (Fig. 9.9), turned relative to the axes starting coordinate system to angle of 90° - $|\theta_C|$, are equal to

$$v_\xi = v_x \sin|\theta_C| + v_y \cos|\theta_C|; \quad v_\eta = -v_x \cos|\theta_C| + v_y \sin|\theta_C|. \tag{9.19}$$

It is possible also to write:

$$\xi = x\sin|\theta_C| + y\cos|\theta_C|, \quad \eta = -x\cos|\theta_C| + y\sin|\theta_C|. \tag{9.20}$$

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During the measurement of accelerations along axes $O\xi$ and $O\eta$ it is possible to write

$$L = F(v_{\xi\kappa}, v_{\eta\kappa}, \xi_\kappa, \eta_\kappa) \tag{9.21}$$

and, correspondingly,

$$\delta L = \frac{\partial L}{\partial v_{\xi\kappa}}\delta v_\xi + \frac{\partial L}{\partial v_{\eta\kappa}}\delta v_\eta + \frac{\partial L}{\partial \xi_\kappa}\delta\xi + \frac{\partial L}{\partial \eta_\kappa}\delta\eta. \tag{9.22}$$

According to the investigations of work [23] with an accuracy to low second order, it is possible to assume

$$\frac{\partial L}{\partial \eta_\kappa} \approx \frac{\partial L}{\partial v_{\eta\kappa}} \approx 0, \tag{9.23}$$

and then

$$\delta L = \frac{\partial L}{\partial \xi_\kappa}\delta\xi + \frac{\partial L}{\partial v_{\xi\kappa}}\delta v_\xi. \tag{9.24}$$

Consequently, to this method of control corresponds the linear binomial steering function, which has the form

$$\frac{\partial L}{\partial \xi_{\kappa}}\xi(t)+\frac{\partial L}{\partial v_{\xi\kappa}}v_{\xi}(t)=\frac{\partial L}{\partial \xi_{\kappa}}\xi_{\kappa}+\frac{\partial L}{\partial v_{\xi\kappa}}v_{\xi\kappa}. \quad (9.25)$$

It is obvious, engine must be switched off upon reaching of equality left and right side (9.25).

Is known the application/use of an autonomous inertial control system with two integrating accelerometers. In this case on board rocket, is establish/installed the gyroscope-stabilized platform, which is oriented along the axes of the starting coordinate system and does not change its position during rocket flight. On gyroscope-stabilized platform are establish/installed two accelerometers. The axis of sensitivity of one of them is directed in parallel to axis $0x_3$, another - in parallel to axis $0y_3$. The schematic of gyroscope-stabilized platform with accelerometers is shown on Fig. 9.10.

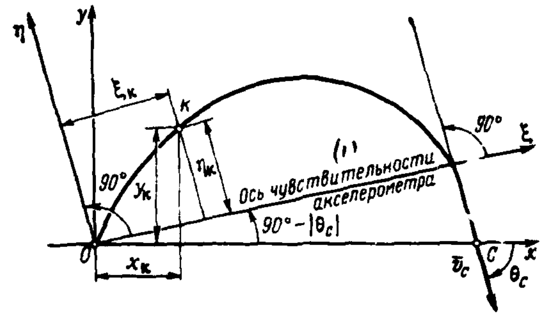

Fig. 9.9. Schematic of coordinates with the inclined location of the axis of the sensitivity of integrating accelerometer.

Key: (1). Axis of the sensitivity of accelerometer.

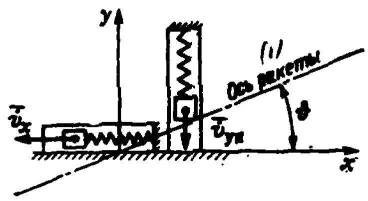

Fig. 9.10. Schematic of gyroscope-stabilized platform with two accelerometers.

Key: (1). Axis of rocket.

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The accelerometers indicated will, correspondingly, measure the comprising accelerations

$$\dot{v}_{x_3} \text{ and } \dot{v}_{y_3\text{н}}.$$

As a result of integrating of dependences $\dot{v}_{x_3}(t)$, $\dot{v}_{y_{3п}}(t)$ and the geometric summation of the projections of the velocity of rocket $v_{x_3}(t)$, $v_{y_{3п}}(t)$ in computer, we determine the value of the apparent velocity

$$v_{п}(t)=\sqrt{v_{x_3}^2+v_{y_{3п}}^2}. \qquad (9.26)$$

The obtained instantaneous value of velocity is compared with value $v_{п.к}$, and upon reaching of equality occurs engine cutoff.

For the increase of the accuracy of the operation of the inertial control system, which contains two integrating accelerometers, direction of the axes of their sensitivity they must be selected on the basis of detailed ballistic investigation [23]. The direction of the axes of sensitivity depends on the flight program, motion characteristics and form of trajectory. For rockets with considerable firing distance, must be considered the rotational effect of the Earth and its asphericity.

§3. Programmed equation of pitch angle.

The programmed equation of pitch angle usually establishes the dependence of tilt angle on time or the over-all payload ratio of

rocket $\mu=\frac{m}{m_0}$. The form of programmed equation depends on the designation/purpose of the rocket, its design-technical parameters and form of start (vertical, inclined, etc.). In programmed equation must be taken into account the character of the expected trajectory - its form, distribution of velocity and accelerations (normal and tangential) according to time and according to the path of the center of mass of rocket. It is necessary to consider the special feature/peculiarities of control - control only on active section or in an entire trajectory, the possible structural/design realization of the control system - autonomous inertial, command, etc.

With the correctly comprised program and conformity to its possibilities of the system of control (limitedness of the deflection of controls) of the characteristic of angular action $\vartheta(t)$, $\dot{\vartheta}(t)$ and $\ddot{\vartheta}(t)$ they must smoothly change in the process of flight so that the moments of control forces and the longitudinal aerodynamic torque/moment M_z would be located within the assigned limits, determined by the rigidity of construction and by the strength of rocket (i.e. and by its weight).

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The degree of curvature of the trajectory, adjustable within reasonable limits for this type of rockets, manifests itself, in

essence, the normal (transverse) accelerations of rocket and barely affects the tangential (axial) accelerations. Normal accelerations are determined by the value of pitching moment.

On (2.86)

$$M_z = qSlm_z^\alpha\alpha \approx qS(l_{ц.м} - l_{ц.д})c_y^\alpha\alpha.$$

Distance between centers of masses and the center of pressure $l_{ц.м} - l_{ц.д}$ barely depends on flight program; also are little affected with flight program (for a defined class of rockets) velocity head q and coefficients m_z^α and c_y^α. The determining value on M_z has an angle of attack $\alpha = \vartheta - \theta$.

Therefore in order to decrease M_z and, therefore, to maximally facilitate the construction of rocket, it is necessary to approach possibly smaller α, especially at vital importance of q.

The trajectory phases with large q must be passed with zero (or minimum, close to zero) angles of attack.

The form of dependence ϑ(t) or α(t) must consider the effectiveness of the work of controls. In the area of transonic speed (M = 0.8-1.2) occurs an abrupt change in the aerodynamic coefficients m_z^α and c_y^α, which negatively affects the operation of the control system. For decreasing the effect of these changes m_z^α and c_y^α it is

necessary that the rocket would pass the nuster domain M indicated with zero angles of attack [2].

The noted conditions satisfies well following program for an angle α during motion in the dense layers of the atmosphere:

$$\alpha=\bar{\alpha}k(k-2), \qquad (9.27)$$

where

$$k=2e^{a(t_1-t)};$$

$\bar{\alpha}$ - a limiting value of angle of attack on the subsonic trajectory phase;

t_1 - time from start to the end/lead of the vertical phase of flight;

a - certain constant coefficient, usually selected for this class of rockets so as upon reaching of numbers Maxa M=0.7-0.8, to obtain angle of attack $\alpha\approx0$.

Important requirement for programmed equation is the need of providing the conduct of firing for the complete range of distances from $x_{c\,min}$ to $x_{c\,max}$. If range $x_{c\,min}-x_{c\,max}$ is narrow or distances x_c are small, then program $\theta(t)$ can be one; with wide-band range, it is necessary to have several programs (it is desirable - possibly less).

The instrument realization of the control system must allow the rapid connection/inclusion of the necessary for the assigned distance program in the process of the preparation of firing.

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During the compilation of the program equation, it is assumed that the axis of rocket is ideally fulfilled outlined by program angular rotations. Typical curve/graphs $\vartheta_{np}(t)$ and $\alpha_{np}(t)$ for a powered flight trajectory with the vertical launching of single-stage long range ballistic missile are represented in Fig. 9.11 [2].

On first section $(0<t<t_1)$ of rocket flight $\vartheta_{np}=90°=\text{const}$. On the second section of program $(t_1<t<t_3)$ the pitch angle smoothly changes from 90° to the value $\vartheta_{np.к}$, which corresponds to the assigned distance x_C, moreover angle α here changes accordingly (9.27); time t_2 characterizes the torque/moment of achieving the numbers M=0.7-0.8. The third section of program $(t_3<t<t_k)$ - this is the section of the action of the rocket in the relatively rarefied layers of the atmosphere with small q, when it is possible to take $\alpha > 0$, necessary for providing the program of action with $\vartheta_{np.к}=\text{const}$. In time interval $t_3'-t_3''$ occurs the transition of program to straight portions with the angles $\vartheta_{кi}$, which ensure the range of distances from $x_{C\,min}$ to $x_{C\,max}$.

Program $\theta_{пр}(t)$ for the curvilinear trajectory phase is described well by the equation

$$\theta_{пр}=90^{\circ}+(90^{\circ}-\theta_{пр.к})(\bar{t}^{2}-2\bar{t}), \qquad (9.28)$$

where $\bar{t}=\frac{t-t_{1}}{t_{11}-t_{1}}$ - relative flight time of rocket on the second section of program whose value changes from zero to unity;

t_1 - flight time of rocket from start to the end/lead of the first section of program;

t_{11} - flight time of rocket from start to the end/lead of the second section;

t - current time of rocket flight, calculated off the torque/moment of start.

For designed calculations the program can be comprised not in the form $\theta(t)$, but as function $\theta(\mu)$. Let to the torque/moment t_1 of the end/lead of the rectilinear vertical flight correspond mass ratio of rocket μ_1, and to the torque/moment of the beginning of rectilinear inclined flight - μ_2. The program of a change of the angle θ in the process of rocket flight on the active curvilinear

trajectory phase can be presented in the form

$$\theta = A(\mu-\mu_0)^2 + B(\mu-\mu_0) + C. \qquad (9.92)$$

For solution would be being concrete/specific/actual, it is necessary to assign values μ_1 and μ_2. It is possible to accept μ_1=0.95 and to count that the curvilinear section of program concludes with μ_2=0.4-0.5. Angle $\theta_к$ must correspond to the angle of maximum range. Taking into account the said program of a change in the angle θ will take form of θ=90° with 1.00>μ>0.95;

$$\theta = 4\left(\frac{\pi}{2}-\theta_к\right)(\mu-0{,}45)^2+\theta_к \ \overset{(1)}{при}\ 0{,}95 \geqslant \mu \geqslant 0{,}45; \qquad (9.30)$$
$$\theta=\theta_к \ \overset{(1)}{при}\ 0{,}45 \geqslant \mu > \mu_к.$$

Key: (1) with.

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For a two-stage rocket the exemplary/approximate program of pitch angle takes the form, presented in Fig. 9.12 [2]. The form of the flight trajectory in the work of first-stage engine is selected according to the program of obtaining maximum angle of attack not more $\bar{\alpha}$ in subsonic range and obtaining α=0 in range of numbers M>0.7-0.8. The programmed equation, which determines the motion of the second stage of rocket, most frequently can be undertaken in the form

$$\theta_{II} = \theta_{II0} - \dot{\theta}(t-t^{m}). \qquad (9.31)$$

where ϑ_{II0} - an initial programmed value of pitch angle for the second step/stage.

Depending on the basic design parameters of the rocket of value ϑ_{II0} and $\dot{\vartheta}$ for providing the maximum of distance, they can take different values.

In a series of the cases when selecting of the most advantageous program, computed values $\dot{\vartheta}_{I\kappa}$ and ϑ_{II0} can not coincide. In this case a change of the derivative $\dot{\vartheta}(t)$ in the work of the second step/stage must provide smooth transition (section t'-t" in Fig. 9.12) from the program, establish/installed for first stage, to the curvilinear section, selected for the second step/stage, during gradual transition to the straight portion, on which is completed the work of second-stage engine. The straight portion of trajectory with the optimum angle of the slope of velocity vector at the end of the work of second-stage engine provides obtaining the distance, close to the greatest.

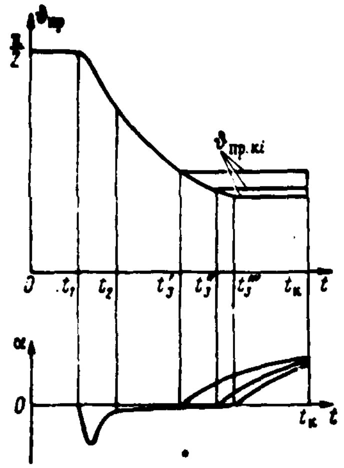

Fig. 9.11.

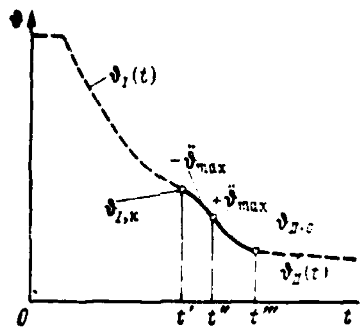

Fig. 9.12.

Fig. 9.11. Flow chart of the program of pitch angle for a single-stage rocket and a change in the angle of attack.

Fig. 9.12. Flow chart of program of pitch angle for two-stage rocket.

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§4. Maneuverability and g-forces.

During completing of the assigned flight program or with missile targeting to the moved target/purpose (i.e. during maneuver accomplishment) change the value and the direction of the velocity of the motion of the center of mass of rocket. In accordance with this under the maneuverability of rockets, is understood the speed of a

change in the flight speed in value and direction. Is estimated the maneuverability of all types of flight vehicles, rockets and projectiles with the aid of the so-called g-forces. G-force, just as velocity, value is vector.

Under the vector of the g-force of rocket $\bar{n}$, is understood the relation to the resultant of all acting on it forces, except gravitational force, to the weight of rocket, i.e.,

$$\bar{n}=\frac{\bar{F}_a+\bar{F}_r}{mg}, \qquad (9.32)$$

where F_a - resultant of all aerodynamic forces;

F_r - resultant of all gas-dynamic forces, including the engine thrust.

The module/modulus of the vector of g-force n is the value of dimensionless, which is made it convenient for comparative evaluations. G-forces are determined by energy resources of rocket, by the possibilities of its aerodynamic configuration, by the control effectiveness and control system as a whole. The total vector of g-force can be determined its those comprise along the axes of the adopted system of coordinates.

Tangential g-force characterizes a change in the velocity in

value, and the g-forces, undertaken along the normal to velocity and called normal, a change in the sense of the vector of the velocity of rocket, i.e., the maneuver of the latter. During the analysis of the conditions, which ensure the maneuverability of the rockets, are examined usually only normal load factors.

Let us establish communication/connection between the form of the completed by rocket maneuver and the g-forces, necessary for its accomplishment.

During the design of the control systems of initial space, is conducting the detailed analysis whose target/purpose - to establish the required forms of trajectories (i.e. the maneuvers), which will allow the designed rocket to satisfy the stated operational requirements.

Figure 9.13 shows the trajectory of approach of rocket for the maneuvering in vertical plane target, constructed for the method of guidance (on pursuit curve) accepted according to the predetermined trajectory of target/purpose and velocity of rocket.

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A similar trajectory, without opening the system of forces, that

caused its form, makes it possible to establish the characteristic which makes it possible to pass to the determination of the acting on rocket forces. This kinematic characteristic of the maneuver of rocket is the radius of curvature of trajectory r.

Figure 9.13 shows that the presented in it trajectory of the approach of rocket for target/purpose is characterized by a gradual increase in the curvature or, that analogously, by the decrease of radius of curvature from value r_0 at initial point in the trajectory to value r_B at the collision point of rocket for target/purpose. In connection with the half-speed coordinate system, it is possible to write following expressions for radii of curvature:

$$r_{y^*}=\frac{dS}{d\theta}=v\frac{dt}{d\theta}; \qquad (9.33)$$

$$r_{z^*}=-\frac{dS}{d\Psi}=-v\frac{dt}{d\Psi}. \qquad (9.34)$$

In this case, it was accepted that $r > 0$ in the case, the code its depicting cut, directed from point in the trajectory toward center of curvature, coincides with positive direction of the corresponding coordinate axis.

Entering expressions (9.33) and (9.34) angular velocities $\frac{d\theta}{dt}$ and $\frac{d\Psi}{dt}$ can be expressed by the appropriate g-forces on the basis of the system of the differential equations, which describe the motion of the center of mass of rocket.

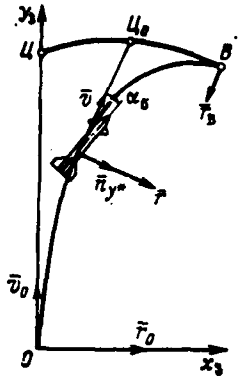

Fig. 9.13.

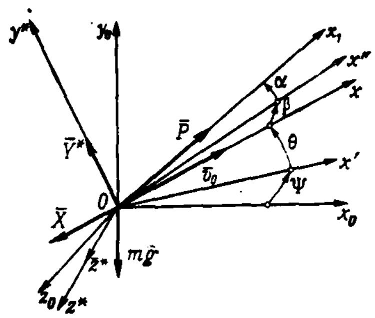

Fig. 9.14.

Fig. 9.13./ Change of the radius of curvature of trajectory during the approach of the guided missile for target/purpose along pursuit trajectory.

Fig. 9.14. Schematic of coordinates during determination of g-forces.

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Let the plane of angle of attack a be vertical, and the plane of angle of slip β - to it is perpendicular; then on Fig. 9.14 it is possible to write the system of equations of the spatial motion of the center of mass of the guided missile in the half-speed coordinate system. In this case, control forces is combined with aerodynamic forces of X, Y*, Z*; the left and right sides of the equations it is subdivided by weight of rocket G=mg.

As a result we will obtain

$$\left.\begin{aligned} \frac{1}{g}\frac{dv}{dt} &= \frac{P\cos\alpha\cos\beta - X}{mg} - \sin\theta; \\ \frac{v}{g}\frac{d\theta}{dt} &= \frac{P\sin\alpha + Y^*}{mg} - \cos\theta; \\ -\frac{v}{g}\cos\theta\frac{d\Psi}{dt} &= \frac{-P\cos\alpha\sin\beta + Z^*}{mg}. \end{aligned}\right| \quad (9.35)$$

From the written equations, according to definition, let us isolate g-forces and it is simplified expressions for them, after replacing as a result of the smallness of the angles

$$\sin\alpha\approx\alpha;\quad \sin\beta\approx\beta;\quad \cos\alpha\approx\cos\beta\approx 1.$$

Then the projections of the vector of g-force on the half-speed coordinate axes are equal to

$$\begin{aligned} n_x &= \frac{P\cos\alpha\cos\beta - X}{mg} \approx \frac{P - X}{mg}; \\ n_{y^*} &= \frac{P\sin\alpha + Y^*}{mg} \approx \frac{P\alpha + Y^*}{mg}; \\ n_{z^*} &= \frac{-P\cos\alpha\sin\beta + Z^*}{mg} \approx \frac{-P\beta + Z^*}{mg}. \end{aligned} \quad (9.36)$$

Inserting the projections of the vector of g-force into system (9.35), let us write

$$n_x = \frac{1}{g}\frac{dv}{dt} + \sin\theta;\quad n_{y^*} = \frac{v}{g}\frac{d\theta}{dt} + \cos\theta;$$
$$n_{z^*} = -\frac{v}{g}\cos\theta\frac{d\Psi}{dt}. \quad (9.37)$$

Hence

$$\frac{d\theta}{dt}=\frac{g}{v}(n_{y^*}-\cos\theta);\quad \frac{d\Psi}{dt}=-\frac{g}{v\cos\theta}n_{z^*}. \qquad (9.38)$$

After substituting with respect (9.38) into equations for radii of curvature (9.33) and (9.34), we will obtain

$$n_{y^*}=\frac{1}{r_{y^*}}\frac{v^2}{g}+\cos\theta, \qquad (9.39)$$
$$n_{z^*}=\frac{1}{r_{z^*}}\frac{v^2\cos\theta}{g}. \qquad (9.40)$$

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The parameters of trajectory v, θ and Ψ determine the value and the sense of the vector of speed, but them derivatives and g-forces characterize the ability of flight vehicle to change value and direction of the flight speed of the center of mass.

Dependences (9.39) and (9.40) show that the rocket can satisfy the maneuver, which is characterized by radius of curvature r, only in such a case, when will be created the necessary g-forces n_{y^*} and n_{z^*}, moreover maneuver with smaller radius of curvature, i.e., sharper, can be realized, other conditions being equal, because of an increase in the normal load factors. During the investigation of g-forces, determine the so-called required g-forces, necessary for obtaining of the trajectories of the assigned form, and the available

g-forces, which can be actually obtained by rocket. G-forces required and arrange/located are compared between themselves.

Rocket can move over programmed trajectory or over the trajectory of the guidance only when the necessary for its obtaining required g-forces will be less or (in the extreme case) equal to the available g-forces which can be created by the rocket

$$n_{y \cdot n} \leqslant n_{y \cdot p}; \quad n_{z \cdot n} \leqslant n_{z \cdot p}. \qquad (9.41)$$

But given conditions (9.41) it is not possible to consider sufficient. The motion of rockets under actual conditions is always accompanied different kind by the short-term or prolonged disturbance/perturbations which deflect/divert rocket from calculated trajectory. For the stabilization of the motion of rocket, its control system must, as already mentioned that to possess possibility to parry these disturbance/perturbations.

Consequently, during setting of communication/connection between the required and available g-forces it is necessary to compulsorily provide for the reserve of the normal load factors $n_{y \cdot s}$ and $n_{z \cdot s}$, which is necessary for realizing assigned rocket flight under the effect on of its random disturbances.

Taking into account this condition (9.41) they take the form

$$n_{y \cdot n} + n_{y \cdot s} \leqslant n_{y \cdot p}; \quad n_{z \cdot n} + n_{z \cdot s} \leqslant n_{z \cdot p}. \qquad (9.42)$$

It is necessary to bear in mind, that setting the maximum sizes of the g-force, both normal and axial, must be conducted taking into account the careful analysis of the effect of g-forces on the work of the onboard measuring and controlling equipment, strength of housing and assemblies of rocket.

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Chapter X.

STUDY OF TRAJECTORIES AND THE CONCEPT OF THE OPTIMUM SOLUTIONS OF THE PROBLEMS OF EXTERNAL BALLISTICS.

The problems of extra-ballistic investigations are very many-sided and depend on the designation/purpose of rocket or projectile and concrete/specific/actual posing of the question. The most frequently encountered investigations can be combined into following basic groups.

1. Investigations, dedicated to setting common/general/total properties of missile trajectories and projectiles in air and vacuum.

2. Investigations, dedicated to finding optimum solutions of problems of theory of rocket flight and projectiles.

3. Investigations of conditions of three-dimensional/space rendezvous of two bodies, i.e., conditions of interception of target/purpose (aircraft, rocket) by zenith or aircraft rockets or defeat of it by artillery shells.

4. Considerable place in ballistic investigations occupy questions, connected with determining of characteristics of scattering trajectories and accuracy of firing (these investigations will examine we in chapter XIII).

Almost for all named groups of extra-ballistic investigations, it is necessary to apply the numerical methods of solutions with the aid of appropriate computer technology; only the part of the simple problems can be solved analytically.

§1. General properties of missile trajectories and of the projectiles of constant mass moving in the atmosphere.

Let us examine this question in connection with the unguided rockets and the projectiles of class "surface - surface". The most important motion characteristics it is possible to consider the form of trajectory, the velocity of the center of mass and g-force.

The velocity of the center of mass of body depends on the acting on it forces and with the constant/invariable composition of the system of forces changes smoothly.

Bigures 10.1 shows the curve/graphs of a change in the rate of motion along the trajectory of two types of rockets - wingless (curve 1) and winged missile (curve 2).

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As can be seen from figure, the velocity of the motion of wingless rocket by active section sharply grow/rises, reaching in its end/lead of the greatest value; then on the free-flight phase (when the system of forces changed) it decreases with the approach/approximation of rocket to peak of the trajectory; after peak of the trajectory, the velocity again grow/rises, and at the end of the trajectory, it again sharply decreases as a result of rocket braking by the dense layers of the atmosphere.

For winged missiles the velocity rapidly grow/rises by the launching phase, and on march it is little affected, but all the same it does not remain constant as a result of a change of the mass of the rocket because of burnout in the operation of engine, nonuniformity of the feed of fuel/propellant into the combustion chamber, etc. During the motion of the rocket in the vacuum, its velocity in the engine operation will continuously grow/rise. In accordance with formula K. E. Tsiolkovskiy the limiting value of velocity is determined by the weight of rocket, by the fuel reserves

and by his energy characteristics (7.146).

Let us examine the properties of the trajectories of the projectiles of constant mass, driving/moving in the atmosphere, characteristic for ground-based artillery pieces.

Let us take the first equation of system (5.8), let us substitute in it value of E from (5.5) and then

$$\frac{du}{dx} = -cH(y)G(v).$$

Since c, H(y) and G(v) are positive, then

$$\frac{du}{dx} < 0 \qquad (10.1)$$

and, therefore, the horizontal projection of speed decreases along trajectory.

Let us take the second equation of system (5.8) $\frac{dp}{dx} = -\frac{g}{u^2}$ and replace in it $dx = \frac{dy}{p}$; then $pdp = -\frac{g}{u^2}dy$. Let us integrate this equation twice; one time on the ascending branch of trajectory from point y_{ni} to peak of the trajectory S and for the second time - on the descending branch of trajectory from the apex/vertex that of the point $y_{ni} = y_{ni}$:

$$\int_{p_{ni}}^{0} pdp = -g\int_{y_{ni}}^{y_S} \frac{dy}{u_n^2}; \quad \int_{0}^{p_{ni}} pdp = -g\int_{y_S}^{y_{ni}} \frac{dy}{u_n^2}.$$

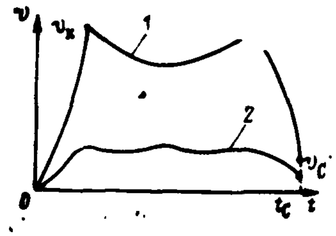

Fig. 10.1. The circuit of a change in the rate of motion along trajectory for: 1 - wingless ballistic missile; 2 - winged missile.

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After integration we can write

$$\frac{p_{\text{в}\,i}^2}{2}=g\int\limits_{y_{\text{в}\,i}}^{y_S}\frac{dy}{u_{\text{в}}^2};\quad \frac{p_{\text{н}\,i}^2}{2}=g\int\limits_{y_{\text{н}\,i}}^{y_S}\frac{dy}{u_{\text{н}}^2}.$$

With the decreasing horizontal projection of speed for any pair of points, which are located on one height/altitude y_i on the ascending and descending branches of trajectory, $u_{\text{в}}>u_{\text{н}}$, therefore

$$\int\limits_{y_{\text{в}\,i}}^{y_S}\frac{dy}{u_{\text{в}}^2}<\int\limits_{y_{\text{н}\,i}}^{y_S}\frac{dy}{u_{\text{н}}^2}$$

and, therefore,

$$p_{\text{в}\,i}<|p_{\text{н}\,i}| \text{ и } \theta_{\text{в}\,i}<\theta_{\text{н}\,i}. \qquad (10.2)$$

Thus, the descending branch of trajectory than steeper ascending and angle of incidence is more angle of departure

$$|\theta_C|>\theta_0. \qquad (10.3)$$

Let us find the change in the kinetic energy of the projectile of constant mass, which corresponds to its displacement/movement from point $y_{в i}$ on the upward leg into point $y_{н i}=y_{в i}$ on the descending branch of trajectory. At the identical height/altitude of the selected points, a change in the kinetic energies is determined by the action only of air resistance R.

Then

$$\frac{Q}{2g}\left(v_{н i}^{2}-v_{в i}^{2}\right)=\int_{S_1}^{S_2} R\cos(\widehat{Rv})\,dS.$$

Since $\cos(\widehat{Rv})=-1$, that $v_{н i}^{2}-v_{в i}^{2}<0$ and, therefore,

$$v_{в i}>v_{н i}. \qquad (10.4)$$

Thus, at points in the trajectory with identical height/altitudes $y_{в i}=y_{н i}$ the velocity of projectile on the descending branch is smaller than the velocity on the upward leg and the velocity of projectile in impact point of the lesser velocity in release point

$$v_C<v_0. \qquad (10.5)$$

Let us examine velocity change along trajectory for the

projectiles of constant mass. In accordance with the first equation of system (5.3) it is possible to write

$$\frac{dv}{dt} = -\frac{X}{m} - g\sin\theta.$$

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Since all values, entering the right side of the last/latter equation, are positive, then, obviously, in the beginning of trajectory the velocity decreases, since $dv/dt<0$. The minimum of velocity will be under condition $dv/dt=0$, when is correct the equality

$$\frac{X}{m} = -g\sin\theta. \qquad (10.6)$$

It is obvious, equality (10.6) can be fulfilled only on the descending branch of the trajectory where $\sin\theta < 0$, i.e., the minimum of velocity will be after peak of the trajectory; after this the velocity of projectile will begin to grow/rise, since will occur inequality $g\sin\theta>X/m$. In the lower layers of the atmosphere, at high air densities, the velocity again will begin to fall (Fig. 10.2). The presented curve/graph is characteristic for high-angle tajectories. For low trajectories, depending on initial conditions, the maximum on the right side of the curve/graph can not be formed to impact point; with direct fire, the velocity of projectile at trajectory continuously decreases.

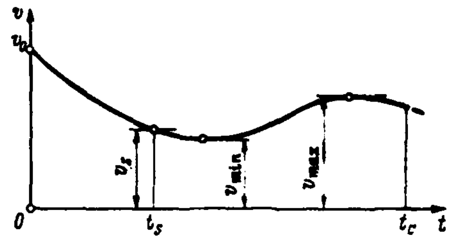

Fig. 10.2. Curve/graph of a change in the velocity of the projectile of constant mass during its motion in the atmosphere.

§2. General properties of the trajectories of the projectiles of constant mass in a vacuum.

Let us give the properties of the trajectory of the motion of projectile, obtained on the basis of parabolic theory (chapter VII §1).

The horizontal projection of speed is constant along trajectory, i.e.,

$$u = v_0 \cos \theta_0 = \text{const}. \qquad (10.7)$$

Parabolic trajectory is symmetrical in form and with respect to velocity change along trajectory relative to ordinate y_s, the passing through the half complete distance. Angle of incidence is equal to angle of departure, final velocity is equal to the initial velocity

$$|\theta_C| = \theta_0; \quad v_C = v_0. \qquad (10.8)$$

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Let us present (7.1) in the form

$$y=xp_0-\frac{gx^2}{2v_0^2}(1+p_0^2), \qquad (10.9)$$

where $p_0=tg\theta_0$ - parameter of family of curves, described by equation (10.9).

We will obtain equation by envelope family of curves. For this, let us take derivative of expression (10.9) from parameter p_0

$$x-\frac{gx^2}{v_0^2}p_0=0$$

let us find that $p_0=\frac{v_0^2}{gx}$. Substituting p_0 in equation (10.9), we will obtain the equation of envelope

$$y=\frac{v_0^2}{2g}-\frac{gx^2}{2v_0^2}. \qquad (10.10)$$

Last/latter equation contains argument x only to positive even degree (x^2) and, therefore, that envelopes - the parabola, symmetrical relative to y axis.

If point (target/purpose) is located out of envelope, i.e., if

$$y>\frac{v_0^2}{2g}-\frac{gx^2}{2v_0^2},$$

then this point is struck be it cannot. Therefore envelope (10.10) they call the parabola of safety. The solution of equation (10.9) relative to $p_0=tg\theta_0$ with

$$y<\frac{v_0^2}{2g}-\frac{gx^2}{2v_0^2}$$

has two real roots, i.e., target/purpose, contained the parabola of safety, can be struck during two adjustments of sight. Target/purpose, which is located on the parabola of safety, can be struck during one adjustment of sight [59]. Last/latter two conclusion/derivations are valid also for enveloping the families of the trajectories in air, which have v_0=const and parameter $p_0=tg\theta_0$.

Let us find a decrease in the trajectory under line of elevation. Let us substitute (7.4) In second term of right side (7.1) and after conversion we will obtain:

$$\frac{gt^2}{2}=x\,\mathrm{tg}\,\theta_0-y. \qquad (10.11)$$

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This there will be lowering the trajectory under line of elevation (Fig. 10.3). In turn,

$$oa=\frac{x}{\cos\theta_0}=v_0t. \qquad (10.12)$$

It is obvious, with firing with one and the same velocity at any angle of elevation the given moment of time it will answer one and the same lowering under line of elevation, which does not depend on angle of elevation. The lines, which connect the points of lowering the trajectories under lines of shots and which correspond to identical flight times, are called isochrones. Within the framework of the parabolic theory of isochrone, will represent by themselves the circumferences, carried out by a radius v_0t from the center, omitted from the origin of coordinates along the axis Oy to value $gt^2/2$. The isochrones of the motion of the projectiles of different designation/purpose in air can be constructed according to the numerical calculations of the family of trajectories.

Certain interest for the theory of corrections represents the

dependence, which determines relative transit time by the projectile of the cuts of the ascending and descending branches of trajectory, limited by ordinates y_i and y_{i-1} (Fig. 10.4).

Let us elevate formula (7.6) into square and is piecemeal divided it into (7.8). As a result we will obtain simple formula for calculation of total flying time of projectile along the trajectory

$$t_C=\sqrt{\frac{8}{g}y_S}. \qquad (10.13)$$

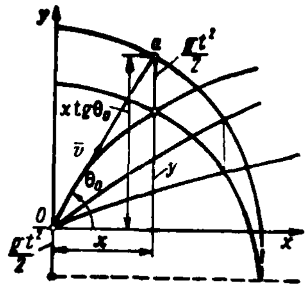

Fig. 10.3. Lowering the trajectory under line of elevation.

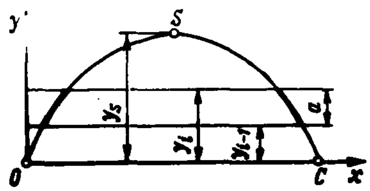

Fig. 10.4. Dividing circuit of trajectory into layers to derivation of dependence for "weight of layer".

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Transit time by the projectile of a layer of the space, limited by ordinates y_i and y_{i-1}, can be found as difference of the missile flight time in trajectory, that has initial ordinates y_i and y_{i-1}.

$$t_{C(i-1)}-t_{Ci}=\sqrt{\frac{8}{g}[(y_S-y_{i-1})}-\sqrt{\frac{8}{g}(y_S-y_i)}.$$

The relative retention time of projectile in a layer is equal

$$\frac{t_{C(i-1)}-t_{Ci}}{t_C}=\sqrt{\frac{(y_S-y_{i-1})}{y_S}}-\sqrt{\frac{(y_S-y_i)}{y_S}}. \quad (10.14)$$

For integer n of layers of the equal thickness a, to which is broken the trajectory, it is possible to write

$$\frac{y_{i-1}}{y_S}=\frac{i-1}{n} \text{ (1) } \frac{y_i}{y_S}=\frac{i}{n}. \qquad (10.15)$$

Key: (1). and.

Then the conditional "weight of a layer", proportional to the retention time of projectile in the i layer, is equal to

$$q_i=\frac{t_{C(i-1)}-t_{Ci}}{t_C}=\frac{\sqrt{n-i+1}-\sqrt{n-i}}{\sqrt{n}}. \qquad (10.16)$$

On last/latter formula can be comprised the table with entries n and i. this table or formula (10.16) can be used for calculating the weights of layers during the approximate (estimated) determination of the effect of meteorological factors on the results of firing.

Within the framework of the elliptical theory, which examines the motion of projectile in central gravitational field without the account of air resistance, the trajectory of projectile is described by the equation of ellipse (7.14). Form trajectory is determined by

eccentricity e. In §2 chapters VII, are examined the trajectories, which correspond to the cases

$$e=0;\ 0<e<1;\ e=1;\ e>1.$$

Let us give several common/general/total properties of the elliptical trajectory, which is closed on the surface of the Earth.

The analysis of equation (7.14) shows that the elliptical trajectory has an axis of symmetry which coincides with radius-vector $\bar{r}_{max}=\bar{r}_S$, turned relative to the beginning of trajectory to vectorial angle φ_S. Consequently, in the points, arrange/located on the ascending and descending branches of trajectory is symmetrical relative to radius-vector $\bar{r}_S$ (i.e. the focal axis of ellipse), will be identical radii r and angles of arrival (on module/modulus).

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Substituting in (7.16) identical values of r and θ, that correspond to the points of the ascending and descending branches of trajectory, we can be convinced of the fact that the velocities in these points are also equal on module/modulus, i.e., elliptical trajectory is symmetrical relatively $\bar{r}_S$ not only in form, but also according to the distribution of velocity along trajectory. The value

of a radius $r_s = r_{max}$ and corresponding to it angle φ_s are determined formulas (7.14): if we take $\varphi = \varphi_s$, then

$$r_S = r_{max} = \frac{p}{1-e};\qquad (10.17)$$

if we take $\varphi = \varphi_n = 0$ and $r = r_n$, that

$$\varphi_S = \arccos\frac{r_n - p}{e r_n}.\qquad (10.18)$$

§3. Kinematic methods of the study of the trajectories of guidance.

In the theory of flight to the kinematic methods of study, it is accepted to relate the methods, which examine the motion of the center of the masses of flight vehicle without the explicit account of the acting on it forces on the assumption that the velocity of its center of mass in the function of time is known. Most frequently kinematic methods are applied during the study of the motion of the guided missiles, which are aimed to moving targets or which are aimed to target/purpose from mobile guidance stations.

We investigate the possible conditions of the encounter of the rocket with target with the different methods of guidance. In the general case for providing the rendezvous, distance r between the rocket and the target/purpose must decreases, but this means that must be fulfilled the equality

$$\frac{dr}{dt} < 0.\qquad (10.19)$$

For an example let us examine the case of guidance with fixed-lead angle $\alpha_p=\alpha_{p0}=\text{const}$ and will proceed from formula (7.91). For providing inequality (10.19) it is necessary to have

$$p\cos\alpha_{p0}>\cos\alpha_{u}.$$

Utilizing (7.88) and transforming, we will obtain

$$p^2(1-\sin^2\alpha_{p0})>(1-p^2\sin^2\alpha_{p0}). \qquad (10.20)$$

Consequently, with the method of guidance in question for providing the encounter of rocket with target it is necessary to have p > 1, i.e., the velocity of rocket must be more than target speed.

The normal acceleration of the rocket with the method of guidance in question is determined by formula (7.98). For providing the rendezvous of rocket for target/purpose $\alpha_{\text{ц}}$ must approach $\alpha_{\text{цв}}$. Then the acceleration limit will be equal

$$\left.\begin{array}{lll} \text{при } k<2 & \lim a_{np}=0; \\ \text{при } k>2 & \lim a_{np}=\infty; \\ \text{при } k=2 & 0<\lim a_{np}<\infty. \end{array}\right\} \quad (10.21)$$

Key: (1). with.

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In practice, obviously, can be used only case k < 2. The analysis of formula (7.98) gives the range of the possible values p depending on a_{p0} (Fig. 10.5). As can be seen from figure, the range of the theoretically possible conditions of the rendezvous (it is cross-hatched) with fixed-lead homing is extremely limited.

For the investigation of the possibilities of parallel approach method, let us conduct the kinematic investigations of the conditions of rendezvous at the constant velocities of the action of target/purpose and rocket (p = const).

For case of $p>1$ rectilinearly driving/moving target/purpose can be intercepted in any relative attitude of rocket and target/purpose during time interval Δt since at angles a_p and a_u is

placed only the limitation of ideal advance/prevention (Fig. 10.6a). In the case of $p < 1$ range of the positions of the rockets, with which is feasible the interception of target/purpose, considerably it is reduced. For providing the interception, must be fulfilled the equality $\Delta t_i = \frac{l_{pi}}{v_{pi}} = \frac{l_{ui}}{v_{ui}}$, however, in addition to this, the rocket before the interception must be located in the space, limited by the cone whose angle μ = arc sin p (Fig. 10.6b).

If in initial position rocket is located on the surface of cone with angle μ, then by the only direction, which ensures interception, it will be the direction of motion, perpendicular generatrix of cone. If rocket is located within cone, then possible different initial lead angles α_{p1} and to α_{p2}, moreover the extreme trajectories of the motion of rocket $\overline{O_2A}$ and $\overline{O_2B}$ will be rectilinear. The trajectories of the motion of rocket, which lie between O_2A and O_2B, for providing the rendezvous must be curvilinear. With the maneuvering of target/purpose, the range, in which is feasible its interception, still decreases. This question requires additional study.

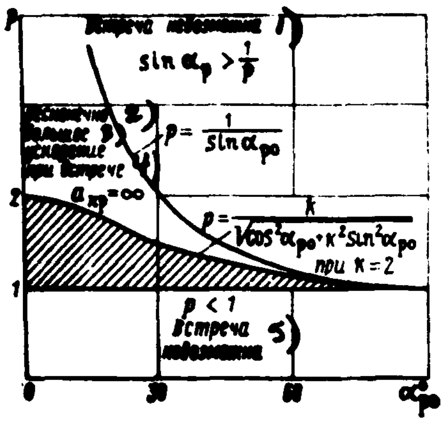

Fig. 10.5. Dependence of the relative velocity cf rocket on fixed-lead angle. The shaded range ccrresponds to the conditions of the encounter of the rocket with target with the finite quantity of normal accelerations.

Key: (1). Rendezvous is impossible. (2). It is infinite. (3). large. (4). upon rendezvous. (5). Rendezvous is impossible.

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Communication/connection between the normal accelerations of rocket and target/purpose is set by formula (7.106). It is obvious that for the case $p > 1$ normal accelerations of rocket will be less than the normal accelerations of target/purpose.

For case of p < 1, if the rocket before the approach is found on the boundary of the region of possible interception - on by generatrix cone with angle μ, we have

$$\alpha_p = \frac{\pi}{2}, \quad \cos\alpha_p = 0 \quad \text{и}^{(1)} \text{ из } (7.105)$$

$$\frac{a_{n\,p}}{a_{n\,u}} = \infty.$$

Key: (1). and from.

Obviously that this case cannot be used. If we designate the ratio of accelerations that is acceptable in practice through

$$\zeta = \frac{a_{n\,p}}{a_{n\,u}},$$

the initial angle α_n must be equal to

$$\alpha_{n0} = \arcsin\sqrt{\frac{p^2(\zeta^2-1)}{\zeta^2-p^2}}. \qquad (10.22)$$

It is obvious that in this case $\alpha_{n0} < \mu$, a this means that the range of the rational positions of the rocket before its approach for target/purpose is less than the range, determined from the condition of the possibility of interception.

We investigate the conditions of the encounter of the rocket with target with pursuit guidance. From formula (7.107) it is evident that for providing the encounter of rocket with target it is necessary to observe inequality $p > \cos\alpha_n$; with this dr/dt < 0 and value r in the process of guidance will decrease.

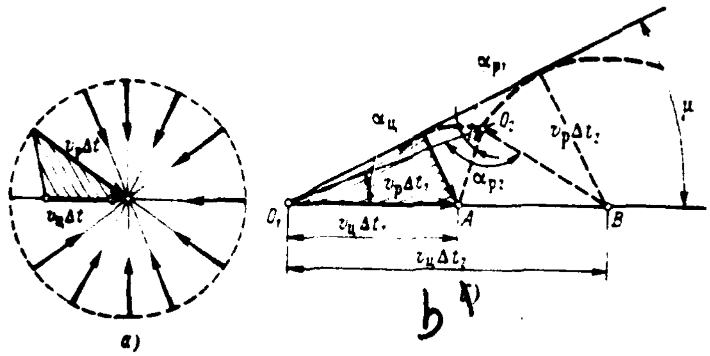

Fig. 10.6. Schematic of the interception of the rectilinearly driving/moving target/purpose with p = const and parallel method of approach.

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For the driven out target/purpose from (7.112) it is evident that when $\sin\alpha_\text{ц}\to 0$ the distance between the rocket and the target/purpose also vanishes. Is obvious, r = 0 when $\sin\alpha_\text{ц}=0$.

For the target/purpose, which flies towards, on (7.116) value r decreases with tendency $\alpha_\text{ц}$ toward π. When $\alpha_\text{ц}=\pi$ let us have r = 0. Let us determine the limits of the possible normal accelerations of the rocket during its approach for purpose according to pursuit curve. For the driven out target/purpose, on the basis of (7.121), with 1 < p < 2 we will obtain

$$\lim_{\gamma\to 0} a_{np} = -\frac{v_p^2}{p\cdot k}\lim_{\gamma\to 0}(\sin\gamma)^{2-p}(1+\cos\gamma)^p = 0,$$

with p = 2

$$\lim_{\gamma\to 0} a_{np} = -\frac{v_p^2}{p\cdot k}\lim_{\gamma\to 0}(1+\cos\gamma)^p = -\frac{4v_p^2}{p\cdot k};$$

with p > 2

$$\lim_{\gamma\to 0} a_{np} = -\frac{v_p^2}{p\cdot k}\lim_{\gamma\to 0}\frac{(1+\cos\gamma)^p}{(\sin\gamma)^{p-2}} = -\infty.$$

For the target/purpose, which flies towards, on the basis of (7.122) it is possible to obtain:

при (1) $1<p<2$ $\lim_{\gamma\to 0} a_{np}=0,$

при (1) $p=2$ $\lim_{\gamma\to 0} a_{np}=-\frac{4v_p^2}{pk'};$

при (1) $p>2$ $\lim_{\gamma\to 0} a_{np}=-\infty.$

Key: (1). with.

Since the real rocket with p > 2 cannot move with infinite normal accelerations, then during approach for target/purpose it will go not along calculated trajectory and can fly wide of the mark. Thus, for providing the reliability of rendezvous with ideal pursuit guidance one should have the condition

$$2>p>1. \qquad (10.23)$$

If we by firing regulations exclude the possibility of rough maneuvers of rocket, then also with p > 2 it is possible to strike target/purpose with the acceptable sizes of the g-force.

We investigate guidance according to the method of coincidence.

On the basis of formula (7.139), let us make several common/general/total derivations. During the approach of rocket for target/purpose r_p it approaches r_n and upon rendezvous $r_p=r_n$, therefore

$$\left(\frac{y_n p}{\sin^2\gamma}\right)^2-r_p^2=\left(r_n\frac{p}{\sin\gamma}\right)^2-r_p^2=r_n^2\left(\frac{p^2}{\sin^2\gamma}-1\right). \quad (10.24)$$

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In the case of p < 1 when at some γ possibly infinite normal acceleration of rocket. Thus, during the design of rocket it is necessary to observe condition p > 1. At the same time a_{np} it is directly proportional to the square of target speed, grows with increase in p and at high values v_n and p can render/show not

admitted. Furthermore, the normal acceleration of the rocket inversely proportional to distance of target/purpose $\frac{v_2}{\sin\gamma}=r_{\text{u}}$ and with low firing distances can also render/show exaggerated. the enumerated factors limit the applicability of guidance method on three-point curved. As a rule, it is applied for missile targeting to comparatively low-speed targets, for example - for the antitank missiles (projectiles), controlled from fixed and mobile guidance stations. The instrument realization of three-point guidance method are usually the command methods of control with the aid of radars or wire communication lines.

The tendency to utilize the best properties also of the command methods of control and homing/self-induction led to the appearance of the combined methods. The trajectory of the motion of the rocket with the combined method of control consists of the separate cuts, which correspond to the method of guidance accepted for each of them. For example, command predicted point guidance can be combined with homing/self-induction during the constant-bearing approach. For the increase of the accuracy of guidance desirably more possible smooth conjugation of the individual sections of trajectories not only in angle 6, but also in angular velocity $\frac{d\theta}{dt}$.

The selection of one or the other method of control and method of guidance to target/purpose is conducted in each specific case by

the design of rocket. On the basis of the study of trajectories, are constructed the zones (spatial domains) of the possible attacks of target/purposes by the projectiles of class "surface - air" and "air - air". Being located at the moment of launching/starting within the zone of possible attacks, projectile can strike target/purpose. Kill probability depends on the technical flight characteristics of target/purpose and rocket. Similar investigations are conducted during ballistic design.

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§4. Concept of the optimum solutions of the problems of external ballistics.

Setting the most advantageous (optimum) conditions of motion is one of the most important and complex problems of the theory of flight. As an example of optimum problem, it is possible to give the determination of angle of departure at which is reached the greatest horizontal flying range at the assigned initial velocity. The obtained from these sine conditions is called the angle of maximum range $\theta_{0x_{max}}$. Most simply the angle of maximum range is determined for the parabolic trajectory of motion (i.e. if we do not consider the air resistance and to accept $\bar{g}$ = const).

Let us take formula (7.5)

$$x_C = \frac{v_0^2 \sin 2\theta_0}{g}.$$

Maximum range $x_C = x_{C\,max}$ will be obtained when $\sin 2\theta_0 = 1$. Consequently, the angle of maximum range $\theta_{0x_{max}} = 45°$.

A characteristic example of the optimal solution is also the determination of the angle of maximum range within the limits of elliptical theory when $\bar{g}_T \neq \text{const}$. Figures 7.5 shows that the maximum range will answer the greatest value of angle φ_S, determined on formula (10.18). From Keplerian equations it is possible to obtain the formula, which connects angles φ_S and θ_H.

$$\operatorname{tg} \varphi_S = \frac{1}{2} \frac{r_H v_H^2 \sin 2\theta_H}{g_{T0} R_3^2 - r_H v_H^2 \cos^2 \theta_H}. \qquad (10.25)$$

Let us find the maximum of function, differentiate $\frac{d}{d\theta_H}(\operatorname{tg}\varphi_S)$ and after making its equal to zero. After conversions we will obtain the formula, which determines the angle of the maximum range:

$$\sin \theta_{H\,max} = \sqrt{\frac{g_{T0} R_3^2 - r_H v_H^2}{2 g_{T0} R_3^2 - r_H v_H^2}}. \qquad (10.26)$$

Figures 10.7 shows curve/graph $\theta_{H\,max} = f(v_H)$, comprised for the case $r_H = R_3$. From curve/graph it is evident that at the low initial

velccities the angle of maximum range is clcse tc 45°. With an increase in the velocity, the angle cf maximum range decreases, reaching zero at orbital velocity.

Is of interest solution by finding of the minimum initial velocity of the projectile cf the ccrstant mass, necessary for the cvercoming of the assigned flying range with known $r_н$. Problem is reduced, thus, to the determination of angle $\theta_{н.опт}$, by which will occur $x_{н\,min}$. It differentiated equation (7.20) for $\theta_н$ and after making obtained expression equal tc zerc, after a series of mathematical conversions, we will obtain the following expression:

$$\theta_{н.опт}=\frac{1}{2}\,\mathrm{arctg}\left(\frac{\sin 2\psi}{\frac{r_н}{R_3}-\cos 2\psi}\right). \tag{10.27}$$

The trajectory, which corresponds to angle $\theta_{н.опт}$, calls the trajectory of minimum speed. The dependence cf optimum angle cr range angle for several values $\theta_{н.опт}$ is visible from Fig. 10.8.

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If we consider air resistance, then problem regarding the angle cf maximum range it is nct sclved in analytical form without

supplementary simplifications. Simplest path - consecutive conducting of the large number of trajectory calculations for different values θ_0 and graphing $x_C = f(\theta_0)$. Value $\theta_{0x_{max}}$ can be determined either from curve/graphs or by reverse/inverse interpolation. Even for the simple case of moving the body of constant mass in air (projectile of cannon-type artillery) the angle of maximum range depends not only on the initial velocity, but also on bore, the weight and the form of projectile, united by the formula of ballistic coefficient. Figures 10.9 shows that depending on bore the angle of maximum range can change over wide limits - from ~30° to ~60°. Great difficulties are encountered during the determination of the optimum parameters, which ensure the greatest firing distance with the complex trajectories of the guided and unguided missiles and projectiles.

Let us take the complex trajectory of the unguided rocket projectile (see Fig. 0.15.1). The determination of the initial angle of the trajectory of maximum range θ_0 and the connection points of jet engine (point N) in the trajectory of the unguided projectile, the passing in dense layers atmosphere, represents by itself multiparametric complex problem. Complete distance depends on $v_0, \theta_0, c_0, v_N, \theta_N, y_N$, aerodynamic drag coefficient (or ballistic coefficient c_{NK}), the thrust P and of the operating time of engine on the phase of trajectory N - K. The distance, which corresponds to the trajectory phase from point K to encounter with target, is determined by values v_K, θ_K, y_K and c_K.

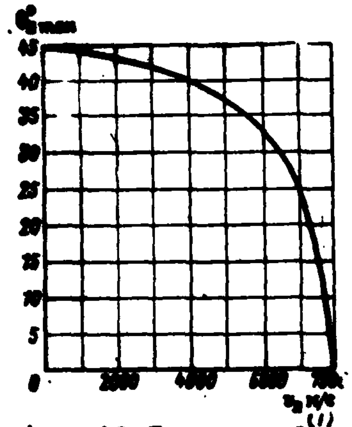

Fig. 10.7. Dependence of the angle of maximum range $\theta_{H\,max}$ on velocity v_H

Key: (1) m/s.

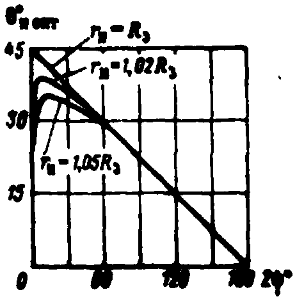

Fig. 10.8. Dependence of optimum initial angle $\theta_{H.opt}$ on range angle 2ψ.

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Thus, total distance will depend on eleven discrete values and two functions $c_x(M)$ and P (y). It is obvious that the identification of

parameters, determining under these conditions the greatest firing distance, represents by itself the very laborious problem, the only method of solution of which - calculation of the family of trajectories.

With the relatively small firing distances, characteristic for barrel artillery pieces, total distance is determined in essence by the second inactive leg. This makes it possible to simplify comparative calculations regarding the initial (gun) angle of maximum range.

For conducting comparative calculations, let us make following assumptions. A velocity increment for the time of the engine operation let us determine from formula K. E. Tsiolkovskiy and the total velocity in the combined point N-K to take as

$$v_{\text{к}}=v_{\text{н}}+v_{\text{рд}},$$

where $v_{\text{рд}}$ — velocity, determined by formula K. E. Tsiolkovskiy;

$v_{\text{н}}$ — velocity of projectile at the moment of firing engine.

The torque/moment of firing engine in trajectory let us select on the angle of maximum range for the second inactive leg. For medium

and high calibers and for velocities $v_к$, the not exceeding ~600 m/s, without large error it is possible to take $\theta_н=45°$.

Let us designate the over-all payload ratio of the rocket charge through

$$\alpha=\frac{Q_p}{Q_0}, \qquad (10.29)$$

where Q_p — weight of rocket charge;

Q_0 - initial weight of projectile.

Then on formula K. E. Tsiolkovskiy the velocity

$$v_{рк}=w_e \ln\left(\frac{1}{1-\alpha}\right). \qquad (10.30)$$

Considering that the factor of the form of projectile will not change with the combustion of rocket charge, it is possible for the second passive section to accept

$$c_к=c_0\frac{1}{1-\alpha}. \qquad (10.31)$$

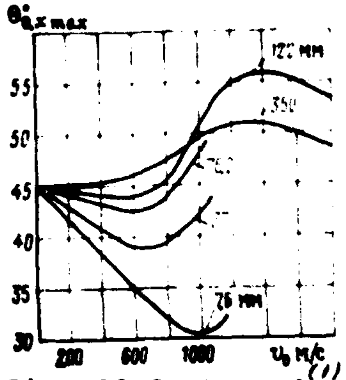

Fig. 10.9. Dependence of the angle of maximum range $\theta_{0x\,max}$ on the initial velocity and the bore of projectile.

Key: (1) m/s.

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With the adopted assumptions the initial angle of the trajectory of maximum range can be found via the comparison of the results of the calculations, conducted employing following procedure. At the assigned values v_0, c_0, a for different θ_0 is calculated the first inactive leg before the achievement of point with angle $\theta_n = 45°$. The calculation of the second passive section with initial conditions $v_K = v_n + v_{pn}$; $\theta_K = \theta_n = 45°$; c_K leads to the determination of flight range x_C. Then calculations are repeated with variation by values v_0, c_0, a. Using the described procedure, we have comprised the tables of the fundamental characteristics of trajectory, which correspond to the point of firings of engine during which one should expect obtaining firing distance, close to the greatest. Tables gives

gun angle of elevation - θ_0;

the coordinate of the point of firing engine - $x_н$ and $y_н$;

time delay of firing engine or the time of the motion of projectile on the first inactive leg - $t_н$;

the velocity of projectile at the moment of firing engine — $v_н$.

For each of these values, are comprised independent tables. The entry into tables are the values:

v_0 within the limits of 50-600 m/s;

c_0 within limits of 0.0-1.5;

α within limits of 0.05-0.30.

Tables gives in appendix.

Data points for the compilation of the tables are designed by the method of numerical integration. The data of tables can be used

as initial for the comparison of the diverse variants of trajectories. Work on tables is reduced to linear interpolation on three entries.

It is necessary to keep in mind that the curve $x_{C\,max}=f(\theta_0)$ within the limits of the angles, close to those given in tables, has flat to Maxim machine gun. Therefore is admissible certain deviation from the values, given in tables, since it does not lead to the noticeable decrease of firing distance. At the low values c_0 and v_0 value x_n, y_n and time t_n they prove to be small (x_n and y_n — an order of several ten meters and even it is less). These numerals are given in tables for the generality of solution in all range of the intake parameters. Taking into account the complexity of ballistic solution, one ought not to select low values t_n, x_n and y_n, since in this case engine will be included on the phase of trajectory where are still considerable angular flutter speed and the amplitude of nutation angle, but this will lead to an increase in scattering trajectories. Furthermore, engine must be included at safe distance from gun crew and on materiel.

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The solution of extreme problems in rocket engineering beginning with the so-called second task of Tsiolkovskiy which consists in the

determination of the law of a change in the mass of rocket and its velocity depending on time, with which it is possible to expect the greatest height/altitude of the rise of rocket. In the setting of Tsiolkovskiy, the problem is solved for vertical ascent of rocket without the account of the air resistance when $\bar{g}$ = const.

Research showed that for the named conditions the maximum climbing range of rocket increases with decreased time of the combustion of the fuel reserve. This law is disrupted upon consideration of the air resistance and variability of the acceleration of gravity with height/altitude. And air resistance, and gravitational force decrease with height/altitude; however, the energy losses of rocket, caused by the action of these forces, depend on different factors. The losses, which depend on the air resistance, are proportional to certain rate of speed of motion and for their decrease it is necessary to approach the limitation of flight speed. The losses, determined by the action of gravitational force, are proportional to the time of motion and it must be decreased, i.e., to approach the decrease of the burn-up time of fuel/propellant and the fastest achievement of larger flight speed. The version of the solution of problem during which total losses will be smallest, determines optimum state of motion and, therefore, greatest climbing range of rocket.

From the given examples it is evident that finding the optimum solutions of the theory of flight is reduced both to determining of the geometric characteristics of the flight trajectories (angles of departure, form of trajectory, etc.) and to the determination of the states of motion of rockets (rate of motion, laws governing the fuel consumption, etc.). The common/general/total formulation of the problem assumes the solution of the problem of optimum motion as a whole, i.e., the simultaneous determination of the form of trajectory and flight conditions. The practical solution of this complex problem encounters great difficulties. By especially complex is represented optimization according to several motion characteristics.

Usually are solved simpler problems with introduction under their conditions of supplementary simplifications. In this respect characteristic are two large groups of problems - determining the geometric characteristics of optimum trajectories under given conditions of flight (engine power rating) and determination of optimum flight conditions under the assigned form of trajectory or the equivalent conditions, specifying geometric characteristics. The substantiated selection of the optimum versions of the solution of different problems in rocket engineering is based on the mathematical methods of variation calculus.

Variation calculus began to be developed from end/lead the XVII

century. As his the founder rightfully they consider the member of the russian Academy of Sciences of L. Euler. The object/subject of variation calculus are investigations for the extremum (maximum or the minimum) of the separate magnitudes, called functionals.

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Functional - variable value whose value is determined by the selection of one or several functions. For example, the area of certain surface is functional, since it is determined by the selection of the function, entering the equation of surface of z = f(x, y). Resistance of medium to the driving/moving in it with the definite velocity body is also functional, since it depends on the function, which lays out of the surface of the driving/moving body. One of the first tasks, solved the method of variation calculus, was the problem of the curve of the faster slope, to the so-called brachistochrone. In this problem were required to determine the form plane curve, the connecting two points, arrange/located on the different height/altitude, on which the body would roll down in shortest time. If we do not consider resistance of medium and friction, then such curved proves to be cycloid.

Let us call/name the typical variational problems of rocket engineering. For the rockets of class "surface - air", "air - air"

and "air - surface" typical are brachistrcctrcne problems by choice cf the program of moticn, causing the minimum time, necessary fcr flight from point with crigin coordinates x_1, y_1 and velocity v_1 to pcint with the final cocrdirates x_2 and y_2. The assignment to velocity v_2 in end point considerably complicates solution. Prcblems can be solved both for the climbing flight and fcr dive. Are of interest variational prcblems with the assigned final conditions feast the minimum fuel ccnsumpticn.

For the rockets of class "surface - surface" the most important variaticnal problem it is pcssible tc ccnsider the problem of the selection of the program of maximum range. For ballistic missiles cptimum solution is connected with the selecticn of value and line of force of the engine thrust. With the assigned magnitude of thrust, the variational problem can be brought to the selection of the program of pitch angle, which ensures maximum flying distance.

Setting the program of maximum range is direct-connected with the prcblems, which appear during the design of rockets. One Of most impcrtant is a questicn ccncerning setting cf the optimum ccnstruction of rocket and conditicns/mcde cf its motions, during which the payload (or warhead) will be supplied up to the assigned distance by the rocket cf minimum launching weight. In this respect mcst characteristic is the problem cf cptimum weight distributicn

between the step/stages of staged rocket and engine power ratings of each step/stage. As evaluation criteria of solution is usually received the minimum total weight of rocket, which ensures satisfaction of conditions for final velocity or flying ranges of the nose cone of the assigned weight.

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For winged missiles characteristic variational problem it is possible to consider the problem of the programming of the thrust of starting and sustainer engines. The problem of the programming of thrust is also characteristic for the high-altitude rockets, intended for the sounding of the atmosphere. Wide distribution received variational methods during the optimization of trajectories and states of motion of space vehicles. Many of the named problems are solved by Soviet scientists [2], [31], [44], [45]. Foreign authors's works are generalized in collection [24].

Let us examine the schematic of solution of one of the variational problems of rocket engineering. Let be required to find the function, which determines the consumption of fuel (change in the mass of the rocket) in time, during realization of which the height/altitude of vertical ascent of rocket will be greatest. It will use the equation of Meshcherskiy for vertical ascent of rocket

(1.9), after replacing for simplification in the investigation the entering it sum of values $\dot{p}_r - \frac{dm}{dt} w_{отн}$ by term $-\frac{dm}{dt} w_e$, where $w_e = const$ — effective discharge velocity.

Replacing also $\dot{x}$ and $\ddot{x}$ on v and $\frac{dv}{dt}$, let us have

$$m\frac{dv}{dt} = -mg - \frac{dm}{dt} w_e - R(v). \qquad (10.32)$$

Let the mass of rocket change on dependence $m = m_0 f$, where f - the function, which characterizes a change of the mass of rocket (fuel consumption) in the process of the engine operation. In the beginning of motion f(0) = 1.

After replacing in equation (10.32) value of m on $m_0 f$ and after dividing all the terms into constant value m_0, we will obtain

$$f\frac{dv}{dt} = -fg - \frac{df}{dt} w_e - \frac{R(v)}{m_0}. \qquad (10.33)$$

Let us conduct the replacement of variables, after introducing the value of the elementary path of rocket dS

$$\frac{dv}{dt} = \frac{dv}{dS}\cdot\frac{dS}{dt} = v\frac{dv}{dS}; \quad \frac{df}{dt} = \frac{df}{dv}\cdot\frac{dS}{dt}\cdot\frac{dv}{dS} = f'_v v\frac{dv}{dS}.$$

Then

$$f\cdot v\cdot\frac{dv}{dS} = -fg - vf'_v w_e\frac{dv}{dS} - \frac{R(v)}{m_0}.$$

After conversion and integration, we will obtain expression for the functional

$$S[f(v)] = \int_{v}^{v_k} \frac{(f + f'_v \cdot w_e) v\, dv}{gf + \frac{1}{m_0} R(v)}. \qquad (10.34)$$

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The target/purpose of the subsequent solution - to find this function f (v), with which the value of integral will be greatest, i.e., to determine this law of mass change from velocity, during which climbing range of the rocket will be maximum.

The determination of this function is called the investigation of integral for the extremum. Investigation must be conducted with specific boundary conditions; at the start of the studied segment the velocity of rocket $v = v_0$, and of $f = 1$, i.e., mass $m = m_0$; at the end of the section respectively $v = v_K$ and $m = m_K$. These conditions are determined by design features of the type in question of rockets.

The formulated problem of variation calculus is solved with the

aid of the differential equations of Euler, which determine necessary the condition of the extremum of the functional, which has the general view

$$L[y(x)]=\int_{x_0}^{x} F[x, y(x), y_x'] dx, \qquad (10.35)$$

where F - the assigned function of the arguments x, y(x) and $y_x'=\frac{dy}{dx}$.

Let be found function y = f(x), that ensures the extremum of functional. For the determination of conditions which must answer this function, is introduced into examination curve, close to f(x), called the curve of comparison. Function $\bar{y}$, which determines the curve of the comparison of the same family of curves, as f(x), little from it differs

$$\bar{y}=f(x)+\varepsilon\delta y(x),$$

where ε — a low number, and δy(x) - the arbitrary function, which turns into zero at the end/leads of the interval/gap of integration, i.e.,

$$\delta y(x_0)=\delta y(x_1)=0.$$

When $\varepsilon=1$ the difference between functions δy(x) = $\bar{y}$(x)-y(x) is called in terms of increase or a variation in argument y(x) of functional L [y(x)].

Understanding of variation (δy) of argument y(x) of functional

L[y(x)] it differs dsignificantly from an increase Δx in argument x of function f(x). An increase in the argument is connected with a change in it for the assigned function, and variation δy is increments of coordinates y because of a change in the form of the function at the fixed value of argument x. In variation calculus it is proven, that the necessary condition of extremum is the inversion in zero variation in the functional. If function $f(x) = F[x, y(x), y']$ within limits from x_0 to x_1 it is single-valued, three times differentiated it is continuous itself and are continuous it partial derivatives, then a variation in the functional it turns into zero, if the unknown function answers the equation of Euler

$$\frac{\partial F}{\partial y}-\frac{d}{dx}\left(\frac{\partial F}{\partial y'}\right)=0. \qquad (10.36)$$

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This condition is necessary. Integral curve equations of Euler $y = y(x, C_1, C_2)$ are called extremals. Only on extremals can be reached the extremum of functional. For determining the function during which can be obtained the extremum, one should integrate the equation of Euler. Arbitrary constants are located from the boundary conditions $y(x_0) = y_0$ and $y(x_1) = y_1$.

The difficulties of practical solution consist in the fact that the differential equation of Euler is second order equation and its

sclution not always can be obtained in final form. If solution takes the final form, then it is must it additionally to check to sufficient conditions. In each specific problem must be stipulated or somehow additionally the defined class of functions within limits of which searches for the extremal.

Finding extremum becomes complicated for the functionals, which contain several functions of independent variable. In this case is comprised and is solved the system of differential equations, in which the number of equations corresponds to the number of functions. Utilizing the aforesaid, let us find the extremum of functional (10.34). In our case the integrand takes the form

$$F=\left[\frac{(f+f'_v w_e)v}{gf+\frac{1}{m_0}R(v)}\right]. \qquad (10.37)$$

For a substitution in the equation of Euler, let us find

$$\frac{\partial F}{\partial f};\ \frac{\partial F}{\partial f'_v}\ \text{и}^{(1)}\ \frac{d}{dv}\left(\frac{\partial F}{\partial f'_v}\right).$$

Key: (1). and

bearing in mind that variables are v, R(v) and function f, which depends on v, but value g = const. Then

$$\frac{\partial F}{\partial f}=\frac{v}{gf+\frac{1}{m_0}R(v)}-\frac{(f+f_v'w_e)gv}{\left[gf+\frac{1}{m_0}R(v)\right]^2};$$

$$\frac{\partial F}{\partial f_v'}=\frac{w_e v}{gf+\frac{1}{m_0}R(v)};$$

$$\frac{d}{dv}\left(\frac{\partial F}{\partial f_v'}\right)=\frac{w_e}{gf+\frac{1}{m_0}R(v)}-\frac{w_e v\left(\frac{1}{m_0}\frac{\partial R(v)}{\partial v}+gf_v'\right)}{\left[gf+\frac{1}{m_0}R(v)\right]^2}.$$

Substituting right sides of the written equations in (10.36) and multiplying after substitution everything by $\left[gf+\frac{R(v)}{m_0}\right]^2$, we will obtain

$$f=f(v)=\frac{1}{gm_0w_e}\left[(v-w_e)R(v)+vw_e\frac{\partial R(v)}{\partial v}\right]. \quad (10.38)$$

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As a result of solution, is obtained the equation of extremal during which occurs the maximum of path S. The obtained dependence cannot be directly used for the solution of our problem, since we obtained function f depending on the velocity of the motion of rocket v, but the majority of systems of equations describes the motion of the rocket with independent by the variable t.

Let us present

$$\frac{df}{dt}=\frac{df}{dv}\cdot\frac{dv}{dt}=f_v'\frac{dv}{dt}$$

and, after conducting the appropriate replacement in (10.33), we will obtain

$$(f+f'_v w_e)\frac{dv}{dt}=-\left[gf+\frac{1}{m_0}R(v)\right],$$

whence

$$t=\int_v^{v_0}\frac{(f+f'_v\cdot w_e)\,dv}{gf+\frac{1}{m_0}\,R(v)}. \qquad (10.39)$$

For taking of integral, it is necessary to substitute in it right side (10.38) and derivative f'_v, which we will preliminarily obtain also from (10.38)

$$f'_v=\frac{d}{dv}(f)=-\frac{1}{gm_0w_e}\left[R(v)+v\frac{\partial R(v)}{\partial v}+vw_e\frac{\partial^2R(v)}{\partial v^2}\right].$$

Accepting, as before g = const, we will obtain from (10.39) working formula for t

$$t=\frac{v_0-v}{g}+\frac{w_e}{g}\int_v^{v_0}\frac{\left(\frac{\partial R(v)}{\partial v}+w_e\frac{\partial^2R(v)}{\partial v^2}\right)dv}{R(v)+w_e\frac{\partial R(v)}{\partial v}}. \qquad (10.40)$$

At the concrete/specific/actual value of function R (v), that determines the air resistance, integral (10.40) there can be taken numerically, after which will be determined the dependence (curve/graph) t(v). Dependence f(v) is located from (10.38) after substitution into it derived $\frac{\partial R(v)}{\partial v}$ of the concrete/specific/actual function R(v). Having dependences f(v) and t(v), it is possible to establish the unknown dependence f(t), that determines a change in the mass of the rocket in time, with which the path of the rocket will be maximum. We focus attention on the fact that the overall consumption of mass (fuel/propellant) remains constant, appears itself only the optimum version of its expenditure during flight. The concrete/specific/actual form of the function f(t), that connects the current and initial masses of the rocket $(m=m_0 f)$, with the stipulated assumptions depends on curve/graph, determining R(v).

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If we introduce for R(v) analytic function during which there

can be taken integral in expression (10.40), then dependence t(v) will obtain final form. For example, in [31] during the determination of the air resistance are accepted by constants air density and drag coefficient. Then with ρ = const and c_x= const we obtain

$$R(v)=\frac{1}{2}\rho v^2 c_x=Bv^2;\quad \frac{\partial R(v)}{\partial v}=2Bv;\quad \frac{\partial^2 R(v)}{\partial v^2}=2B.$$

Carrying out the replacement of functions under integral in (10.40), let us have

$$t=\frac{1}{g}\left[v_0-v-w_e\int_v^{v_0}\frac{2(v+w_e)dv}{v(v+2w_e)}\right]. \qquad (10.41)$$

Integration gives the known formula, which connects time with the velocity of the motion of the rocket

$$t=\frac{1}{g}\left[v_0-v-w_e\ln\frac{v_0(v_0+2w_e)}{v(v+2w_e)}\right]. \qquad (10.42)$$

Recall that this relatively simple solution of variational problem is obtained with assumptions g = const, ρ = const, c_x= const and R(v) = Bv². Simultaneous failure of these assumptions leads to the problem, virtually not solved at present within the framework of classical variation theory.

In the examined above example the curves among which one should

search for the solutions of variational problem, had the fixed/recorded end points, determining integration limits of the functional. More great possibilities have variation of solution under alternating/variable boundary conditions. They frequently assume that the end-points are located on the specific lines or even surfaces. In these cases the determination of arbitrary constants during the solution of the equation of Euler requires supplementary conditions. These conditions are called transversality conditions.

The extremum of functional can be reached on extremals with points of inflection. Between points of inflection separate smooth cuttings off must be the integral curves of the equation of Euler. The coordinates of points of inflection must satisfy Weierstrass-Erdmann's supplementary conditions. Besides those named, were obtained the considerable propagation and other necessary conditions, for example, of the condition of Clebsch, Jacobi, etc.

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Many of the problems of the optimization of the states of motion of rockets can be referred to the class of the problems, called in variation calculus isoperimetric. This name is appropriated to the problems, in which is placed supplementary isoperimetric condition in the form of an auxiliary functional. As example of the similar

problem in the overall theory of variation calculus to usually serve problem for finding of the geometric figure of maximum area with the assigned perimeter. A characteristic example of the isoperimetric problem of rocket dynamics can be obtained, after using the given above formulas, which describe vertical ascent of rocket. It is necessary to supply supplementary condition about the minimization of the time of the motion of rocket from point O to point K, which lies on of direct/straight vertical climb. On the basis of (10.39), let us write expression for the functional, which determines the minimum time of the motion of rocket from point O to point K:

$$T[f(v)]=\int_{v_K}^{v_0}\frac{(f+f'_v w_e)\,dv}{gf+\frac{1}{m_0}R(v)}. \qquad (10.43)$$

Problem consists in finding of this function f(v), that determines the fuel consumption, by which the rocket will move from point O into point K for minimum time. The unknown function must answer the supplementary condition, written in accordance with (10.34) and to that determining the path of rocket from point O to point K

$$S_{OK}=\int_{v_K}^{v_0}\frac{(f+f'_v\cdot w_e)\,v\,dv}{gf+\frac{1}{m_0}R(v)}. \qquad (10.44)$$

Furthermore, as before must be maintained boundary conditions. In the beginning of the section (with $v = v_0$) being investigated f = 1; at the end of the section (at point K, with $v=v_K$) $f=f_K$, t.e. the mass of rocket $m=m_K$. The solution of a similar problem is given in [31]. The fundamental difficulties, which are encountered when solving variational problems by classical methods, and the imperative need for practical solutions led to the appearance of different approximation methods, to which can be attributed finite-difference method, a Ritz's method and a series of others.

Recently in connection with the development of electronic digital computers for the solution of variational problems, won acceptance the method of dynamic programming [64].

We focus reader's attention to the fact that in practical work on the creation of the specimen/samples of rocket and ordnance theoretical variational methods are always combined with design studyings and the comparison of the design/projected specimen/samples with existing well recommended themselves by systems.

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Chapter XI.

CORRECTION FORMULAS OF EXTERNAL BALLISTICS.

The basic problems of external ballistics, examined above, they were solved with the initial data, corresponding to technical specifications for rocket (projectile) and to the characteristics of standard atmosphere.

The trajectory, designed at the "normal" values of its specifying factors indicated, is called basic or that not disturbed. During the description of the complex process of flight on mathematical means for simplification in the calculations, the part of the acting factors is not considered, but some of them are taken by average values. However, the conditions of firing in the majority of the cases differ from calculated theoretical. As an example of the nonconformity of calculated and actual conditions, it is convenient to exile to the account of the effect of weather factors. As is known, the problem of external ballistics is solved for the normal (standard) atmosphere, and the meteorological conditions with firing, as a rule, differ from the standard. Calculations are conducted for still air (dead calm), but in actuality very frequently the atmosphere is not calm and wind can substantially change the results

of firing. They affect the parameters of trajectory and deviation of other weather factors from their normal values.

Desiring to establish effect on the trajectory of any cell/element, than not earlier examine/considered, it is necessary to comprise the new system of differential equations, including the which interests us value. For example, solving one time the system of equations of the motion of the center of mass of projectile, comprised without the account of the curvature of the Earth and its rotation, but for the second time - comprised with their account, it is possible to rate/estimate the effect of the named factors on the motion characteristic of the center of mass of projectile. It is necessary to comprise the new systems of differential equations for the account of the deviations of some weather factors, for example for the account alternating/variable with respect to height/altitude and wind direction.

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In many instances the effect of the perturbation factors can be considered without the compilation of new differential equations. Most simple this to make when the which interests us factor is already taken into account in fundamental differential equations and it is necessary to establish the effect of its change to the results

of calculation or firing. If the parameter being investigated changes substantially, then the effect of this change to the results of performance calculation of motion and trajectory elements should find by the solution of the basic system of differential equations with the new changed data. The comparison of the results of the solutions, obtained with those changed and "normal" data, gives allowance.

In certain cases it proves to be possible to introduce into the results of the solution of interference correction of the factors, which do not contain in the basic system of differential equations, and without comprising the new system of equations, which includes the which interests us value. For example, effect on the distance of the form of the Earth can be established by indirect method from geometric considerations, without solving the system of the differential equations of motion of projectile, comprised for the spherical or other model of the Earth.

In practice, as a rule, it is necessary to meet the small deviations of the determining parameters from their normal (drawing rooms and standard) values. In the majority of the cases, the low deviations of parameters lead to small changes in the trajectory elements. This makes it possible to establish the effect of the perturbation factors on the characteristics of the undisturbed trajectory by determining the corrections in these cell/elements with

the aid of various short-cut methods and dependences.

Corrections are called changes in the motion characteristics or trajectory elements, which correspond to the deviations of its determining parameters. Corrections, as a rule, are calculated on the motion characteristics of the center of mass of rocket or projectile for any fixed/recorded point in the trajectory. Figures 11.1 shows active sections not disturbed 1 and the disturbed 2 trajectories of the unguided rocket. The corrections, which correspond to the torque/moment of the end/lead of the engine operation, caused by the deviation of any determining parameter or group of the parameters, will be: δx_k — correction for actual conditions into coordinate x_k; δy_k — a correction coordinate y_k; δv_k — a correction into the velocity of the motion of the center of mass of projectile; δt_k — correction during the operation of engine and so forth

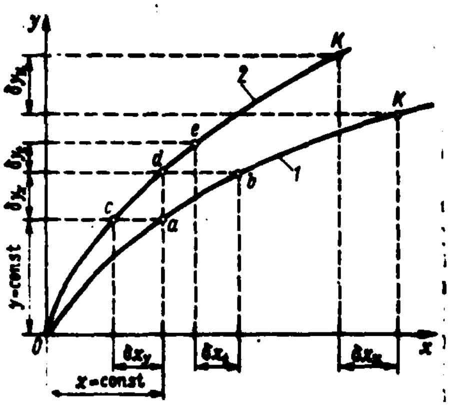

Fig. 11.1. A change in the trajectcry cf the mction of the center of mass cf rocket because cf a change in the determining parameter: 1 - undisturbed trajectcry; 2 - disturted trajectcry.

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Corrections can also te determined for the point, assigned by any motion characteristic (ty time, velccity, the abscissa, ordinate, etc.), identical fcr basic and that agitated cf trajectories. For example, to point b of the undisturted trajectcry from the condition cf the constancy of flight time will ccrrespond point e of the trajectory of the disturbed moticn; in this case, corrections for a trajectcry in point b will te values $\delta x_t, \delta y_t, \delta v_t$ and so forth. During the assignment of conditicn y = const tc point a corresponds pcint c

and for a trajectory at point a we will obtain corrections δx_y, δv_y, $\delta\theta_y$...

If we take condition x = const, then to point a corresponds point d, moreover corrections for a trajectory in point a are equal to δy_x, δv_x, δt_x and the like. For the projectiles of barrel terrestrial artillery and surface-to-surface missiles usually is determined correction into complete firing distance and the deviation of the impact point in the projectile in side direction. When conducting of direct/straight ballistic calculations, i.e., during the solution of the first problem of external ballistics, correction they are introduced into the cell/elements of the undisturbed trajectory, designed under standard conditions (drawing rooms and meteorological). During processing of the results of the firings, obtained under actual conditions, the corrections are introduced into experimental data with the fact in order to lead them to standard conditions.

In the latter case the sign of correction will be reverse/inverse to the sign, determined when conducting of direct/straight ballistic calculations.

§1. Correcting formulas and correction factors.

In general form, for the trajectory element either motion characteristic it is possible to write

$$A=f(\xi_1, \xi_2, \xi_3, \ldots, \xi_n), \qquad (11.1)$$

where A - a trajectory element or motion characteristic;

ξ_i — i specifying parameter.

The deviations of the parameters from computed values, designated $\delta\xi_1, \delta\xi_2, \ldots, \delta\xi_n$, will cause a change in the trajectory element A which let us designate δA. In accordance with the common/general/total formula of series expansion of Taylor, the correction will be determined by formula (8.11). The number of terms of expansion, held in calculations, depends on the required accuracy of the determination of correction. Most frequently during the solution of the practical problems of the theory of corrections, hold the linear terms of expansion. The obtained with this formula corresponds to the formula of total differential from functional dependence (11.1).

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If one assumes that the parameters, united by functional dependence (11.1), do not depend on each other and in the formula of total

differential infinitesimal increases to replace with certainly low, then it is possible to obtain

$$\delta A = \frac{\partial f}{\partial \xi_1}\delta\xi_1 + \frac{\partial f}{\partial \xi_2}\delta\xi_2 + \ldots + \frac{\partial f}{\partial \xi_n}\delta\xi_n. \qquad (11.2)$$

In last/latter formula are not considered the terms of expansion of higher orders, than the first. The error in concrete/specific/actual calculations, which depends on neglect of the remainders of expansion, is determined by special investigations.

Correcting formulas of type (11.2) are called differential, and the specific by them values δA are called the corrections, calculated according to the method of differentials.

Ballistic derivatives $\frac{\partial f}{\partial \xi_i} = \frac{\partial A}{\partial \xi_i}$ in the theory of corrections are called correction factors for a cell/element A of trajectory to the deviation of the parameter ξ_i. The correction factor (ballistic derivative) it is numerical equal to a change in the trajectory element in question with an increase in the corresponding determining parameter by the unit of its measurement accepted. In the theory of the corrections of cannon-type artillery the correction factors, which characterize changes in the distance during the deviations of ballistic coefficient $-\frac{\partial x_C}{\partial c}$, the initial velocity $-\frac{\partial x_C}{\partial v_0}$ and angle of departure $-\frac{\partial x_C}{\partial \theta_0}$, is conventionally designated as basic correction coefficients.

As an example let us examine obtaining correcting formulas for a complete horizontal firing distance from artillery piece under conditions of the vacuum. In the named case complete distance to be determined, as is known, by dependence (7.5)

$$x_C = \frac{v_0^2 \sin 2\theta_0}{g}.$$

We will obtain correcting formula taking into account the terms of the expansion of the first and second orders. Taking into account that in the last/latter formula only two arguments (v_0 and θ_0), in general designations on (8.11) let us have

$$\delta x_C = \frac{1}{1!}\left(\frac{\partial}{\partial\xi_1}\delta\xi_1 + \frac{\partial}{\partial\xi_2}\delta\xi_2\right) f(\xi_1, \xi_2) +$$
$$+\frac{1}{2!}\left[\frac{\partial^2}{\partial\xi_1^2}(\delta\xi_1)^2 + 2\frac{\partial^2}{\partial\xi_1\partial\xi_2}\delta\xi_1\delta\xi_2 + \frac{\partial^2}{\partial\xi_2^2}(\delta\xi_2)^2\right] f(\xi_1, \xi_2). \quad (11.3)$$

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In the designations of formula (7.5) we will obtain

$$\delta x_C = \frac{\partial x_C}{\partial v_0}\delta v_0 + \frac{\partial x_C}{\partial\theta_0}\delta\theta_0 +$$
$$+\frac{1}{2}\left[\frac{\partial^2 x_C}{\partial v_0^2}(\delta v_0)^2 + 2\frac{\partial^2 x_C}{\partial v_0\partial\theta_0}\delta v_0\delta\theta_0 + \frac{\partial^2 x_C}{\partial\theta_0^2}(\delta\theta_0)^2\right].$$

Opening the values of partial derivatives, let us have

$$\delta x_C = \frac{2v_0 \sin 2\theta_0}{g}\delta v_0 + \frac{2v_0^2 \cos 2\theta_0}{g}\delta\theta_0 + \frac{\sin 2\theta_0}{g}(\delta v_0)^2 + \frac{4v_0 \cos 2\theta_0}{g}\delta v_0 \delta\theta_0 - \frac{2v_0^2 \sin 2\theta_0}{g}(\delta\theta_0)^2. \quad (11.4)$$

Last/latter three terms are determined by quadratic terms of expansion and are the values of the higher order of smallness, than two first, determined by the linear terms of expansion. In the majority of the cases, are utilized only linear terms and correction is determined so

$$\delta x_C = \frac{2v_0 \sin 2\theta_0}{g}\delta v_0 + \frac{2v_0^2 \cos 2\theta_0}{g}\delta\theta_0, \quad (11.5)$$

where the ballistic derivatives (correction factors) are equal to

$$\frac{\partial x_C}{\partial v_0} = \frac{2v_0 \sin 2\theta_0}{g}; \quad \frac{\partial x_C}{\partial \theta_0} = \frac{2v_0^2 \cos 2\theta_0}{g}. \quad (11.6)$$

Having a value of correction factor, it is easy to find correction in equivalent component of the trajectory

$$\delta A_l = \frac{\partial A_l}{\partial \xi_l}\delta\xi_l. \quad (11.7)$$

If one assumes that changes only initial velocity, then

$$\delta x_{Cv} = \frac{\partial x_C}{\partial v_0} \delta v_0. \tag{11.8}$$

Utilizing (7.5) and first formula (11.6), we will obtain

$$\frac{\delta x_{Cv}}{x_C} = 2\frac{\delta v_0}{v_0},$$

i.e. with the assumptions, which correspond to parabolic theory, the low relative deflection of the initial velocity causes the doubled relative change in the distance. Proceeding in like manner, it is possible to obtain correcting formulas also for other cell/elements of the trajectory.

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The derivation of the formulas of corrections and formula of the ballistic derived for trajectories different types of rockets and projectiles is considerably more complex than this is shown higher based on simple example, and it requires special methods.

§2. Qualitative effect of the determining parameters and the signs of ballistic derivatives.

The sign of ballistic derivative (correction factor) for the complete flight distance of the projectiles of rocket and artillery pieces is set depending on the effect of an increase in the determining parameter by firing distance. If an increase in the parameter leads to an increase in the firing distance, then correction factor has positive value; if firing distance decreases with an increase in the determining parameter, then correcting coefficient has negative sign. In many instances the sign of correction factor can be established that on the basis of the qualitative effect of the parameter on distance, before conducting of

calculations according to its determination.

Let us conduct qualitative analysis regarding the signs of basic correction factors. The initial velocity of the unguided projectile of constant mass - artillery shell and the velocity of nose cone in the beginning of inactive leg they are some of the basic parameters, which estimate distance of firing. With an increase in the initial velocity, the distance grows; therefore correction factor has positive value. For rockets with inclined start, the velocity of descent from starter (the initial velocity) is, as a rule, small the part of the velocity of the center of mass of projectile at the end of powered flight trajectory $-v_к$. An increase in the velocity of descent from guides leads to increase of the ordinates of an entire trajectory and an increase $v_к$. Thus other conditions being equal an increase in the initial velocity of projectile leads to an increase in the firing distance, therefore, $\frac{\partial x_c}{\partial v_0}>0.$

The lift-drag ratios of projectile affect the firing distance through the area of midsection S and the drag coefficient $c_x(M)$. An increase in the midsection and an increase in the integral value of drag coefficient $\bar{c}_x(M)$ for the time of action along trajectory will lead to the decrease of firing distance.

It is obvious that

$$\frac{\partial x_c}{\partial S}<0 \text{ and } \frac{\partial x_c}{\partial s_x(M)}<0.$$

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If we substitute (2.95) into the equation

$$\frac{dv}{dt}=-\frac{X}{m}-g\sin\theta,$$

that we will obtain

$$\frac{dv}{dt}=-cH(y)F(v)-g\sin\theta.$$

Designating integral characteristics for the time of the motion of projectile along trajectory through H(y) and F(v), let us have

$$\frac{\partial x_c}{\partial H(y)}<0 \text{ and } \frac{\partial x_c}{\partial F_v}<0.$$

For the projectiles of the constant mass c = const and one of the basic correction factors $\frac{\partial x_c}{\partial c}<0$.

For the unguided projectiles of variable and constant mass, initial angle of departure is one of the basic determining parameters. The angle of arrival in the beginning of the inactive leg of unguided rocket flight also significantly affects the firing distance. Fig. 11.2 shows the dependence of complete distance x_c on angle θ_k and the dependence of correction factor $\frac{\partial x_c}{\partial \theta_k}$ on initial

angle of departure θ_0. For barrel systems and unguided rockets $\theta_a = \theta_0$. For the inactive legs of the guided and unguided missiles $\theta_a = \theta_k$.

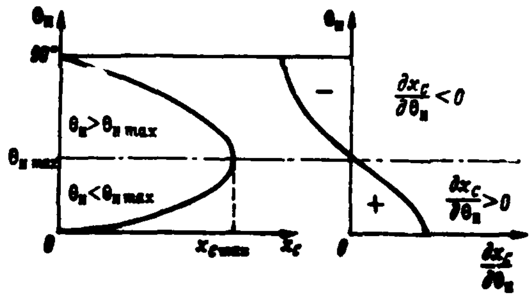

Fig. 11.2. Curve/graph of a change of the correction factor $\frac{\partial x_c}{\partial \theta_н}$ in function from initial angle of departure.

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If $\theta_н$ is lesser than the angle of maximum range, then is correction factor $\frac{\partial x_c}{\partial \theta_н}>0$, if $\theta_н$ more angle $\theta_{н\,max}$, then $\frac{\partial x_c}{\partial \theta_н}<0$. In general form it is possible to write $\frac{\partial x_c}{\partial \theta_н}\gtreqless 0$. The sign of coefficient changes in the dependence from value $\theta_н$ and the angle of maximum range $\theta_{н\,max}$.

Of the fixed/recorded initial velocity $v_н$ correcting coefficient $\frac{\partial x_c}{\partial \theta_н}=0$ corresponds to maximum range and angle of departure, with which this distance is obtained. With firing with the angles, close to the angle of maximum distance small changes in the angle $\theta_н$ barely affect the firing distance.

The cell/elements of the end/lead of powered flight trajectory

$x_к, y_к, v_к$ and $\theta_к$ can be accepted as the independent parameters, which estimate distance of firing. Effect $v_к$ and $\theta_к$ cn firing distarce the projectiles of constant mass is examined above. An increase in the abscissa of powered flight trajectory $x_к$ when $y_к$=const displaces trajectory to the right; with this an increase $x_к$ and firing distances x_C are equal, i.e., $\frac{\partial x_C}{\partial x_к}=1.$

An increase in the ordinate $y_к$ leads to an increase of the distance as a result of the action of rocket in the higher and less dense layers of atmosphere and certain elongation of final trajectory. Correction factor $\frac{\partial x_C}{\partial y_к}>0.$

A change in the mass of projectile affects differently the firing distance. For the projectiles of constant mass from equation $\dot{v} = -X/m - g \sin \theta$ is evident that with an increase in the mass decreases the acceleration of projectile from the air resistance X/m, the velocity of projectile decreases more slowly and firing distance increases; consequently,

$$\frac{\partial x_C}{\partial m}>0.$$

The artillery shell of smaller weight under the identical conditions of throwing (identical charge) from instrument obtains high initial velocity v_0. However, in flight light/lung projectile faster loses its velocity than identical to it in form heavy projectile. The total effect of a change in the mass (weight) of

projectile to firing distance from artillery instrument is set by the special correcting formula (see further).

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For rockets on powered flight trajectory

$$\dot{v} = \frac{P - X}{m} - g \sin \theta$$

with P > X with an increase in the mass, the acceleration decreases, which, other conditions being equal, will bring to smaller $v_к$ and, consequently, also to smaller distance. Thus, the sign of correction factor must be determined during calculation according to concrete/specific/actual data.

A change in the weather factors will entail a change in the air resistance. An increase in the air density increases velocity head and decreases firing distance; consequently,

$$\frac{\partial x_c}{\partial \rho} < 0.$$

An increase of the barometric pressure at launch point and in trajectory leads, in accordance with (2.45), to an increase in the air density; consequently,

$$\frac{\partial x_c}{\partial h_0} < 0 \text{ and } \frac{\partial x_c}{\partial h_x} < 0.$$

Complex proves to be the temperature effect of air on firing distance. In accordance with (2.45) an increase in the temperature leads to the decrease of air density. simultaneously an increase T leads to an increase in the speed of sound in air and a change in Mach number. An increase in Mach number can lead to an increase of function $c_x(M)$ in the range of increase curve $c_x(M)$, and can give decrease $c_x(M)$ on descending leg of a curve $c_x(M)$. The temperature effect of air on firing distance and, consequently, also the sign of correction factor they are set for artillery projectiles of constant mass on correcting formulas and special tables; for rockets - by one of the methods, presented below.

For the gyroscopically stable projectiles of constant and variable mass, the tailwind increases firing distance and

$$\frac{\partial x_c}{\partial w_x} > 0.$$

§3. Methods of calculating the corrections and of ballistic derivatives.

The methods of determining of ballistic derivatives and corrections substantially change depending on the form of trajectory and designation/purpose of calculation.

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Find a use method of the direct/straight solution of the systems of differential equations of motion, method of the solution of the differential equations of corrections, method of calculating the corrections with the aid of the conjugated/combined system of equations, method of determining the corrections with the aid of the law of similarity of trajectories, different analytical methods of calculating of corrections and the correction factors, difference and tabular methods. For the complex trajectories, subdivided during calculations into individual sections (for example, active and passive the controlled and unguided flight) proves to be advisable the application/use of different combined methods.

3.1. Determination of corrections and correction factors by the integration of the systems of equations of motion.

Method is applied during a substantial change in the parameter, which determines the trajectory element being investigated, or if necessary to determine correction for the parameter, which does not

contain in the basic system of equations, comprised for the undisturbed trajectory.

In the first case basic system is solved with n the discrete values of the determining parameter or at n the functional dependences, correction for change in which it is possible to determine. Let, for example, as a result of structural/design changes in exterior form of rocket or its control organ/controls change the form of the curve/graphs of drag coefficient. Let us designate functional dependence for the undisturbed trajectory c_x(M, Re) and for the missile trajectory of the changed form c_{xt}(M, Re). For determining the effect of a change in the form of rocket to the cell/elements of the end/lead of powered flight trajectory, we will use the system of equations (3.59), after directing thrust along the axis of rocket and after drop/omitting control forces. For the undisturbed trajectory the system can be solved at the values

$$X=qSc_x(\mathrm{M},\ \mathrm{Re});\quad Y=qSc_y^{\alpha}(\mathrm{M},\ \mathrm{Re})\alpha. \qquad (11.9)$$

For the rocket whose form changed,

$$X_t=qSc_{xt}(\mathrm{M},\ \mathrm{Re});\quad Y_t=qSc_{yt}^{\alpha}(\mathrm{M},\ \mathrm{Re})\alpha \qquad (11.10)$$

system of equations will take the form

$$\left.\begin{array}{l} \dot{v}=\dfrac{P-X_t}{m}-g\sin\theta;\quad \dot{\theta}=\dfrac{P+Y_t^{\alpha}}{mv}\alpha-\dfrac{g}{v}\cos\theta;\\ \dot{y}=v\sin\theta;\quad \dot{x}=v\cos\theta;\quad m=m(t);\quad \alpha=\theta_{np}(t)-\theta. \end{array}\right\} \qquad (11.11)$$

If during a change in the form of the rocket (or the construction of control devices) besides X and Y changes even axial

component of the engine thrust, then during the solution of last/latter system one should use the changed dependence P_t. The comparison of the results of solutions for the undisturbed and disturbed trajectories will give allowance for a change in the form of rocket (its aerodynamic characteristics).

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Are possible the cases, when the parameter, which determines the motion characteristics of rocket, is not produced change other initial conditions of basic system of equations. For example, let with the firing the unguided rocket change the angle of departure θ_0. In this case the corresponding system of equations - (3.80) or (3.81) - is solved the necessary number once at the changed values θ_{0i}. The number of solutions depends on the boundaries of change θ_0 and of the character of value being investigated change depending on θ_0. If $x_c=f(\theta_0)$ it changes smoothly and within the limits of the section being investigated from θ_{0i} to $\theta_{0(i+1)}$ this dependence can be accepted for linear, are sufficient two solutions at the extreme values θ_0. If third solution, which corresponds, approximately, to the middle of interval θ_0, will give noticeable deviation from straight line, then the number of solutions must be increased. The constructed according to the results of solutions curve/graph or the smoothed tabular dependence $x_c=f(\theta_0)$ will make it possible to determine correction for

distance Δx_c, corresponding to the deviation of angle of departure $\Delta\theta_0$. ¶ The advantage of the described method is the possibility of the account of mutual and indirect effect on the allowance of a change in the values, which depend on the deviation of the basic parameter and presented in basic system differential equations. Furthermore, method makes it possible to determine corrections for each computed motion characteristic in any assigned point in the trajectory.

The determination of correction factors according to the results of the solutions of the differential equations of motion of the center of mass is expedient when the dependence of the value being investigated from the determining parameter can be accepted as linear within the limits of the expected change in the determining parameter. In this case at first the system of equations is integrated under standard conditions and for a second time - at the changed value of the parameter whose effect is investigated. After this is computed the unknown correction factor. for example, for determination correction factor. For example, for determining the correction factor into the firing distance of barrel system to the deviation of the initial velocity is must by the solution of the corresponding system of equations to find firing distance at value v_0, accepted as normal, and with $v_{01} = v_0 + \Delta v_0$, after which to find the correction factor

$$\frac{\partial x_c}{\partial v_0} \approx \frac{\Delta x_c}{\Delta v_0} = \frac{x_{c v_{01}} - x_{c v_0}}{\Delta v_0}. \qquad (11.12)$$

This method requires conducting calculations with high accuracy (i.e. by the provision for a large quantity of accurate significant digits), since with low Δv_0 are possible the large errors in the determination of difference [illegible] and, therefore, correction factor.

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Figures 11.3 gives examples of the graph/diagrams of the dependences $\frac{\partial x_c}{\partial v_0}$ on v_0 for a barrel system. From curve/graphs it is evident that the value of correction factor can depend substantially on the determining parameter and the use of discrete values of correction factors is admissible only within the narrow limit of a change in the determining parameters.

The system of equations, which describes the undisturbed motion, and the system of equations, written taking into account the perturbation factors, they must be comprised depending on specific problem. The systems of equations, comprised with the frequently utilized assumptions, are obtained in chapter III. The perturbation factors can be represented in the form of additional accelerations along the axes of the coordinates, in which is comprised basic system

of equations. For the inactive legs of the unguided rockets and trajectories of artillery shells with independent alternating/variable t (time) as the system of equations, which describes the undisturbed flight, can be undertaken system (5.7) with the addition of one equation, which describes action in side direction along coordinate z

$$\ddot{x}=-E\dot{x};\quad \ddot{y}=-E\dot{y}-g;\quad \ddot{z}=-E\dot{z}. \qquad (11.13)$$

The additional accelerations along the axes, caused by the effect of the perturbation factor, let us designate respectively ε_x, ε_y and ε_z. Then the system of equations, written in general form taking into account the influence of the perturbation factor, will take the form

$$\ddot{x}=-E\dot{x}+\varepsilon_x;\quad \ddot{y}=-E\dot{y}-g+\varepsilon_y;\quad \ddot{z}=-E\dot{z}+\varepsilon_z. \qquad (11.14)$$

For example, with the wish to consider the rotational effect of the Earth on rocket flight, it is necessary to compare (2.33), (2.35) and (3.75). After comparison we will obtain

$$\varepsilon_x=\Omega^2 x-2\Omega v_z;\quad \varepsilon_y=0;\quad \varepsilon_z=\Omega^2 z+2\Omega v_x.$$

For the systems in which on the left side of the equations do not stand directly the acceleration, the terms, which consider the perturbation factors, will be different from ε_x, ε_y and ε_z.

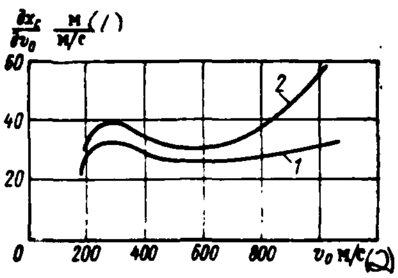

Fig. 11.3. Curve/graphs $\frac{\partial x_c}{\partial v_0} = f(v_0)$ for the barrel system whose projectile has the ballistic coefficient of c = 0.2: 1 - at θ_0 = 25°; 2 - at θ_0 = 55°.

Key: (1). m/(m/s). (2). m/s.

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For example, with independence alternating/variable x as the system of equations, which describes the undisturbed flight, let us take system (5.8)

$$u'_x = -E;\quad p'_x = -\frac{g}{u^2};\quad y'_x = p;\quad t'_x = \frac{1}{u}.$$

The system of equations written taking into account the perturbation factors, will take the form

$$u'_x = -E + \varepsilon_u;\quad p'_x = -\frac{g}{u^2} + \varepsilon_p;\quad y'_x = p;\quad t'_x = \frac{1}{u}. \qquad (11.15)$$

We convert values u'_x and p'_x with consideration of (11.14)

$$u'_x = \ddot{x} t'_x = \frac{\ddot{x}}{u} = -E + \frac{\varepsilon_x}{u};$$

$$p'_x = \dot{p} t'_x = \frac{d}{dt}\left(\frac{\dot{y}}{\dot{x}}\right)\frac{dt}{dx} = \frac{\ddot{y}\dot{x} - \ddot{x}\dot{y}}{\dot{x}^3}.$$

Remembering that $\dot{x} = u$ and by utilizing the first two equations of system (11.14), we will obtain

$$p'_x = -\frac{g}{u^2} + \frac{1}{u^2}(\varepsilon_y - \varepsilon_x p).$$

Thus, from the comparison of the obtained equations with (11.15) we have

$$\varepsilon_u = \frac{\varepsilon_x}{u} \text{ and } \varepsilon_p = \frac{1}{u^2}(\varepsilon_y - \varepsilon_x p). \quad (11.16)$$

For example, for a planar trajectory the terms, which consider correction for a change in value and direction of the acceleration of gravity in accordance with [9] are approximately equal to

$$\varepsilon_u = -\frac{g}{u}\frac{x}{R_3} \text{ and } \varepsilon_p = \frac{1}{R_3}(2y + xp)\frac{g}{u^2}. \quad (11.17)$$

3.2. Differential equations of corrections.

In the case of the low deviations of the determining parameters, the correction factors can be determined during the solution of differential equations of corrections. For the compilation of the equations of corrections, we will use the general method of the linearization of the differential equations, which describe the motion of the center of mass of rocket or projectile (Chapter VIII). Usually are considered only first members of expansion in Taylor

series and then for the linearization of equations, it is possible to use formula (4.75). the basic system of differential equations is selected depending on specific problem.

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As an example we will obtain the differential equations of corrections for the powered flight trajectory of the unguided rocket. In the named case we utilize a system of equations (3.79)

$$\dot{v}=\frac{P-X}{m}-g\sin\theta;\quad \dot{\theta}=-\frac{g\cos\theta}{v};\quad \dot{y}=v\sin\theta;\quad \dot{x}=v\cos\theta.$$

Let us designate right part one equation through a_v and will discover the entering it values. Let us consider that the drag coefficient depends only from Mach number and does not change with height/altitude. Let us also assume that for design data point in the trajectory a change in thrust P does not depend on change in altitude y

$$a_v=\frac{1}{m_0-|\dot{m}|t}\left[|\dot{m}|J_1g-\frac{S\varrho v^2}{2}c_x(\mathrm{M})-g\sin\theta\right]. \quad (11.18)$$

last/latter equality can be rewritten in the form of the functional dependence

$$a_v=f_1(v, \theta, y, J_1, S, \varrho, c_x(\mathrm{M}), m_0, |\dot{m}|).$$

Designating right Part Two equation through a_θ, let us write the functional dependence

$$a_\theta=f_2(v, \theta).$$

Entering similarly, we can write for the third and fourth

equations

$$a_y = f_3(v, \theta) \text{ and } a_x = f_4(v, \theta).$$

Utilizing formulas of linearization (8.15), we will obtain the system of the differential equations of the corrections

$$\left.\begin{aligned}
&\frac{d}{dt}(\delta v) = \frac{\partial a_v}{\partial v}\delta v + \frac{\partial a_v}{\partial \theta}\delta\theta + \frac{\partial a_v}{\partial y}\delta y + \frac{\partial a_v}{\partial J_1}\delta J_1 + \frac{\partial a_v}{\partial Q}\delta Q + \\
&\qquad + \frac{\partial a_v}{\partial c_x}\delta c_x + \frac{\partial a_v}{\partial m_0}\delta m_0 + \frac{\partial a_v}{\partial |\dot m|}\delta|\dot m|; \\
&\frac{d}{dt}(\delta\theta) = \frac{\partial a_\theta}{\partial v}\delta v + \frac{\partial a_\theta}{\partial \theta}\delta\theta; \\
&\frac{d}{dt}(\delta y) = \frac{\partial a_y}{\partial v}\delta v + \frac{\partial a_y}{\partial \theta}\delta\theta; \\
&\frac{d}{dt}(\delta x) = \frac{\partial a_x}{\partial v}\delta v + \frac{\partial a_x}{\partial \theta}\delta\theta.
\end{aligned}\right\} \quad (11.19)$$

The determination of ballistic derivatives $\frac{\partial a_i}{\partial j}$ in many instances represents although not complex, laborious problem.

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For the adopted by us system we will obtain

$$
\left.
\begin{array}{l}
1.\ a_{vv}=\frac{\partial a_v}{\partial v}=-\frac{S\rho v}{m}\left[c_x(M)+\frac{M}{2}\,\frac{\partial c_x(M)}{\partial M}\right];\\
2.\ a_{v\theta}=\frac{\partial a_v}{\partial \theta}=-g\cos\theta;\\
3.\ a_{vy}=\frac{\partial a_v}{\partial y}=-\frac{1}{m}\,S\,\frac{v^2}{2}\,c_x(M)\frac{\partial \rho}{\partial y};\\
4.\ a_{vJ_1}=\frac{\partial a_v}{\partial J_1}=\frac{P}{m}\cdot\frac{1}{J_1};\\
5.\ a_{v\rho}=\frac{\partial a_v}{\partial \rho}=-\frac{X}{m}\cdot\frac{1}{\rho};\\
6.\ a_{vc_x}=\frac{\partial a_v}{\partial c_x}=-\frac{X}{m}\cdot\frac{1}{c_x(M)};\\
7.\ a_{vm_0}=\frac{\partial a_v}{\partial m_0}=-\frac{P-X}{m}\cdot\frac{1}{m};\\
8.\ a_{v|\dot m|}=\frac{\partial a_v}{\partial |\dot m|}=\frac{1}{|\dot m|}\left[\frac{P}{m}+\frac{|\dot m|\,t}{m}\left(\frac{P-X}{m}\right)\right];\\
9.\ a_{\theta v}=\frac{\partial a_\theta}{\partial v}=\frac{g\cos\theta}{v^2};\\
10.\ a_{\theta\theta}=\frac{\partial a_\theta}{\partial \theta}=\frac{g\sin\theta}{v};\\
11.\ a_{yv}=\frac{\partial a_y}{\partial v}=\sin\theta;\\
12.\ a_{y\theta}=\frac{\partial a_y}{\partial \theta}=v\cos\theta;\\
13.\ a_{xv}=\frac{\partial a_x}{\partial v}=\cos\theta;\\
14.\ a_{x\theta}=\frac{\partial a_x}{\partial \theta}=-v\sin\theta.
\end{array}
\right\}\quad (11.20)
$$

After this the system cf the differential equations of corrections cr as cccasicnally referred to as, system of equations in deviaticns let us write in this fcrm:

$$
\left.
\begin{array}{l}
\frac{d}{dt}(\delta v)=a_{vv}\delta v+a_{v\theta}\delta\theta+a_{vy}\delta y+a_{vJ_1}\delta J_1+a_{vc_x}\delta c_x+\\
\qquad +a_{vm_0}\delta m_0+a_{v|\dot m|}\delta|\dot m|;\\
\frac{d}{dt}(\delta\theta)=a_{\theta v}\delta v+a_{\theta\theta}\delta\theta;\\
\frac{d}{dt}(\delta y)=a_{yv}\delta v+a_{y\theta}\delta\theta;\\
\frac{d}{dt}(\delta x)=a_{xv}\delta v+a_{x\theta}\delta\theta.
\end{array}
\right\}\quad (11.21)
$$

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System of equations in deviations with independent alternating/variable t, suitable for the calculation of corrections and correction factors for passive phases of trajectories of the unguided rockets and trajectories of the projectiles of barrel systems, we will obtain from system of equations (11.21). With the constant mass of projectile $P = J_1 = |\dot{m}| = 0$ and the first equation of system (11.19) can be written thus:

$$\frac{d}{dt}(\delta v) = \frac{\partial a_v}{\partial v}\delta v + \frac{\partial a_v}{\partial \theta}\delta\theta + \frac{\partial a_v}{\partial y}\delta y + \frac{\partial a_v}{\partial Q}\delta Q + \\ + \frac{\partial a_v}{\partial c_x}\delta c_x + \frac{\partial a_v}{\partial m_0}\delta m_0.$$

Respectively in the formulas of coefficients (11.20) it is necessary to take

$$a_{vm_0} = \frac{X}{m_0^2}$$

and in the first, third, by the heel, the sixth formulas to equate $m = m_0$. The remaining formulas of coefficients will remain without change. The system of the differential equations of corrections

taking into account this will take the form

$$\left.\begin{aligned}&\frac{d}{dt}(\delta v)=a_{vv}\delta v+a_{v\theta}\delta\theta+a_{vy}\delta y+a_{vq}\delta q+\\&\qquad+a_{vc_x}\delta c_x+a_{vm_0}\delta m_0;\\&\frac{d}{dt}(\delta\theta)=a_{\theta v}\delta v+a_{\theta\theta}\delta\theta;\\&\frac{d}{dt}(\delta y)=a_{yv}\delta v+a_{y\theta}\delta\theta;\\&\frac{d}{dt}(\delta x)=a_{xv}\delta v+a_{x\theta}\delta\theta.\end{aligned}\right\}\qquad(11.22)$$

We will obtain the system of equations in deviations, suitable for determining the corrections into the trajectory elements of the projectiles of constant mass with independent alternating/variable x. As basic system let us take system (5.8), which describes the motion of the center of mass of artillery shell or unguided rocket on inactive leg in the dense layers of the atmosphere

$$u'_x=-E;\quad p'_x=-\frac{g}{u^2};\quad y'_x=p;\quad t'_x=\frac{1}{u}.$$

Let us recall that $E=cH_\tau(y)G(v_\tau)$, where

$$v_\tau=u\sqrt{1+p^2}\sqrt{\frac{\tau_{0N}}{\tau}}.$$

Since $H_\tau=f(y)$ and $\tau = f_1(y)$, the $E = f(u, p, y)$.

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Let us designate by analogy with previous

$$a_u=f_1(u,\ p,\ y);\quad a_p=f_2(u);\quad a_y=f_3(p);\quad a_t=f_4(u),$$

and then

$$u'_x=a_u;\quad p'_x=a_p;\quad y'_x=a_y;\quad t'_x=a_t.$$

Carrying out the linearization of the equations of system, accepted by us as basic, we will obtain the system of differential equations in the deviations

$$\left.\begin{aligned}&\frac{d}{dx}(\delta u)=+a_{uu}\delta u+a_{up}\delta p+a_{uy}\delta y;\\&\frac{d}{dx}(\delta p)=a_{pu}\delta u;\ \frac{d}{dx_1}(\delta y)=a_{yp}\delta p;\ \frac{d}{dx}(\delta t)=a_{tu}\delta u.\end{aligned}\right\} \quad (11.23)$$

Let us find first the values of the coefficients a_{ij}, entering the first equation of system (11.23),

$$a_{uu}=\frac{\partial a_u}{\partial u}=\frac{-\partial E}{\partial u};\ a_{up}=\frac{\partial a_u}{\partial p}=\frac{-\partial E}{\partial p};\ a_{uy}=\frac{\partial a_u}{\partial y}=\frac{-\partial E}{\partial y}.$$

The values of ballistic derivatives let us find, utilizing (5.6) and (11.23). Differentiating E on u, from (5.6) we will obtain

$$\frac{\partial E}{\partial u}=\frac{\partial E}{\partial v_\tau}\cdot\frac{\partial v_\tau}{\partial u}=cH_\tau(y)G'(v_\tau)\sqrt{1+p^2}\sqrt{\frac{\tau_{0N}}{\tau}}. \quad (11.24)$$

Multiplying numerator and denominator on $uG(v_\tau)$ and designating

$$\frac{G'(v_\tau)}{G(v_\tau)}v_\tau=f(v_\tau), \quad (11.25)$$

we will obtain

$$a_{uu}=-\frac{E}{u}f(v_\tau). \quad (11.26)$$

Differentiating E on p, we will obtain

$$\frac{\partial E}{\partial p}=\frac{\partial E}{\partial v_\tau}\cdot\frac{\partial v_\tau}{\partial p}=cH_\tau(y)G'(v_\tau)u\frac{p}{\sqrt{1+p^2}}\sqrt{\frac{\tau_{0N}}{\tau}}.$$

After multiplying numerator and denominator on $G(v_\tau)\sqrt{1+p^2}$, we will obtain

$$a_{up}=-\frac{Ep}{1+p^2}\cdot f(v_\tau). \quad (11.27)$$

More laborious proves to be determination a_{uy}, since E) depends on y through values $H_\tau(y)$ and $v_\tau=f(y)$:

$$\frac{\partial E}{\partial y}=\frac{\partial E}{\partial[H_\tau(y)]}\cdot\frac{\partial[H_\tau(y)]}{\partial y}+\frac{\partial E}{\partial v_\tau}\cdot\frac{\partial v_\tau}{\partial y}.$$

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From dependence for E, let us have

$$\frac{\partial E}{\partial [H_\tau(y)]}=cG(v_\tau)=\frac{E}{H_\tau(y)}.$$

For determination $\frac{\partial [H_\tau(y)]}{\partial y}$ we will use formula (2.54). after substituting it into expression for $H_\tau(y)$

$$H_\tau(y)=\sqrt{\frac{\tau_{0N}}{\tau}}\,e^{-\frac{1}{R}\int\limits_0^y\frac{dy}{\tau}}. \qquad (11.28)$$

Let us take the logarithm expression (11.28):

$$\ln H_\tau(y)=\frac{1}{2}\ln\tau_{0N}-\frac{1}{2}\ln\tau-\frac{1}{R}\int\limits_0^y\frac{dy}{\tau}.$$

Differentiating on y converting, we will obtain

$$\frac{\partial [H_\tau(y)]}{\partial y}=-H_\tau(y)\left(\frac{\tau'_y}{2\tau}+\frac{1}{R\tau}\right). \qquad (11.29)$$

Furthermore,

$$\frac{\partial E}{\partial v_\tau}=cH_\tau(y)G'(v_\tau)=\frac{E}{v_\tau}f(v_\tau); \qquad (11.30)$$

$$\frac{\partial v_\tau}{\partial y}=u\sqrt{1+p^2}\left(\frac{-\tau'_y\sqrt{\tau_{0N}}}{2\tau\sqrt{\tau}}\right)=-v_\tau\frac{\tau'_y}{2\tau}. \qquad (11.31)$$

Substituting the value of partial derivatives, we will obtain after the conversions

$$a_{uy}=-\frac{\partial E}{\partial y}=\frac{E}{\tau}\left(\frac{f(v_\tau)+1}{2}\tau'_y+\frac{1}{R}\right). \qquad (11.32)$$

The coefficients of last/latter three equations of system (11.28) will be equal to

$$a_{pu}=\frac{2p}{u^2};\ a_{yp}=1;\ a_{uu}=-\frac{1}{u^2}. \qquad (11.33)$$

3.3. Determination of correction factors by the integration of differential equations of corrections.

The integration of equations in deviations makes it possible to obtain correction factors and corrections into the motion characteristics, presented in the left sides of the equations, for any fixed/recorded point in the trajectory. Without supplementary simplifications the system of equations of corrections in the general case is solved only by numerical integration or in the analog electronic mathematical computers of continuous action.

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Coefficients a_{ij} are calculated from the data of the undisturbed trajectory.

As an example let us examine the solution of system (11.21). Let us determine correction factors for the end/lead of powered flight trajectory in the following hypothetical initial data, necessary for the solution of assigned mission:

midsection - $S = 0.2289\ m^2$;

initial mass - $m_0 = 177.18\ kg{\cdot}s^2/m$;

the consumption of mass - $|\dot{m}|$ = 23.2 kg·s²/m·s;

thrust - P = 45.4 t;

the operating time of engine - $t_k = 2.4$ s;

the initial velocity - v_0 = 35.5 m/s;

initial angle of departure θ_0 = 25°00';

spot height of start - y_0 = 0.

Cell/elements of the undisturbed trajectory, which correspond to initial data, placed in Table 11.1.

For convenience in the introduction of some deviations the first equation of system (11.21) let us convert after isolating in its right side separate dimensionless quantities - ratios of deviation of the determining parameter to its complete value:

$$\frac{\Delta J_1}{J_1};\ \frac{\Delta q}{q};\ \frac{\Delta c_x}{c_x};\ \frac{\Delta m_0}{m_0}\ \text{and}\ \frac{\Delta|\dot{m}|}{|\dot{m}|}.$$

Table 11.1. Cell/elements of the undisturbed trajectory.

(1) № по пор.	(2) Элемент				
	t, c	v, (3) м/с	θ	y, м	$c_x(M)$
0	0	35,5	25°00′	0	0,3060
1	0,2	86,6	23°04′	3,2	0,3060
2	0,4	139,2	22°11′	4,3	0,3060
3	0,6	193,2	21°34′	9,8	0,3066
4	0,8	248,5	21°05′	19,6	0,3180
5	1,0	305,4	20°44′	33,8	0,3700
6	1,2	363,5	20°25′	56,6	0,5933
7	1,4	422,9	20°09′	88,7	0,6182
8	1,6	483,5	19°55′	128,1	0,5950
9	1,8	546,5	19°43′	171,3	0,5415
10	2,0	610,0	19°32′	219,8	0,5100
11	2,2	675,0	19°22′	268,1	0,4850
12	2,4	742,5	19°13′	322,2	0,4580

Key: (1). on pores. (2). Cell/element. (3). m/s.

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In this case, instead of the coefficients a_{ij} into equation, will enter the following values:

instead of a_{vJ_1} $f_{vJ_1}=a_{vJ_1}J_1=\frac{P}{m}$;

instead of a_{vQ} and a_{vc_x} $f_{vQ}=a_{vQ}Q=f_{vc_x}=a_{vc_x}c_x=-\frac{X}{m}$;

instead of a_{vm_0} $a_{vm_0}m_0=f_{vm_0}=-\frac{P-X}{m}\,\frac{m_0}{m}$;

instead of $a_{v|\dot{m}|}$ $a_{v|\dot{m}|}|\dot{m}| = f_{v|\dot{m}|} = \frac{P}{m} + \frac{|\dot{m}|t(P-X)}{m^2}$.

Taking into account this equation will obtain the form

$$\frac{d}{dt}(\delta v) = a_{vv}\delta v + a_{v\theta}\delta\theta + a_{vy}\delta y + f_{vJ_1}\frac{\delta J_1}{J_1} + f_{vQ}\frac{\delta Q}{Q} +$$

$$+ f_{vc_x}\frac{\delta c_x}{c_x} + f_{vm_0}\frac{\delta m_0}{m_0} + f_{v|\dot{m}|}\frac{\delta|\dot{m}|}{|\dot{m}|}.$$

Coefficients (11.20) of the differential equations of corrections taking into account new values f_{ij} let us present in the form of curve/graphs and tables, which contain the average values of coefficients on the section of approximation. Since the cell/elements of supporting trajectory usually are assigned on argument t, and partial derivatives it proves to be necessary to calculate according to argument M or y, then it is necessary to introduce into examination intermediate derivatives. For example, for the determination

$$a_{vv} = \frac{\partial a_v}{\partial v} = -\frac{S\varrho v}{m}\left[c_x(M) + \frac{M}{2}\frac{\partial c_x(M)}{\partial M}\right]$$

it is necessary to calculate

$$\frac{\partial c_x(M)}{\partial M} = \frac{\partial c_x(M)}{\partial t}\cdot\frac{\partial t}{\partial M}.$$

for determining the partial derivatives from t, we will use the formulas of counted differentiation (6.28)

$$\left(\frac{\partial c_x(M)}{\partial t}\right)_n = \frac{c_x(M)_{n+1} - c_x(M)_{n-1}}{2h_t};$$

$$\left(\frac{\partial M}{\partial t}\right)_n = \frac{M_{n+1} - M_{n-1}}{2h_t},$$

where h_t — a space on time.

For determining the values of derivative at the first and last/latter points $\left(\frac{\partial M}{\partial t}\right)_0$ and $\left(\frac{\partial M}{\partial t}\right)_{12}$ it is necessary to use Newton's first and second interpolation formulas.

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The results of calculation $\frac{\partial c_x(M)}{\partial t}$ and $\frac{\partial M}{\partial t}$ are given in Table 11.2. An example according to the calculation of coefficients a_{vv} and a_{xt} is given in Table 11.3.

In terms of the calculated values a_{ij} we construct the curve/graphs of coefficients and carry out stepped approximation through equality the areas, limited by section curved curve/graph and by straight line. The averaged values of coefficients for a numerical example in question are given in Table 11.4. The characteristic curve/graphs of coefficients are given to Fig. 11.4-11.9.

The values of the voltages of electron analogue by which with transition to machine system of equations are replaced the real values, are connected with each other by the appropriate scale factors, for example:

$U_{\delta v}=\mu_{\delta v}\delta v,\ U_{\delta\theta}=\mu_{\delta\theta}\delta\theta, \ldots,$ where

$\mu_{\delta v}$ V/m/s, $\mu_{\delta\theta}$ V/rad. - scales on voltages for a velocity increment and flight path angle.

The numerical values of the scales of supporting/reference values are selected from the condition of obtaining the maximum voltage, which characterizes real value, are not more than 80-100 V.

Table 11.2. Derivatives of $c_x(M)$ and M on time.

(1) № по пор.	t	$\frac{\partial c_x(M)}{\partial t}$	$\frac{\partial M}{\partial t}$	$\frac{\partial c_x(M)}{\partial M}$
0	0	0	0,724	0
1	0,2	0	0,762	0
2	0,4	0,0015	0,785	0,00191
3	0,6	0,0300	0,802	0,0374
4	0,8	0,1585	0,822	0,193
5	1,0	0,6882	0,845	0,815
6	1,2	0,6205	0,865	0,718
7	1,4	0,0042	0,882	0,00477
8	1,6	−0,1918	0,908	−0,211
9	1,8	−0,2125	0,930	−0,2284
10	2,0	−0,1412	0,945	−0,1494
11	2,2	−0,1425	0,970	−0,1469
12	2,4	−0,3810	1,022	−0,373

Key: (1). on pores.

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The numerical values of the scales of the variable coefficients of the realizable with the aid of units variable coefficients, must not exceed 1.0. For further use scales it is expedient to round off to the nearest convenient number.

Let us rewrite system of equations (11.21) in the form of machine system with the introduction of stresses in dimensional values, bearing in mind that functions $f_{vJ_1}, f_{vm_0}, f_{v|\dot{m}|}, f_{vQ}$ and f_{vc_x} in our case represent by themselves supporting/reference values, but functions $a_{vv}, a_{v\theta}, a_{vy}, a_{\theta v} \ldots a_{x\theta}$ - variable coefficients when $\delta v, \delta\theta, \delta y$ and so forth

$$
\begin{aligned}
&\frac{d}{d\tau}(U_{\delta v}) = -k_{vv}|\overset{*}{a}_{vv}|U_{\delta v} - k_{v\theta}|\overset{*}{a}_{v\theta}|U_{\delta\theta} + \\
&+ k_{vy}\overset{*}{a}_{vy}U_{\delta y} + k_{vJ_1}U_{f_{vJ_1}}\frac{\delta J_1}{J_1} - k_{vQ}U_{|f_{vQ}|}\frac{\Delta Q}{Q} - \\
&- k_{vc_x}U_{|f_{vc_x}|}\frac{\delta c_x}{c_x} - k_{vm_0}U_{|f_{vm_0}|}\frac{\delta m_0}{m_0} + k_{v|\dot{m}|}U_{f_{v|\dot{m}|}}\frac{d|\dot{m}|}{|\dot{m}|}; \\
&\frac{d}{d\tau}(U_{\delta\theta}) = k_{\theta v}\overset{*}{a}_{\theta v}U_{\delta v} + k_{\theta\theta}\overset{*}{a}_{\theta\theta}U_{\delta\theta}; \\
&\frac{d}{d\tau}(U_{\delta y}) = k_{yv}\overset{*}{a}_{yv}U_{\delta v} + k_{y\theta}\overset{*}{a}_{y\theta}U_{\delta\theta}; \\
&\frac{d}{d\tau}(U_{\delta x}) = k_{xv}\overset{*}{a}_{xv}U_{\delta v} - k_{x\theta}|\overset{*}{a}_{x\theta}|U_{\delta\theta}.
\end{aligned}
$$

In this system $\overset{*}{a}_{ij} = \mu_{a_{ij}}a_{ij}$.

Table 11.3. Example of the calculation of coefficients $|a_{vv}|$ and $|a_{x\theta}|$

№ по пор.(1)	t	$\frac{M}{2}$	$\frac{M}{2}\frac{\partial c_x(M)}{\partial M}$	$c_x\left(M+\frac{M}{2}\frac{\partial c_x(M)}{\partial M}\right)$	$q\left[c_x(M)+\frac{M}{2}\frac{\partial c_x(M)}{\partial M}\right]$	$m=m_0-\lvert\dot m\rvert t$	$\lvert a_{vv}\rvert$	$\lvert a_{x\theta}\rvert$
0	0	0,052	0	0,3060	10,86	177,18	0,00173	15,00
1	0,2	0,127	0	0,3060	26,51	172,54	0,00438	33,93
2	0,4	0,204	0,00039	0,3064	42,6	167,90	0,00726	52,55
3	0,6	0,284	0,0105	0,3172	61,3	163,26	0,0107	71,02
4	0,8	0,365	0,0704	0,3884	96,5	158,62	0,0174	89,39
5	1,0	0,448	0,3650	0,7350	224,3	153,98	0,0416	108,1
6	1,2	0,534	0,3838	0,9771	355,0	149,34	0,068	126,8
7	1,4	0,622	0,0030	0,6212	262,8	144,70	0,0518	145,7
8	1,6	0,710	−0,15	0,4450	215,0	140,06	0,0438	164,7
9	1,8	0,803	−0,1835	0,3580	195,7	135,42	0,0413	184,4
10	2,0	0,896	−0,1338	0,3762	229,4	130,78	0,0502	203,9
11	2,2	0,992	−0,1455	0,3395	229,0	126,14	0,0519	223,9
12	2,4	1,090	−0,406	0,0470	34,9	121,50	0,00822	244,4

Key: (1). on pores.

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Table 11.4. Averaged values of coefficients a_{ij} and f_{ij}

t	с	0—0,2	0,2—0,4	0,4—0,6	0,6—0,8	0,8—1,0	1,0—1,2	1,2—1,4	1,4—1,6	1,6—1,8	1,8—2,0	2,0—2,2	2,2—2,4
$\lvert a_{vv}\rvert$	1/с	0,003	0,00565	0,00884	0,0137	0,0278	0,0578	0,0605	0,0468	0,0422	0,0444	0,0516	0,0435
$\lvert a_{x\theta}\rvert$	м/с	24	43	61,6	80	99	117,6	137	155	174,5	194	214	234
$a_{\theta v}\cdot 10^5$	1/м	375	75	34	19	12	8	6	4,5	3,5	3	2,5	1,8
$\lvert f_{vm_0}\rvert$	м/с²	263	279	293	308	328	347	362	381	403	429	457	487
$f_v\lvert \dot m\rvert$	м/с²	260	275	290	310	330	350	370	400	420	440	470	510
a_{xv}	—	0,9138	0,923	0,928	0,932	0,934	0,936	0,938	0,939	0,940	0,941	0,943	0,944
$\lvert a_{v\theta}\rvert$	м/с²	8,978	9,057	9,105	9,138	9,164	9,184	9,200	9,216	9,230	9,242	9,252	9,259
$a_{vy}\cdot 10^3$	1/с²	0,00	0,03	0,06	0,20	0,38	0,56	0,84	1,10	1,40	1,80	2,50	3,40
f_{vJ_1}	м/с²	259	266	274	282	290	299	308	317	328	341	353	367
$a_{y\theta}$	м/с	56	101	152	206	258	312	363	425	476	545	601	687
a_{yv}	—	0,425	0,386	0,375	0,365	0,356	0,351	0,346	0,342	0,339	0,335	0,332	0,330
$\lvert f_{v\alpha}\rvert = \lvert f_{vc_x}\rvert$	м/с²	0,10	0,30	0,76	1,40	2,40	4,80	8,80	12,25	15,55	18,70	22,00	26,00
$a_{\theta\theta}$	1/с	0,0806	0,0345	0,022	0,016	0,0125	0,0105	0,0085	0,0075	0,0065	0,0055	0,0050	0,0045

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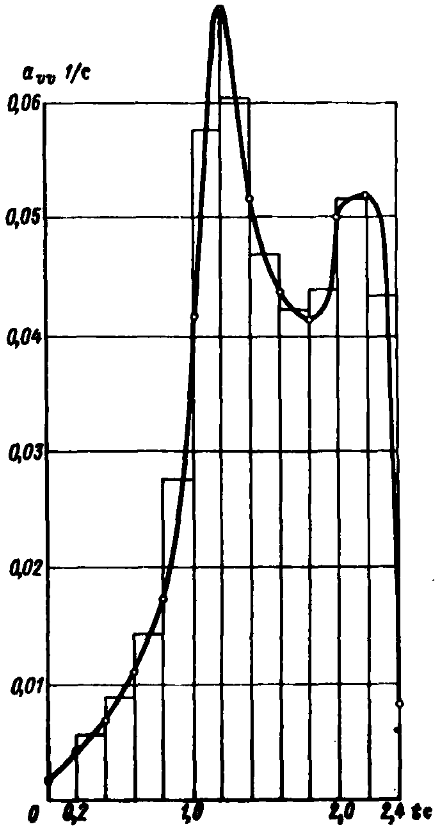

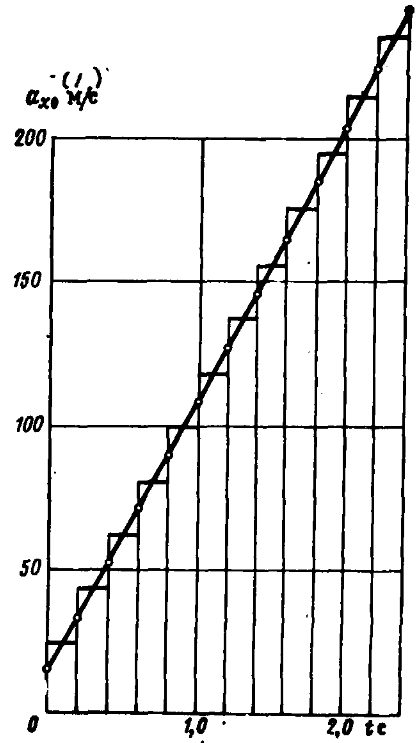

Fig. 11.4. Stepped approximation of function $a_{vv}(t)$

Fig. 11.5. To approximation of function $\dot{a}_{x0}(t)$

Key: (1). m/s.

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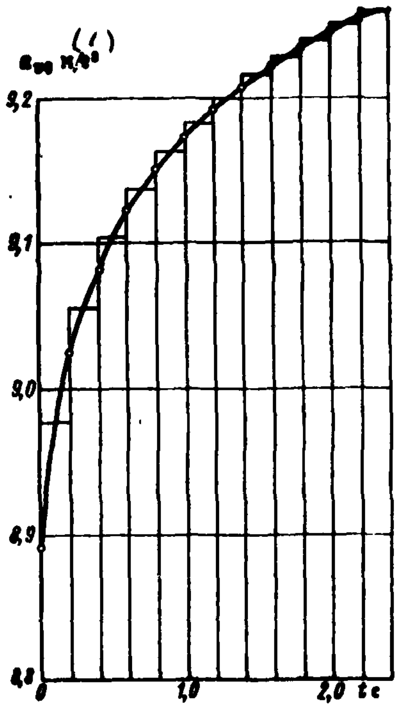

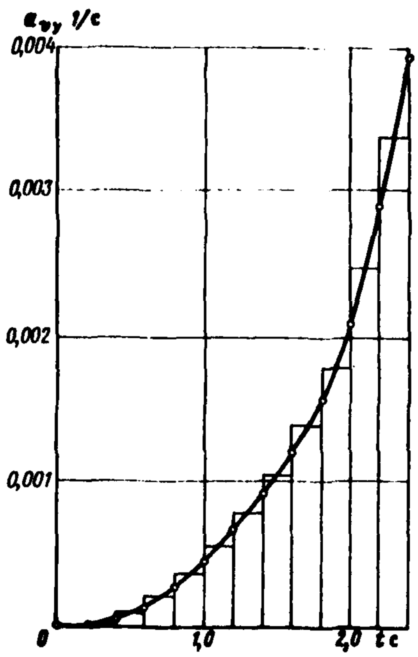

Fig. 11.6. To approximation of function $a_{v0}(t)$

Key: (1). m/s².

Fig. 11.7. To approximation of function $a_{vy}(t)$

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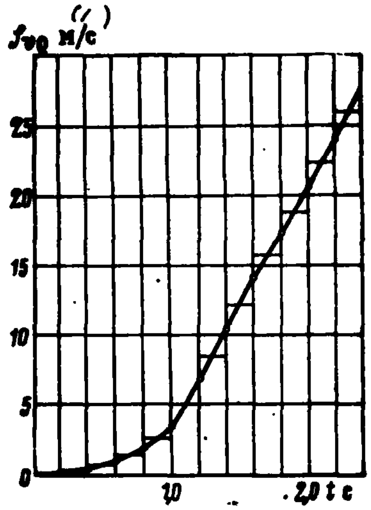

Fig 11.8

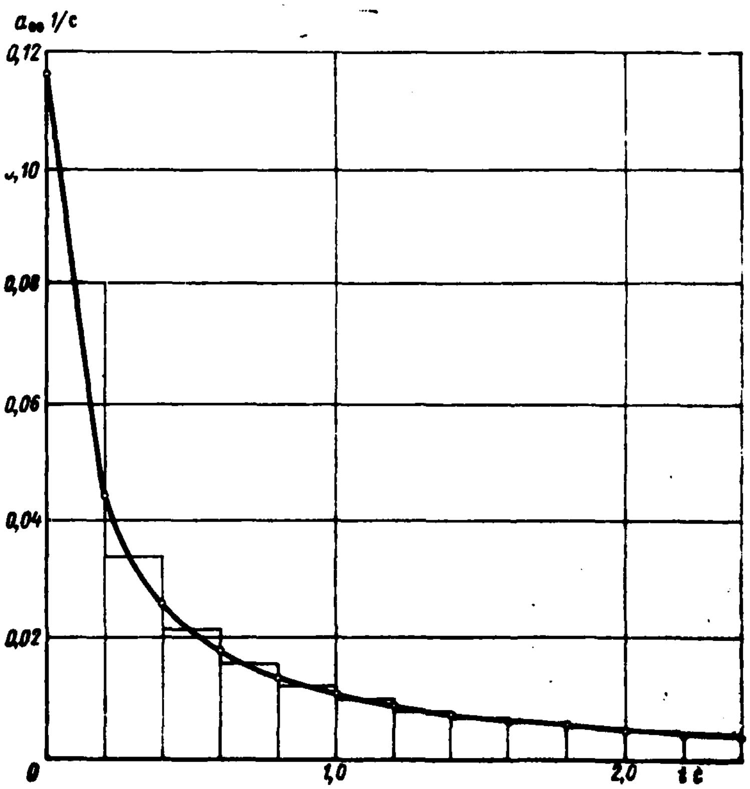

Fig 11.9.

Fig. 11.8. To approximation of function $\dot{l}_{v0}(t)$

Key: (1). m/s.

Fig. 11.9. To approximation of function $a_{u}(t)$

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Coefficients k_{ij} establish communication/connection between scales with transition from physical quantities to voltages. With some values of the scale factors, determined for an example in question,

$$\mu_{a_{vv}} = 16 \frac{1}{1/c}; \quad \mu_{bv} = 10 \frac{B}{м/c};$$
$$\mu_{a_{v\theta}} = 200 \frac{1}{1/c^2}; \quad \mu_{b_{\theta}} = 10 \frac{B}{м};$$
$$\mu_{f_{vJ_1}} = 0{,}25 \frac{B}{м/c^2}; \quad \mu_t = 5.$$

Key: (1). 1/s. (2). V/(m/s). (3). V/m.

The coefficients, which establish communication/connection between scales, will be equal to

$$k_{vv} = \frac{1}{\mu_{a_{vv}}\mu_t} = \frac{1}{16\cdot 5} = 0{,}012 \frac{1}{c};$$
$$k_{v\theta} = \frac{\mu_{bv}}{\mu_{a_{v\theta}}\mu_{b_{\theta}}\mu_t} = \frac{10}{200\cdot 10\cdot 5} = 0{,}001 \frac{1}{c};$$
$$k_{vJ_1} = \frac{\mu_{bv}}{\mu_{f_{vJ_1}}\mu_t} = \frac{10}{0{,}25\cdot 5} = 8 \frac{1}{c}.$$

On the basis of the obtained system of equations, is comprised

the block diagram of the set of problem in the simulating electronic computer. Figures 11.10 depicts one of the possible versions of the block diagram of solution on widespread Soviet model MPT-9-2. The block diagram of the solution of problem is installed in the setting field of machine. Initially assigning (possible deviations of the parameters), let us determine the effect of each of them on change $v_к, x_к, y_к, \theta_к$ individually. Let us take the initial deviations:

$$\delta x_0=\delta y_0=\delta J_1=\delta m_0=\delta|\dot{m}|=\delta c_x=\delta Q=0;$$

δv_0 = 5 m/s, which corresponds to voltage $U_{v_0}=50$ V;

$\delta\theta_0$ = 0.458 deg $\approx$ 0.46 deg, which corresponds to voltage $U_{\theta_0}=80$V. Obtained in machine values $\delta v_к, \delta x_к, \delta y_к$ and $\delta\theta_к$ in voltages are translate/transferred according to scale factors into physical quantities. For the increase of accuracy, the solution is repeated several times with positive and negative deviations δv_0 and $\delta\theta_0$. Correction factors are calculated from average values $\delta v_к, \delta x_к, \delta y_к, \delta\theta_к$ located in Table 11.5.

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Table 11.5.

(1) Параметр	$v_к$		$x_к$		$y_к$		$\theta_к$	
	$U_{\delta v_к}$, в	$\delta v_к$, м/с (2)	$U_{\delta x_к}$, в	$\delta x_к$, м	$U_{\delta y_к}$, в	$\delta y_к$, м	$U_{\delta\theta_к}$, в	$\delta\theta_к$, град. (3)
$\delta v_0=5$ м/с (2)	47	4,7	[illegible]	8,3	75	75	56	0,32
$\delta\theta_0=0,46$ град (3)	−1,[illegible]	−0,1[illegible]	[illegible]	−2,[illegible]	[illegible]	5,8	[illegible]	0,47

Key: (1). Parameter. (2). m/s. (3). deg.

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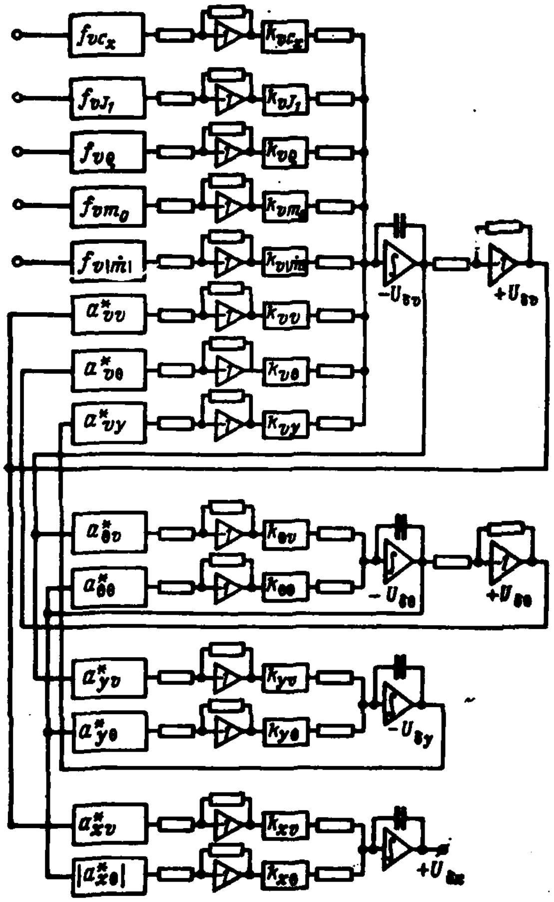

Fig. 11.10. Unit schematic of solution of system of equations in deviations on electron analogue.

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Using the data of Table 11.5 let us determine correction factors. Let us cite the calculation of some of them

$$\frac{\partial v_к}{\partial v_0} \approx \frac{\delta v_к}{\delta v_0} = \frac{4,7}{5,0} = 0,94 \text{ м/с/м/с;}$$ (1)

$$\frac{\partial x_к}{\partial v_0} \approx \frac{\delta x_к}{\delta v_0} = \frac{8,8}{5,0} = 1,76 \text{ м/м/с;}$$ (2)

$$\frac{\partial y_к}{\partial v_0} \approx \frac{\delta y_к}{\delta v_0} = \frac{7,5}{5,0} = 1,5 \text{ м/м/с;}$$ (2)

$$\frac{\partial \theta_к}{\partial v_0} \approx \frac{\delta \theta_к}{\delta v_0} = \frac{0,32}{5,0} = 0,064 \text{ град/м/с;}$$ (3)

$$\frac{\partial v_к}{\partial \theta_0} \approx \frac{\delta v_к}{\delta \theta_0} = -\frac{0,16}{0,46} = -0,35 \text{ м/с/град;}$$ (4)

$$\frac{\partial x_к}{\partial \theta_0} \approx \frac{\delta x_к}{\delta \theta_0} = -\frac{2,0}{0,46} = -4,36 \text{ м/град;}$$ (5)

$$\frac{\partial y_к}{\partial \theta_0} \approx \frac{\delta y_к}{\delta \theta_0} = \frac{5,8}{0,46} = 12,65 \text{ м/град;}$$ (5)

$$\frac{\partial \theta_к}{\partial \theta_0} \approx \frac{\delta \theta_к}{\delta \theta_0} = \frac{0,47}{0,46} = 1,02 \text{ град/град.}$$ (6)

Key: (1). m/s/m/s. (2). m/m/s. (3). degree/m/s. (4). m/s/deg. (5). m/deg. (6). deg/deg.

Similarly can be found correction factors, also, to other determining parameters. The obtained values are utilized for calculating of corrections and characteristics of scattering trajectories.

3.4. Conjugated/combined system of equations of corrections and its solution.

The system of the nonhomogeneous linear differential equations

group of which includes the systems of the differential equations of corrections, can have interconnected circuit of linear homogeneous equations. The number of variables conjugate/combined and basic of systems they coincide. For the rule of the compilation of interconnected circuit of linear homogeneous equations, the coefficients of the first equation are taken equal to the coefficients of the first members of basic system of equations, the coefficients of the second equation are taken equal to the coefficients of the second members of fundamental equations, etc. In new system all coefficients are taken with opposite signs with respect to basic. On the basis of the determination of homogeneous differential equations, in interconnected circuit let us drop/omit the absolute terms, available into basic. For an explanation let us take the system of the differential equations of corrections (11.23) and let us rewrite three first joint of equation, after introducing terms s_u and s_p, the considering disturbance/perturbations.

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For generality let us write the terms, which have zero coefficients; coefficients a_{ij} let us take by absolute values, after introducing into equations their signs in an explicit form.

$$
\left.\begin{aligned}
\frac{d}{dx}(\delta u) &= -a_{uu}\delta u - a_{up}\delta p + a_{uy}\delta y + \varepsilon_u;\\
\frac{d}{dx}(\delta p) &= a_{pu}\delta u + 0\cdot\delta p + 0\cdot\delta y + \varepsilon_p;\\
\frac{d}{dx}(\delta y) &= 0\cdot\delta u + a_{yp}\delta p + 0\cdot\delta y.
\end{aligned}\right\} \quad (11.34)
$$

Let us accept system (11.34) for basic, Dependent variables in it are u, p and y. Let us write the conjugated/combined with (11.34) system of equations, after designating variables through λ_1, λ_2 and λ_3:

$$
\left.\begin{aligned}
\frac{d\lambda_1}{dx} &= a_{uu}\lambda_1 - a_{pu}\lambda_2 - 0\lambda_3;\\
\frac{d\lambda_2}{dx} &= a_{up}\lambda_1 - 0\lambda_2 - a_{yp}\lambda_3;\\
\frac{d\lambda_3}{dx} &= -a_{uy}\lambda_1 - 0\lambda_2 - 0\lambda_3.
\end{aligned}\right| \quad (11.35)
$$

In the general case the physical sense of variables λ_i depends on the content of the coefficients of fundamental equation a_{ij} and of terms ε_u and ε_p. Let us establish communication/connection between variables basic and by that conjugate/combined systems of equations for our case. Let us multiply the equation of basic system respectively by λ_1, λ_2, and λ_3, but the equation of interconnected circuit on δu, δp and δy let us add them. After conversions let us have

$$
\lambda_1\frac{d}{dx}(\delta u) + \lambda_2\frac{d}{dx}(\delta p) + \lambda_3\frac{d}{dx}(\delta y) +
+\delta u\frac{d\lambda_1}{dx} + \delta p\frac{d\lambda_2}{dx} + \delta y\frac{d\lambda_3}{dx} = \lambda_1\varepsilon_u + \lambda_2\varepsilon_p
$$

and, further,

$$
\frac{d}{dx}(\lambda_1\delta u + \lambda_2\delta p + \lambda_3\delta y) = \lambda_1\varepsilon_u + \lambda_2\varepsilon_p. \qquad (11.36)
$$

Let us integrate the right and left of the part of the last/latter equation from x_1 to x_2. Then

$$\lambda_{12}\delta u_2+\lambda_{22}\delta p_2+\lambda_{32}\delta y_2-\lambda_{11}\delta u_1-\lambda_{21}\delta p_1-\lambda_{31}\delta y_1=$$
$$=\int_{x_1}^{x_2}(\lambda_1 \varepsilon_u+\lambda_2 \varepsilon_p)dx. \qquad (11.37)$$

The obtained equation is common/general/total and it is correct under any initial conditions.

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For the basic trajectory in question integration limits of interconnected circuit can be selected in the manner that this is convenient for the solution of the specific problem of the theory of corrections. In many instances it proves to be advisable to integrate conjugate system of differential equations, beginning with final point in the trajectory.

For example, during the determination of corrections for the inactive legs of the unguided rockets of class "surface - surface" and the trajectories of the projectiles of terrestrial artillery interconnected circuit (11.35) is integrated from the impact point in the projectile to the beginning of the trajectory phase in question. Let us write equation (11.37) for the passive section of ground-based

missile trajectory

$$\lambda_{1C}\delta u_C+\lambda_{2C}\delta p_C+\lambda_{3C}\delta y_C-\lambda_{1\text{н}}\delta u_\text{н}-$$
$$-\lambda_{2\text{н}}\delta p_\text{н}-\lambda_{3\text{н}}\delta y_\text{н}=\int_{x_\text{н}}^{x_C}(\lambda_1\varepsilon_u+\lambda_2\varepsilon_p)\,dx. \quad (11.38)$$

In order to obtain from (11.38) the formula, which determines correction into distance, it is necessary to select one of the coefficients λ_{iC} so that it would contain δx_C, the remaining coefficients of interconnected circuit they are taken equal to zero. For an impact point (Fig. 11.11) it is possible to take

$$\delta x_C=\frac{\delta y_C}{|p_C|};$$

after comparison with the third term of equation (11.38) we will obtain

$$\lambda_{3C}=\frac{1}{|p_C|}.$$

Furthermore,

$\lambda_{1C}=0,\ \lambda_{2C}=0.$ From (11.38) let us have

$$\delta x_C=\lambda_{1\text{н}}\delta u_\text{н}+\lambda_{2\text{н}}\delta p_\text{н}+\lambda_{3\text{н}}\delta y_\text{н}+\int_{x_\text{н}}^{x_C}(\lambda_1\varepsilon_u+\lambda_2\varepsilon_p)\,dx. \quad (11.39)$$

The physical sense of values $\lambda_{1\text{н}}, \lambda_{2\text{н}}$ and $\lambda_{3\text{н}}$ is explained, after making equal to zero ε_u and ε_p and in pairs deviations $\delta u_\text{н}, \delta p_\text{н}, \delta y_\text{н}$: for example, by taking $\delta p_\text{н}=\delta y_\text{н}=0$ and $\delta u_\text{н}=1$, let us find

$$\lambda_{1\text{н}}=\frac{\delta x_C}{\delta u_\text{н}}.$$

Similarly we will obtain

$$\lambda_{2\text{н}}=\frac{\delta x_C}{\delta p_\text{н}} \text{ and } \lambda_{3\text{н}}=\frac{\delta x_C}{\delta y_\text{н}}.$$

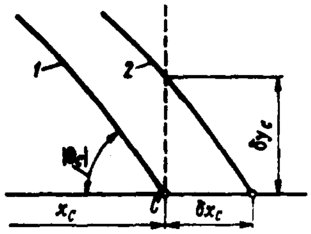

Fig. 11.11. The cuts of trajectories, adjacent to impact point: 1 - not disturbed (calculated); 2 - deformed.

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Instantaneous values λ_1, λ_2, λ_3 in function from x, including $\lambda_{1\kappa}$, $\lambda_{2\kappa}$ and $\lambda_{3\kappa}$, are obtained by the numerical integration (by ETsVM or with manual count) of interconnected circuit (11.35) under the initial conditions

$$\lambda_{1C}=0;\ \lambda_{2C}=0;\ \lambda_{3C}=\frac{1}{|p_C|}.$$

The space of integration for x is taken with minus sign, since integration is realized from one x_C to x_κ.

During the determination of correction into distance from (11.39) values $\delta u_\kappa, \delta p_\kappa$ and δy_κ they must be determined during the study of the preceding/previous trajectory phase or be specified by assignment.

If we assume $\delta u_{\kappa}=\delta p_{\kappa}=\delta y_{\kappa}=0,$ then

$$\delta x_C=\int_{x_\kappa}^{x_C}(\lambda_1\varepsilon_u+\lambda_2\varepsilon_p)\,dx. \qquad (11.40)$$

Last/latter formula makes it possible to determine correction into distance for the factors whose effect is considered through ε_u and ε_p (rotational effect of the earth/ground, a change in the meteorological conditions, etc.).

During determination δx_C for the trajectories of the projectiles of ground-based cannon-type artillery the integration of interconnected circuit (11.35) it is necessary to carry out from x_C to x = 0. Furthermore, for the point of flight it is possible to accept δy_0 = 0 and then from (11.39)

$$\delta x_C=\lambda_{10}\delta u_0+\lambda_{20}\delta p_0+\int_0^{x_C}(\lambda_1\varepsilon_u+\lambda_2\varepsilon_p)\,dx. \qquad (11.41)$$

Last/latter formula is common/general/total correcting formula for the complete horizontal firing distance ground-based artillery piece.

3.5. Determination of correction factors from the similarity of trajectories.

For calculating the corrections into the trajectory elements of the projectiles of constant mass, frequently are utilized the dependences, obtained from similarity conditions of trajectories.

Let us take the first equation of system (7.141) and let us

determine correction factors for complete distance for a change in pressure and temperature of air in the point of flight (and further along trajectory). If we assume $\tau_{0}=\tau_{0N}$, then $v_{\tau}=v_{0}$ and

$$x_C=\Phi_1(v_0,\ c^*,\ \theta_0). \qquad (11.42)$$

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After recalling that $c^*=c\frac{h_0}{h_{0N}}$, and, considering v_0 and θ_0 in (11.42) constant/invariable, let us take alternately derivative of x_C on c and on h_0.

$$\frac{\partial x_C}{\partial c}=\frac{\partial\Phi_1}{\partial c^*}\frac{\partial c^*}{\partial c}=\frac{\partial\Phi_1}{\partial c^*}\frac{h_0}{h_{0N}};$$

$$\frac{\partial x_C}{\partial h_0}=\frac{\partial\Phi_1}{\partial c^*}\frac{\partial c^*}{\partial h_0}=\frac{\partial\Phi_1}{\partial c^*}\frac{c}{h_{0N}}.$$

Comparing two last/latter equalities, we will obtain

$$\frac{\partial x_C}{\partial h_0}=\frac{\partial x_C}{\partial c}\frac{c}{h_0}. \qquad (11.43)$$

If to accept $h_0=h_{0N},\ \tau_0\neq\tau_{0N}$, the first equation of system (7.141) will take the form

$$x_C=\frac{\tau_0}{\tau_{0N}}\Phi_1(v_{\tau 0},\ c,\ \theta_0). \qquad (11.44)$$

Accepting in last/latter equality θ_0 constant/invariable and differentiating alternately on v_0 and τ_0, we will obtain after the conversions

$$\frac{\partial x_C}{\partial \tau_0}=\frac{1}{\tau_0}\left(x_C-\frac{1}{2}v_0\frac{\partial x_C}{\partial v_0}\right). \qquad (11.45)$$

From (11.43) and (11.45) it is evident that the correcting coefficients, which consider effect on the firing distance of the deviation of barometric pressure from given standard atmosphere and the temperatures of air in release point (and further along

trajectory), can be expressed through basic correction factors to the deviations of the ballistic coefficient and the initial velocity.

3.6. Analytical method can be applied in the case when the basic value being investigated is determined by the function, which allow/assumes obtaining derivatives in analytical form. Method, as a rule, is utilized during the study of the trajectories of the projectiles of constant mass. an example of the use of the named method have examined we in §1 present chapter with the assumptions, which correspond to the parabolic theory of the motion of projectile (11.6).

Obtaining ballistic derivatives in connection with the elliptical theory of the motion of the projectiles of constant mass is examined in detail in works [2], [22], [35], [46]. It is presented here the basis of the common/general/total procedure of obtaining correcting formulas and correction factors.

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Let us take the equation of elliptical trajectory, which contains in an explicit form the parameters, which correspond to the beginning of inactive leg and determining all the motion characteristics, including firing distance [2], and let us write it for point C' with

a radius $r=r_{C'}>R_з$, accepting it for the beginning of the entry of projectile into the dense layers of the atmosphere

$$\left[2r_{C'}(1+\mathrm{tg}^2\theta_н)-\frac{r_н v_н^2}{k}(r_н+r_{C'})\right]\mathrm{tg}^2\phi-$$

$$-\frac{r_н v_н^2}{k}[2r_{C'}\,\mathrm{tg}\,\theta_н\,\mathrm{tg}\,\phi+(r_н-r_{C'})]=0. \qquad (11.46)$$

The written equation can be presented in the form of the functional dependence

$$F(\theta_н,\ v_н,\ r_н,\ r_{C'},\ 2\phi)=0$$

and to write

$$\left[2r_{C'}(1+\mathrm{tg}^2\theta_н)-\frac{r_н v_н^2}{k}(r_н+r_{C'})\right]\mathrm{tg}^2\phi-$$

$$-\frac{2r_н v_н^2}{k}r_{C'}\,\mathrm{tg}\,\theta_н\,\mathrm{tg}\,\phi-\frac{r_н v_н^2}{k}(r_н-r_{C'})=F(\theta_н,\ v_н,\ r_н,\ r_{C'},\ 2\phi).$$

Hence it follows that $2\phi=f(\theta_н,\ v_н,\ r_н,\ r_{C'})$.

In connection with the spherical model of the Earth, communication/connection between angular and by linear circular by distances is determined by the relationship/ratio

$$L=L_н+2R_з\phi. \qquad (11.47)$$

It is obvious,

$$L=\varphi(v_н,\ \theta_н,\ r_н,\ r_{C'},\ L_н), \qquad (11.48)$$

and the correcting formula, obtained taking into account only first terms of expansion, in accordance with (11.2) it will take the form

$$\delta L=\frac{\partial L}{\partial v_н}\delta v_н+\frac{\partial L}{\partial \theta_н}\delta\theta_н+\frac{\partial L}{\partial r_н}\delta r_н+\frac{\partial L}{\partial r_{C'}}\delta r_{C'}+\frac{\partial L}{\partial L_н}\delta L_н.$$

Ballistic derivatives $\frac{\partial L}{\partial x_i}$ in accordance with (11.47) and (11.48) will be equal to

$$\frac{\partial L}{\partial v_{\rm н}}=\frac{\partial(2\psi)}{\partial v_{\rm н}}R_{g}^{3},\quad \frac{\partial L}{\partial \theta_{\rm н}}=\frac{\partial(2\psi)}{\partial \theta_{\rm н}}R_{g}^{3},$$

$$\frac{\partial L}{\partial r_{\rm н}}=\frac{\partial(2\psi)}{\partial r_{\rm н}}R_{g}^{3},\quad \frac{\partial L}{\partial r_{C'}}=\frac{\partial(2\psi)}{\partial r_{C'}}R_{g}^{3},$$

$$\frac{\partial L}{\partial t_{\rm н}}=1.$$

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Equation (11.46) is written in implicit form and, without isolating the determining parameters, we will obtain ballistic derivatives (correcting coefficients), using the obvious formula

$$\frac{\partial(2\psi)}{\partial \xi_{i}}=-\frac{\partial F/\partial \xi_{i}}{\partial F/\partial(2\psi)}.$$

Let us give as example the derivation of the dependence for a correction factor $\frac{\partial L}{\partial v_{\rm н}}$.

Using (11.46), we will obtain

$$\frac{\partial F}{\partial v_{\rm н}}=-\frac{2r_{\rm н}v_{\rm н}}{k}(r_{\rm н}+r_{C'})\,{\rm tg}^{2}\psi-$$

$$-4\frac{r_{\rm н}v_{\rm н}}{k}r_{C'}\,{\rm tg}\,\theta_{\rm н}\,{\rm tg}\,\psi-2\frac{r_{\rm н}v_{\rm н}}{k}(r_{\rm н}-r_{C'});\qquad(11.49)$$

$$\frac{\partial F}{\partial(2\psi)}=\left[2r_{C'}\,(1+{\rm tg}^{2}\,\theta_{\rm н})-\frac{r_{\rm н}v_{\rm н}^{2}}{k}(r_{\rm н}+r_{C'})\right]{\rm tg}\,\psi\,\frac{1}{\cos^{2}\psi}-$$

$$-\frac{r_{\rm н}v_{\rm н}^{2}}{k}r_{C'}\,{\rm tg}\,\theta_{\rm н}\,\frac{1}{\cos^{2}\psi}.\qquad(11.50)$$

Dale (11.49) to (11.50), let us have

$$\frac{\partial(2\psi)}{\partial v_н} =$$

$$= \frac{2\frac{r_н v_н}{k}\left[(r_н + r_{C'})\sin^2\psi + r_{C'}\operatorname{tg}\theta_н\sin(2\psi) + (r_н - r_{C'})\cos^2\psi\right]}{\left[2r_{C'}(1+\operatorname{tg}^2\theta_н) - \frac{r_н v_н^2}{k}(r_н + r_{C'})\right]\operatorname{tg}\psi - \frac{r_н v_н^2}{k} r_{C'}\operatorname{tg}\theta_н}. \tag{11.51}$$

Formula for the calculaticn of correcticn intc firing distance during the deviaticn only cf initial velccity takes the form

$$\delta L_{v_н} = \frac{\partial L}{\partial v_н}\delta v_н = \frac{\partial(2\psi)}{\partial v_н}R_з\delta v_н. \tag{11.52}$$

By form can be written correcting formulas also fcr ctler determining parameters

$$\delta L_{r_н} = \frac{\partial(2\psi)}{\partial r_н}R_з\delta r_н. \tag{11.53}$$

$$\delta L_{\theta_н} = \frac{\partial(2\psi)}{\partial\theta_н}R_з\delta\theta_н. \tag{11.54}$$

$$\delta L_{r_{C'}} = \frac{\partial(2\psi)}{\partial r_{C'}}R_з\delta r_{C'}. \tag{11.55}$$

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Frequently they introduce intc examinaticn value $\varkappa = rv^2/k$ which for initial conditions is equal to $\varkappa_н = \frac{r_н v_н^2}{k}$. Utilizing value $\varkappa_н$, it is necessary to bear in mind, that $\varkappa_н = f(r_н, v_н)$ and

$$\delta\varkappa_н = \frac{\partial f}{\partial r_н}\delta r_н + \frac{\partial f}{\partial v_н}\delta v_н. \tag{11.56}$$

Let us give without derivaticn the ballistic derivatives, expressed through $\varkappa_н$ [2]. When $r_{C'} = R_з$

$$\left.\begin{aligned}
\frac{\partial L}{\partial r_n} &= R_3 \frac{\varkappa_n + \frac{2R_3}{r_n}(1+\operatorname{tg}^2\theta_n)\sin^2\psi\operatorname{tg}\psi}{\varkappa_n(r_n - R_3 + R_3\operatorname{tg}\theta_n\operatorname{tg}\psi)};\\
\frac{\partial L}{\partial v_n} &= \frac{4R_3^2}{v_n}\,\frac{(1+\operatorname{tg}^2\theta_n)\sin^2\psi\operatorname{tg}\psi}{\varkappa_n(r_n - R_3 + R_3\operatorname{tg}\theta_n\operatorname{tg}\psi)};\\
\frac{\partial L}{\partial \theta_n} &= 2R_3^2\frac{(1+\operatorname{tg}^2\theta_n)(\varkappa_n - 2\operatorname{tg}\theta_n\operatorname{tg}\psi)\sin^2\psi}{\varkappa_n(r_n - R_3 + R_3\operatorname{tg}\theta_n\operatorname{tg}\psi)}.
\end{aligned}\right\} \quad (11.57)$$

For a symmetrical trajectory in the case $r_C = r_n$ we will obtain

$$\left.\begin{aligned}
\frac{\partial(2\psi)}{\partial r_n} &= \frac{\varkappa_n + 2\sin^2\psi\cdot(1+\operatorname{tg}^2\theta_n)}{\varkappa_n r_n\operatorname{tg}\theta_n};\\
\frac{\partial(2\psi)}{\partial v_n} &= \frac{4\sin^2\psi\cdot(1+\operatorname{tg}^2\theta_n)}{\varkappa_n v_n\operatorname{tg}\theta_n};\\
\frac{\partial(2\psi)}{\partial \theta_n} &= \frac{(\varkappa_n - 2\operatorname{tg}\theta_n\operatorname{tg}\psi)(1+\operatorname{tg}^2\theta_n)\sin(2\psi)}{\varkappa_n\operatorname{tg}\theta_n}.
\end{aligned}\right\} \quad (11.58)$$

Range angle in case when $r_C = R_3$, can be represented in the form of the functional dependence

$$2\psi = F_\psi(v_n, \theta_n, h_n),$$

where h_n — a height/altitude of the initial point of the elliptical trajectory above the surface of the spherical model

$$h_n = r_n - R_3.$$

earth/ground. In this case, a change in the range angle taking into account only linear terms of expansion in terms of (11.2) is equal

$$\delta(2\psi) = \frac{\partial(2\psi)}{\partial v_n}\delta v_n + \frac{\partial(2\psi)}{\partial \theta_n}\delta\theta_n + \frac{\partial(2\psi)}{\partial h_n}\delta h_n. \qquad (11.59)$$

Utilizing (7.19), I. N. Lysenko was converted (11.59) and obtained the equation, connecting in implicit form of deviation δv_n, $\delta\theta_n$ and δh_n [18].

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$$\frac{2\delta v_{\text{н}}}{v_{\text{н}}}\left(\frac{1-\cos 2\psi}{\varkappa_{\text{н}}\cos^2\theta_{\text{н}}}\right)+\frac{\delta h_{\text{н}}}{R_3}\left[1+\frac{R_3(1-\cos 2\psi)}{\varkappa_{\text{н}}r_{\text{н}}\cos^2\theta_{\text{н}}}\right]=$$
$$=\delta(2\psi)\left[\frac{\sin 2\psi}{\varkappa_{\text{н}}\cos^2\theta_{\text{н}}}-\frac{\sin(\theta_{\text{н}}+2\psi)}{\cos\theta_{\text{н}}}\right]-\delta\theta_{\text{н}}\left[\frac{\sin(\theta_{\text{н}}+2\psi)}{\cos\theta_{\text{н}}}-\right.$$
$$\left.-\frac{\sin\theta_{\text{н}}}{\cos^2\theta_{\text{н}}}\cos(\theta_{\text{н}}+2\psi)-\frac{2\sin\theta_{\text{н}}}{\varkappa_{\text{н}}\cos^3\theta_{\text{н}}}(1-\cos 2\psi)\right]. \qquad (11.60)$$

Remembering that $\delta L=R_3\delta(2\psi)$, from (11.60) we will obtain when $\delta\theta_{\text{н}}=\delta h_{\text{н}}=0$:

$$\frac{\partial L}{\partial v_{\text{н}}}=2\frac{R_3}{v_{\text{н}}}\frac{1-\cos 2\psi}{\sin 2\psi-\varkappa_{\text{н}}\cos\theta_{\text{н}}\sin(\theta_{\text{н}}+2\psi)}.$$

If one assumes that the elliptical trajectory is symmetrical and begins on the surface of the Earth, i.e., $r_{\text{н}}=r_C=R_3$ and projectile it has velocity $v_{\text{н}}=v_0$, then

$$\frac{\partial L}{\partial v_0}=2\frac{R_3}{v_0}[\sin 2\psi+\operatorname{ctg}\theta_0(1-\cos 2\psi)]. \qquad (11.61)$$

Figures 11.12 gives curve/graphs $\frac{\partial L}{\partial v_0}=f_{v_0}(\theta_0)$ with different 2ψ. From curve/graphs it is evident that in all cases the ballistic derivative $\frac{\partial L}{\partial v_0}$ has positive value. With identical angular firing distance, it decreases with an increase of the flight path angle in the beginning of passive elliptical section.

From equation (11.60) accepting $\delta v_{\text{н}}=\delta h_{\text{н}}=0$, we will obtain

$$\frac{\partial L}{\partial\theta_0}=2R_3\left[\frac{\sin 2(\theta_0+\psi)}{\sin 2\theta_0}-1\right]. \qquad (11.62)$$

Dependence $\frac{\partial L}{\partial\theta_0}=f_{\theta_0}(\theta_0)$ is given to Fig. 11.13. with an increase in the angle of departure, ballistic derivative $\frac{\partial L}{\partial\theta_0}$ decreases, transfer/converting from positive values to negative. The zero value of derivatives corresponds to the optimum flight path

angles in the beginning of passive section. With assigned the flying range the zero value of derivative answers the trajectory, obtained at the minimum initial velocity v_0.

The ballistic derivative of a charge in the distance in height/altitude gave inactive leg was equal to

$$\frac{\partial L}{\partial h_k}=2\,\mathrm{ctg}\,\theta_k-\frac{\cos(\theta_k+2\beta)}{\sin\theta_k}. \qquad (11.63)$$

The curve/graphs of dependence $\frac{\partial L}{\partial h_k}=f_h(\theta_k)$ are given to Fig. 11.14. In all cases the derivative has positive value.

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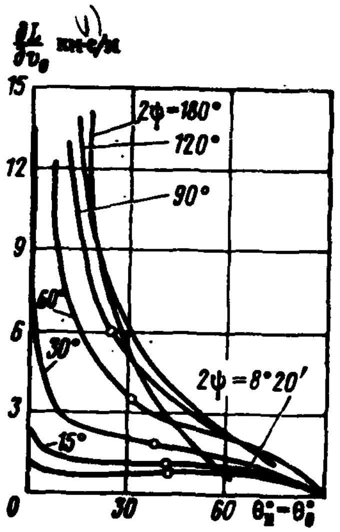
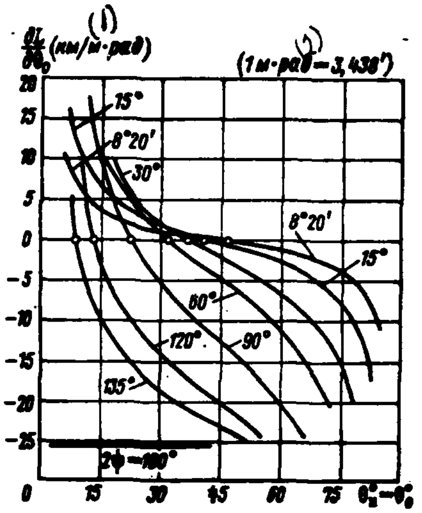


Fig. 11.12. Change of ballistic derivatives $\frac{\partial L}{\partial v_0}$ in function of angle of departure $\theta_н - \theta_0$ at different values of range angle 2ψ

Key: (1). km·s/m.



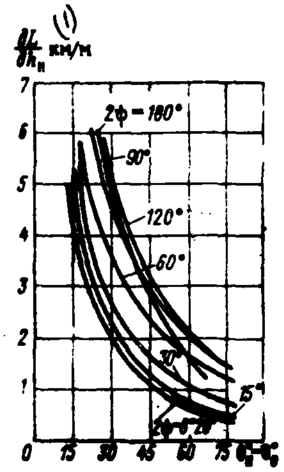


Fig. 11.14.

Fig. 11.13. Change of ballistic derivatives $\frac{\partial L}{\partial \theta_n}$ in function of angle of departure $\theta_n - \theta_0$ at different values of range angle

Key: (1). (km/m•rad). (2). m•rad.

Fig. 11.14. Change of ballistic derivatives $\frac{\partial L}{\partial h_n}$ in function of angle of departure $\theta_n - \theta_0$ at different values of range angle 2φ.

Key: (1). km/m.

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With identical distance it decreases with an increase in angle θ_n.

The analytical method of determining the correction factors can be also used in connection with different approximate solutions of the problem of external ballistics of the projectiles of constant mass, driving/moving in the dense layers of the atmosphere. Let us examine obtaining formulas and correcting coefficients for the method of pseudovelocity. We utilize formulas (7.63) and (7.65), the connecting complete firing distance with the parameters, which determine the trajectory:

$$x_C=\frac{1}{c'}(D_C-D_0); \qquad (11.64)$$

$$\sin 2\theta_0=\frac{1}{c'}\left(\frac{A_C-A_0}{D_C-D_0}-J_0\right). \qquad (11.65)$$

Substituting (11.64) in (11.65), we will obtain the formula, which connects x_C c c', θ_0 and v_0:

$$c'x_C(c'\sin 2\theta_0+J_0)=A_C-A_0. \qquad (11.66)$$

For obtaining the differential correcting formula of form (11.2) let us differentiate last/latter equality, considering variables x_C, c', v_0, θ_0. It is obvious that the terms, which contain u_C, will also be variables

$$c'x_C d(c'\sin 2\theta_0+J_0)+(c'\sin 2\theta_0+J_0)d(c'x_C)=dA_C-dA_0. \qquad (11.67)$$

Utilizing a basic conclusion of the method of pseudovelocity, let us write

$$dA=JdD.$$

Then the right side of equation (11.67) can be represented in the form

$$dA_C-dA_0=J_CdD_C-J_0dD_0.$$

Value dD_C can be expressed by the initial parameters, it differentiated (11.64); then

$$dA_C-dA_0=J_Cd(c'x_C)+J_CdD_0-J_0dD_0.$$

Carrying out the replacement of right side in (11.67), after conversion we will obtain

$$c'x_Cd(c'\sin 2\theta_0+J_0)+[c'\sin 2\theta_0-(J_C-J_0)]d(c'x_C)= \\ =(J_C-J_0)dD_0. \qquad (11.68)$$

Difference (J_C-J_0) can be determined from the basic conclusion of the method of pseudovelocity. If we in formula (7.60) replace

$$\operatorname{tg}\theta=\operatorname{tg}\theta_C=-\operatorname{tg}|\theta_C|,$$

that we will obtain

$$J_C-J_0=2c'\cos^2\theta_0(\operatorname{tg}\theta_0+\operatorname{tg}|\theta_C|)= \\ =c'\sin 2\theta_0+2c'\cos^2\theta_0\operatorname{tg}|\theta_C|.$$

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After substituting J_C-J_0 in (11.68), after conversion let us have

$$x_Cd(c'\sin 2\theta_0+J_0)-2\cos^2\theta_0\operatorname{tg}|\theta_C|d(c'x_C)= \\ =(\sin 2\theta_0+2\cos^2\theta_0\operatorname{tg}|\theta_C|)dD_0. \qquad (11.69)$$

Realizing differentiation and carrying out the replacement

$$dJ_0=-\frac{2g\,dv_0}{v_0^2 G(v_0)};\quad dD_0=-\frac{dv_0}{G(v_0)},$$

we will obtain

$$x_C\sin 2\theta_0 dc'+2c'x_C\cos 2\theta_0 d\theta_0-2gx_C\frac{dv_0}{v_0^2 G(v_0)}-$$
$$-2\cos^2\theta_0\,\mathrm{tg}\,|\theta_C|\,x_C dc'-2\cos^2\theta_0\,\mathrm{tg}\,|\theta_C|\,c'dx_C=$$
$$=-(\sin 2\theta_0+2\cos^2\theta_0\,\mathrm{tg}\,|\theta_C|)\frac{dv_0}{G(v_0)}.$$

After passing from differentials to finite low increments, we convert last/latter equality and we will obtain the differential correcting formula

$$\delta x_C=\frac{\partial x_C}{\partial c'}\delta c'+\frac{\partial x_C}{\partial v_0}\delta v_0+\frac{\partial x_C}{\partial \theta_0}\delta\theta_0.$$

After conversion let us have

$$\delta x_C=-x_C\left(1-\frac{\mathrm{tg}\,\theta_0}{\mathrm{tg}\,|\theta_C|}\right)\frac{\delta c'}{c'}+\frac{x_C\cos 2\theta_0}{\cos^2\theta_0\,\mathrm{tg}\,|\theta_C|}\delta\theta_0+$$
$$+\frac{1}{c'G(v_0)\,\mathrm{tg}\,|\theta_C|}\left[\mathrm{tg}\,\theta_0+\mathrm{tg}\,|\theta_C|-\frac{gx_C}{v_0\cos^2\theta_0}\right]\delta v_0.\quad (11.70)$$

Frequently is introduced substitution $\gamma=\frac{\mathrm{tg}\,\theta_0}{\mathrm{tg}\,|\theta_C|}$, and then the formulas of basic correction factors will take the form

$$\left.\begin{aligned}&\frac{\partial x_C}{\partial c'}=-(1-\gamma)\frac{x_C}{c'};\quad \frac{\partial x_C}{\partial\theta_0}=\frac{x_C\cos 2\theta_0}{\cos^2\theta_0\,\mathrm{tg}\,|\theta_C|};\\&\frac{\partial x_C}{\partial v_0}=\frac{1}{c'G(v_0)}\left[1+\gamma-\frac{gx_C}{v_0^2\cos^2\theta_0\,\mathrm{tg}\,|\theta_C|}\right].\end{aligned}\right\}\quad (11.71)$$

Analytical method is applicable for determining the ballistic derived and higher orders, than the first.

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Within the framework of parabolic theory, the ballistic derivatives, which characterize principal part of the change in the function, are determined by formulas (11.5). The refinement of correction is determined by three last/latter terms of formula (11.4), from which the second derivatives are equal to

$$\frac{\partial^2 x_c}{\partial v_0^2}=2\frac{\sin 2\theta_0}{g};\quad \frac{\partial^2 x_c}{\partial \theta_0^2}=-4\frac{v_0^2\sin 2\theta_0}{g};\quad \frac{\partial^2 x_c}{\partial v_0\partial\theta_0}=4\frac{v_0\cos 2\theta_0}{g};\tag{11.72}$$

Similarly can be obtained derivatives $\frac{\partial^2 L}{\partial v_n^2}$, $\frac{\partial^2 L}{\partial \theta_n^2}$ and $\frac{\partial^2 L}{\partial v_n\partial\theta_n}$ on the basis of elliptical theory.

However, the mathematical complexities of obtaining the second derivatives, the low value of the standard deviations $(\delta v_n)^2$, $(\delta\theta_n)^2$ and of product $\delta v_n\delta\theta_n$ and, as a result, the insignificant refinement of correction itself, given by these terms, lead to the fact that in the practical calculations of corrections are considered the usually only linear terms of expansion.

3.7. Tabular methods of calculating the correction factors.

Under tabular methods let us understand the methods of determining the correction factors with the aid of the various kinds of the precomputed tables. The use of tables won acceptance during

the calculation of corrections to the trajectory elements of the projectiles of cannon-type artillery. Is known the use of ballistic collections (ballistic tables) for calculating the correction factors in the method of differences and the use of tables of quite correction factors. The majorities of ballistic collections contain the cell/elements of characteristic points in the trajectory for normal meteorological conditions depending on the determining parameters c, v_0 and θ_0.

For zenith trajectories the number of determining parameters, includes time t (see page 286). The tables are comprised as a rule, with the constant space of the determining parameters and this makes it possible to utilize for the calculation of correction factors a known formula of numerical differentiation (6.28):

$$y'_x = \frac{y_{n+1} - y_{n-1}}{2h_x}. \qquad (11.73)$$

Alternately substituting in the numerator of last/latter formula the firing distance, undertaken depending on a change in the determining parameter, and into denominator - the tabular interval (space) of the determining parameter, it is possible to obtain the values of basic correction factors for the calculation of corrections into distance.

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For the ground-based cannon-type artillery

$$\left.\begin{aligned} \frac{\partial x_C}{\partial c} &= \frac{x_{C(c+h_c)} - x_{C(c-h_c)}}{2h_c}; \\ \frac{\partial x_C}{\partial v_0} &= \frac{x_{C(v_0+h_{v_0})} - x_{C(v_0-h_{v_0})}}{2h_{v_0}}; \\ \frac{\partial x_C}{\partial \theta_0} &= \frac{x_{C(\theta_0+h_{\theta_0})} - x_{C(\theta_0-h_{\theta_0})}}{2h_{\theta_0}}. \end{aligned}\right\} \quad (11.74)$$

Last/latter correction factor in a series of the cases must be calculated when h_{θ_0}, undertaken in radians.

For the trajectories of antiaircraft artillery with the assigned constant/invariable time t of formula for calculating the correction factors, they take the following form:

$$\left.\begin{aligned} \frac{\partial x}{\partial c} &= \frac{x(c+h_c) - x(c-h_c)}{2h_c}; \\ \frac{\partial y}{\partial c} &= \frac{y(c+h_c) - y(c-h_c)}{2h_c}; \\ \frac{\partial x}{\partial v_0} &= \frac{x(v_0+h_{v_0}) - x(v_0-h_{v_0})}{2h_{v_0}}; \\ \frac{\partial y}{\partial v_0} &= \frac{y(v_0+h_{v_0}) - y(v_0-h_{v_0})}{2h_{v_0}}; \\ \frac{\partial x}{\partial \theta_0} &= \frac{x(\theta_0+h_{\theta_0}) - x(\theta_0-h_{\theta_0})}{2h_{\theta_0}}; \\ \frac{\partial y}{\partial \theta_0} &= \frac{y(\theta_0+h_{\theta_0}) - y(\theta_0-h_{\theta_0})}{2h_{\theta_0}}. \end{aligned}\right\} \quad (11.75)$$

The tables of correction factors can be calculated by one of the described above methods.

Since the trajectory is determined by parameters c, v_0 and θ_0,

also the tables of correction factors have the same entries.

Are known the correction cards of artillery academy, calculated by the method of the numerical integration of the differential equations of corrections [59].

In tables for barrel artillery pieces, they are given:

$Q_\tau = \frac{\partial x_c}{\partial \tau}$ — change in the complete distance with an increase in the temperature of air on 1° along an entire trajectory;

$Q_{v_0} = \frac{\partial x_c}{\partial v_0}$ — change in the complete distance as a result of an increase in the initial velocity on 1 m/s;

$Q_{\frac{\Delta c}{c}} = \left|\frac{\partial x_c}{\partial c}\right| \cdot \frac{c}{100}$ — change in the complete distance as a result of an increase in the ballistic coefficient or barometric pressure on 1o/o;

Q_{w_x} — change in the complete distance as a result of action on the projectile of longitudinal tailwind with a velocity of 1 m/s, constant along an entire trajectory;

Q_{w_z} — lateral deviation of the impact point in the projectile as a result of the action of cross wind at a rate of 1 m/s, constant

along an entire trajectory.

Practical work on tables is reduced to linear interpolation in terms of the assigned intake values of c, v_0 and θ_0.

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§4. Correction into firing distance artillery shells during a change in the initial weight.

The weight of projectile affects both the motion of projectile in the bore of artillery instrument, determining its initial velocity and to the trajectory of its free flight in the dense layers of the atmosphere. Firing distance depends on three values: v_0, c and θ_0; the weight of projectile is determined first two. Consequently, at the fixed angle of departure it is possible to write $x_c=f(v_0, c)$.

Let us make an assumption about independence of action v_0 and c on firing distance and we will use the differential correcting formula

$$\Delta x_{cq}=\frac{\partial x_c}{\partial v_0}\Delta v_0+\frac{\partial x_c}{\partial c}\Delta c. \qquad (11.76)$$

The dependence of the initial velocity on the weight of projectile we will obtain on the correcting formula, known from interior

ballistics,

$$\delta v_0 = -l_Q v_0 \frac{\delta Q}{Q}.$$

where l_Q — correction factor.

From formula for the ballistic coefficient $c=(id^2/Q)10^3$, accepting product id^2 constant, taking the logarithm of, differentiating and replacing infinitesimal increases by final low, we will obtain $\delta c = -c\frac{\delta Q}{Q}$. Substituting δv_0 and δc in (11.76), let us have

$$\delta x_{cQ} = -\frac{\partial x_c}{\partial v_0} l_Q v_0 \frac{\delta Q}{Q} - \frac{\partial x_c}{\partial c} c \frac{\delta Q}{Q}. \qquad (11.77)$$

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Since correction factor $\frac{\partial x_c}{\partial c}$ value negative, is convenient to take it by absolute value and then

$$\delta x_{cQ} = \left[\left|\frac{\partial x_c}{\partial c}\right| c - l_Q v_0 \frac{\partial x_c}{\partial v_0}\right] \frac{\delta Q}{Q}, \qquad (11.78)$$

whence correction factor into distance directly by weight of projectile is equal to

$$\frac{\partial x_c}{\partial Q} = \frac{1}{Q}\left[\left|\frac{\partial x_c}{\partial c}\right| c - l_Q v_0 \frac{\partial x_c}{\partial v_0}\right]. \qquad (11.79)$$

Taking into account this designation we will obtain

$$\delta x_{cQ} = \frac{\partial x_c}{\partial Q} \delta Q.$$

Depending on specific conditions, the correction factor and correction into distance for the deviation of the weight of projectile can have different signs.

§5. Account of the effect of the atmospheric parameters on rocket flight and of artillery shells.

Theoretical studies and firings confirm the essential effect of the deviations of weather factors from their normal values for rocket flight and projectiles. The effect of weather factors is considered during determining of motion characteristics, during the calculation of scattering trajectories and the determination of the accuracy of firing, in the calculations, connected with the design of the system of the flight control, and during calculations for strength.

In ballistics the effect of the deviations of the atmospheric parameters of rocket flight and projectiles can be taken into account by three methods. First method - calculation of corrections according to the ground-based deviations of pressure, humidity and temperature from the normal under the assumption of the validity of hypothesis about the vertical equilibrium of the atmosphere and the preservation/retention/maintaining of the character of a change in

the temperature with the height/altitude of the similar of the function $\tau(y)$, accepted in standard atmosphere. An example of obtaining correction factors into distance to the deviation of ground-based barometric pressure and virtual temperature is placed in §3 present chapters [formula (11.43) and (11.45)].

The second method consists in determination and use of ballistic mean deviations of weather factors - average, conditionally constant with respect to an entire trajectory of the deviation of virtual temperature from normal law and conditionally constant neutral wind. The first and by the second methods are applied most frequently in practical work during preparation of firings and processing of their results.

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Theoretically stricter is the third method according to which into the systems of differential equations of motion are introduced the equations, which directly determine a change of the weather factors in the function of any coordinate (most frequently height/altitude) or in the function of time. With a strict account of the effect of a change in the atmospheric parameters to rocket flight and artillery shells in calculation, must be utilized the concrete/specific/actual (experimental) functions, obtained according

to the results of the sounding of the atmosphere.

During forecasting of a change in the weather factors, are used the statistical processings of the results of meteorological investigations. The results of statistical processing can be represented in the form of the random functions, comprised on coordinates and on time. Statistical processing on coordinates is utilized usually in ballistic calculations, statistical processing on time is utilized predominantly during setting of wind load on flight vehicle in stress analyses, in the stability analyses of motion and flight dynamics in the restless atmosphere.

5.1. Account of a barometric change and temperature.

A barometric change and temperature on height/altitude for a standard atmosphere is examined in chapter II, §3. With the wish to consider in ballistic calculations the concrete/specific/actual realization, experimental or calculated, it is necessary to have the data on a barometric change, humidity and temperatures of air on height/altitude, presented in the form of formulas, tables or curve/graphs. Air humidity easily is considered through virtual temperature on formula (2.49). Pressure and temperature are introduced into calculation according to formulas for an air density (2.43) or (2.51), temperature is considered also in formula for the

speed of sound. During the use of curve/graph $c_x(M)$, comprised for ground-based standard conditions, the effect of a change in the speed of sound in height/altitude for drag can be considered on formula (2.108). A change in the thrust in height/altitude must be calculated for concrete/specific/actual meteorological realization according to common/general/total equation (2.115).

It is obvious, each realization of the charged weather constituents on height/altitude requires the independent solution of the selected system of equations.

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5.2. Equations of motion of the projectile of constant mass upon consideration of a barometric change and temperature.

During the definition of air drag with the aid of coefficient $c_x(M)$ the account of a change in the pressure and temperature is conducted, just as it is described in the preceding/previous section for the projectiles of variable mass. If we for determining the drag use value E (5.6), then the system of differential equations, which describes the motion of the projectile of constant mass taking into account a change in the pressure and temperature, it can be given to the general view of equations (11.14). Let us take changed value

$(E+\delta E)$, then

$$\ddot{x}=-E\dot{x}-\frac{\delta E}{E}E\dot{x};\quad \ddot{y}=-E\dot{y}-g-\frac{\delta E}{E}E\dot{y}. \qquad (11.80)$$

In accordance with (5.6) it is possible to present E=f(c, h and τ) and to obtain [9]

$$\frac{\delta E}{E}=\frac{\delta c}{c}+\frac{\delta h}{h}-\frac{f(v_\tau)+1}{2}\cdot\frac{\delta\tau}{\tau}. \qquad (11.81)$$

Value $f(v_\tau)$ is determined from (11.25).

During a change only in ballistic coefficient in accordance with (11.80) and (11.81) we will obtain the following system of differential equations:

$$\ddot{x}=-E\dot{x}-\frac{\delta c}{c}E\dot{x};\quad \ddot{y}=-E\dot{y}-g-\frac{\delta c}{c}E\dot{y}. \qquad (11.82)$$

With the constant ballistic coefficient

$$\begin{aligned}\ddot{x}&=-E\dot{x}-\left(\frac{\delta h}{h}-\frac{f(v_\tau)+1}{2}\cdot\frac{\delta\tau}{\tau}\right)E\dot{x};\\ \ddot{y}&=-E\dot{y}-g-\left(\frac{\delta h}{h}-\frac{f(v_\tau)+1}{2}\cdot\frac{\delta\tau}{\tau}\right)E\dot{y}.\end{aligned} \qquad (11.83)$$

After integration can be obtained the motion characteristics of the projectile of constant mass during the deviations of pressure δh and of virtual temperature δτ from the values, determined by standard atmosphere. If we use hypothesis about the vertical equilibrium of the atmosphere, then the changed motion characteristics can be obtained depending on surface pressure and the function of a change in the temperature with height/altitude [9].

Taking the logarithm of and differentiating equation for h

[barometric function $\tau(y)$] and transfer/converting from infinitesimal to finite low increments, we will obtain

$$\frac{\delta h}{h}=\frac{\delta h_0}{h_0}-\delta\left[\frac{1}{R}\int\limits_0^y\frac{dy}{\tau}\right]$$

or, by differentiating second term on τ, let us have

$$\frac{\delta h}{h}=\frac{\delta h_0}{h_0}+\frac{1}{R}\int\limits_0^y\frac{\delta\tau}{\tau^2}\,dy. \qquad (11.84)$$

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Introducing replacement in (11.83), we will obtain

$$\left.\begin{aligned}\ddot{x}&=-E\dot{x}-\left(\frac{\delta h_0}{h_0}+\frac{1}{R}\int\limits_0^y\frac{\delta\tau}{\tau^2}\,dy-\frac{f(v_\tau)+1}{2}\,\frac{\delta\tau}{\tau}\right)E\dot{x};\\ \ddot{y}&=-E\dot{y}-g-\left(\frac{\delta h_0}{h_0}+\frac{1}{R}\int\limits_0^y\frac{\delta\tau}{\tau^2}\,dy-\right.\\ &\left.-\frac{f(v_\tau)+1}{2}\,\frac{\delta\tau}{\tau}\right)E\dot{y}.\end{aligned}\right\} \qquad (11.85)$$

Last/latter system of equations does not require for its solution of functioning the barometric change with height/altitude and it is convenient for theoretical studies.

5.3. Account of wind effect on the flight of the unguided rockets and artillery shells.

Basic problems during the study of the action of wind on rocket

flight and artillery shells are: setting the mechanism of the direct action of wind on the driving/moving rocket and projectile, setting networks of the motion of the masses of air and the proof of the legitimacy of the diagram accepted during the solution of one or the other problem. Meteorological investigations show that for the time interval, which exceeds the time of one group of firings, the motion of air masses in sufficiently large territory can be considered as relatively steady and rectilinear with preferred horizontal flows. In such flows wind velocity gradients in time and horizontal direction prove to be unessential. Figures 11.15 shows an example of a change in wind velocity in horizontal direction.

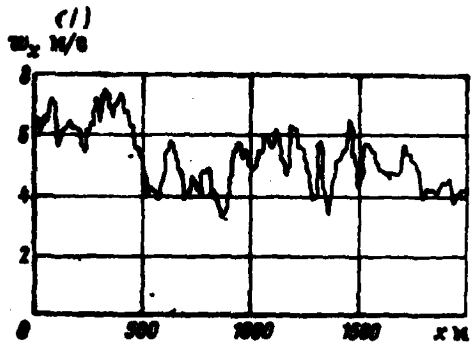

Fig. 11.15. Change in wind velocity in horizontal direction.

Key: (1). m/s.

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Together with the relatively tranquil flows of air masses, there is chaotic vortex/eddy, intermittent air action, which is called of gustiness. The investigation of the eddying effect of the atmosphere on rocket flight and projectiles is complex independent problem [20].

During ballistic calculations and during the preparation of firing usually are accepted following assumptions about the character of the motion of the atmosphere: they do not consider vertical wind component; they assume that a change in the wind in height/altitude remains constant/invariable within the limits of the entire trajectory in question, i.e., it is assumed that wind does not depend

cn horizontal coordinates. With the adopted assumptions it proves to be most convenient to represent the velccity vector of wind being of two comprising: the constant $\bar{w}_0(y)$ and randcm $\bar{w}_c(y)$

$$\bar{w}(y)=\bar{w}_0(y)+\bar{w}_c(y). \qquad (11.86)$$

Constant component corresponds to the mathematical expectation of upper wind y and characterizes the continuous uniform displacement/movement of the masses of air. Random comprising characterizes a change in the wind from one shot to the next to height/altitude y. When conducting of calculations the velocity vector of wind they expand in the direction of firing (so-called longitudinal wind), also, in standard to it (cross wind).

The special feature/peculiarity of the motion of flight vehicle to the moved atmosphere consists in the fact that its velocity relative to the earth/ground and relative to the driving/moving masses of air is different. The velocity of flight vehicle (aircraft, rocket, projectile) relative to the Earth it is accepted to call ground speed, velocity relative to the atmosphere - air. If the ground speed of projectile (absolute) is designated by vector $\bar{v}$, the velocity of the motion of the atmosphere relative to the Earth (translational speed - wind velocity) by vector $\bar{w}$ and the airspeed of projectile (relative) through $\bar{v}_r$, then

$$\bar{v}=\bar{w}+\bar{v}_r, \qquad (11.87)$$

and the velocity of projectile in motion relative to the atmosphere

is equal to

$$\bar{v}_r = \bar{v} - \bar{w}. \quad (11.88)$$

The acceleration of projectile in motion relative to the Earth is equal to $\dot{\bar{v}}$, and relative to the atmosphere $-\dot{\bar{v}}_r$. In this case, it is necessary to bear in mind, that the aerodynamic forces and torque/moments are determined by the airspeed, i.e., $\bar{v}_r$ whose module/modulus can be designed on the approximate dependence

$$v_r = \sqrt{v^2 + w_x^2 + w_z^2 - 2vw_x\cos\theta}. \quad (11.89)$$

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The mechanism of the effect of wind in the general case during the solution of spatial problem taking into account the angles of attack and slip, determined with respect to airspeed, is very complex. Therefore in practical work frequently is examined the action of longitudinal and cross wind separately.

Let us write by analogy with (3.44), (3.49) and (3.50) the systems of equations, which describe the separate longitudinal and yawing motions of rocket, taking into account in them respectively only longitudinal or only cross wind and counting that thrust $\bar{P}$ is directed along the axis Ox_1 of rocket. For the axial motion of the projection of vector equality (11.88) on the axis of the starting coordinate system, combined with the center of mass of projectile,

are equal (Fig. 11.16):

$$v_{rx} = v\cos\theta - w_x; \quad v_{ry} = v\sin\theta. \qquad (11.90)$$

The flight path angle in relative axial motion is determined from the dependence

$$\operatorname{tg}\theta_r = \frac{v\sin\theta}{v\cos\theta - w_x}. \qquad (11.91)$$

The fundamental equations, which describe the axial motion of the unguided rocket taking into account the action of wind $W_x = \text{const}$, will take the form

$$\left.\begin{aligned}
&\dot v = \frac{1}{m}\left[P - X_r\cos(\theta_r - \theta) + Y_r\sin(\theta_r - \theta) - Q\sin\theta\right];\\
&\dot\theta = \frac{1}{mv}\left[P\alpha - X_r\sin(\theta_r - \theta) + Y_r\cos(\theta_r - \theta) - Q\cos\theta\right];\\
&J_z\ddot\vartheta = \sum M_{zi};\ \text{[illegible]}\ \dot y = v\sin\theta;\ \dot x = v\cos\theta.
\end{aligned}\right\} \qquad (11.92)$$

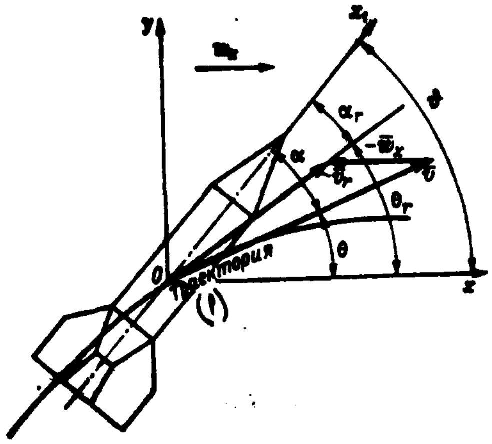

Fig. 11.16. Schematic of the construction of angle of attack in relative motion upon consideration of longitudinal tailwind.

Key: (1). Trajectory.

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Supplementary equation for determining the angle of attack in relative motion

$$\alpha_r = \vartheta - \theta_r. \qquad (11.93)$$

In accordance with (2.85), (2.86) and (2.89) aerodynamic forces and torque/moment taking into account the action of longitudinal wind are approximately equal to

$$X_r = \frac{\rho v_r^2}{2} S c_x(M_r); \quad Y_r = \frac{\rho v_r^2}{2} S c_y^{\alpha} \alpha_r; \qquad (11.94)$$
$$M_{z_r} = \frac{\rho v_r^2}{2} S l m_z^{\alpha} \alpha_r,$$

where

$$c_x(M_r)=c_{x0}(M_r)+c_{xi}(\alpha_r);$$
$$c_{x0}(M_r)=c_{x0}\left(\frac{v_r}{a}\right).$$

The system of equations, which describes the yawing motion of the unguided rocket taking into account the action of constant cross wind $\bar{w}_z$, can be written in the form

$$\left.\begin{aligned}\dot{\Psi}&=-\frac{1}{mv\cos\theta}(-P\beta+Z_r);\\ J_{y_1}\ddot{\psi}=M_{y_1r};\quad \psi&=\Psi+\beta;\quad \dot{z}=-v\cos\theta\sin\Psi;\end{aligned}\right\}\quad (11.95)$$

Recall that velocity v must be determined during the solution of the system of equations, which describes axial action.

During the determination of the direction of the yawing action of rocket on powered flight trajectory, has a value the mutual location of the center of mass and resultant pressure of rocket. The effect of cross wind $\bar{w}_z$ on rocket will create supplementary aerodynamic force R_w, applied in resultant pressure (Fig. 11.17). Of finned statically stable rocket the resultant pressure is arrange/located after the center of gravity nearer to tail assembly; therefore under the action of torque/moment from force R_w rocket will turn itself by nose section towards wind so that its longitudinal axis will coincide with the vector of airspeed. In this case, will appear lateral thrust component P_B, the directed against

wind and when $P_z > R_z$ rocket will be moved also against wind. The spin-stabilized missiles as a result of derivation, composite action of the force of Magnus and cross wind can be moved on powered flight trajectory in wind direction. On inactive leg with P=0 the rocket just as the projectile of barrel system, will be moved in the direction of cross wind. Figures 11.18 depicts the schematic of yawing motion against wind of the statically stable fin-stabilized rocket on powered flight trajectory under the assumption that the rocket is inertia-free.

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Let us designate the horizontal projection of the velocity of the center of mass in the starting coordinate system through u=v cos θ, the horizontal projection of the speed in relative motion through u_r. For Fig. 11.18

$$u_r = \sqrt{u^2 + w_z^2 + 2uw_z \sin\Psi}; \qquad (11.96)$$

$$\cos\Psi_r = \frac{v\cos\theta\cos\Psi}{u_r}, \qquad (11.97)$$

supplementary equation for determining the slip angle in the relative yawing motion

$$\beta_r = \psi - \Psi_r. \qquad (11.98)$$

Aerodynamic forces and torque/moment taking into account the action of cross wind will be equal to

$$Z_r = \frac{\rho v_r^2}{2} S c_z^\beta \beta_r; \quad M_{yr} = \frac{\rho v_r^2}{2} S l m_y^\beta \beta_r. \qquad (11.99)$$

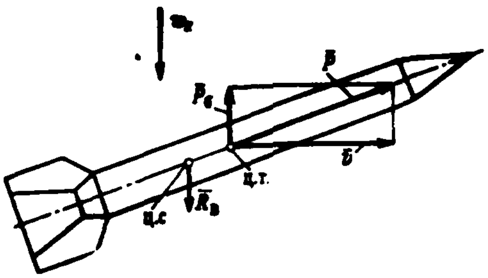

Fig. 11.17. Rotation of the unguided fin-stabilized rocket against wind.

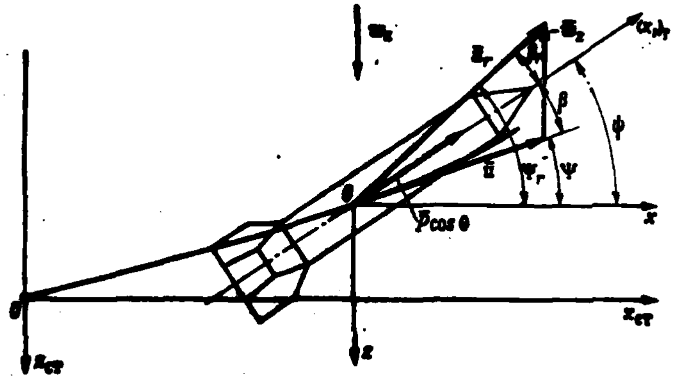

Fig. 11.18. Schematic of construction of slip angles during yawing action of rocket against wind.

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Figures 11.19 depicts the schematic of yawing motion stabilized by rolling of the rocket (projectile) in powered flight trajectory in the case when lateral thrust component acts in wind direction. According to Fig. 11.19

$$w_r = \sqrt{w^2 + w_z^2 - 2ww_z \sin\Psi}; \qquad (11.100)$$

$$\cos\Psi_r = \frac{v\cos\theta\cos\Psi}{w_r} \qquad (11.101)$$

and supplementary equation

$$\beta_r = \phi - \Psi_r. \qquad (11.102)$$

For spatial motion with constant wind from vector equality (11.88) we will obtain for modulus of velocity

$$v_r = \sqrt{(v_x - w_x)^2 + (v_y - w_y)^2 - (v_z - w_z)^2}, \qquad (11.103)$$

where v_x, v_y, v_z and w_x, w_y and w_z — projection of the velocity of the center of mass of projectile and projection of wind velocity on the axis of the starting coordinate system, combined with the center of mass. Upon consideration only of horizontal component wind

$$v_r = \sqrt{(v_x - w_x)^2 + v_y^2 + (v_z - w_z)^2}. \qquad (11.104)$$

Considering stable rocket as material point of variable mass and taking into account that with wind force vector of thrust coincides with the airspeed of the center of mass, we will obtain the cosines of the angles between the direction of airspeed and the corresponding axes of the starting system of coordinates

$$\frac{v_{rx}}{v_r} = \frac{v_x - w_x}{v_r}; \quad \frac{v_{ry}}{v_r} = \frac{v_y - w_y}{v_r}; \quad \frac{v_{rz}}{v_r} = \frac{v_z - w_z}{v_r}. \qquad (11.105)$$

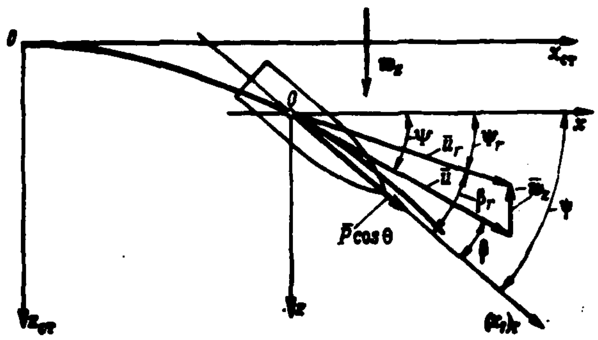

Fig. 11.19. Schematic of the construction of slip angles during the yawing motion of rocket in wind direction.

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For a statically stable fin-stabilized rocket on powered flight trajectory, the system of equations of motion taking into account wind will take the form

$$\left.\begin{aligned}\dot{v}_x&=\frac{Pv_{rx}}{mv_r}-\frac{\varrho v_r}{2m}(v_x-w_x)Sc_x(M_r);\\ \dot{v}_y&=\frac{Pv_{ry}}{mv_r}-\frac{\varrho v_r}{2m}(v_y-w_y)Sc_x(M_r)-g;\\ \dot{v}_z&=\frac{Pv_{rz}}{mv_r}-\frac{\varrho v_r}{2m}(v_z-w_z)Sc_x(M_r).\end{aligned}\right\}\quad(11.106)$$

During the study of the action of the gyroscopically stable rocket (TRS) on powered flight trajectory in the right side of the last/latter equation of first term the sign must be changed by reverse/inverse:

$$\dot{v}_z=-\frac{Pv_{rz}}{mv_r}-\frac{\varrho v_r}{2m}(v_z-w_z)Sc_x(M_r).\qquad(11.107)$$

For the projectile of constant mass, system (11.106) will take the form

$$\left.\begin{aligned}\dot{v}_x &= -\frac{\varrho v_r}{2m}(v_x - w_x) S c_x(M_r);\\ \dot{v}_y &= -\frac{\varrho v_r}{2m}(v_y - w_y) S c_x(M_r) - g;\\ \dot{v}_z &= -\frac{\varrho v_r}{2m}(v_z - w_z) S c_x(M_r).\end{aligned}\right\} \quad (11.108)$$

If we determine the drag through the ballistic coefficient of c and function $G(v_r)$, then value E in (5.6) must be calculated through velocity $v_{\tau r}$

$$E_r = cH_\tau(y) G(v_{\tau r}). \quad (11.109)$$

For the projectile of constant mass the system of equations, which considers wind effect, can be written in the form of common/general/total system (11.14)

$$\ddot{x} = -E_r(\dot{x} - w_x);\ \ddot{y} = -E_r\dot{y} - g;\ \ddot{z} = -E_r(\dot{z} - w_z). \quad (11.110)$$

Using (11.104), it is possible to obtain

$$v_r = v\sqrt{1 + \frac{w^2}{v^2} - \frac{2\dot{x}w_x}{v^2} - \frac{2\dot{z}w_z}{v^2}}, \quad (11.111)$$

where

$$w = \sqrt{w_x^2 + w_z^2} \text{ and } v = \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}.$$

The velocity of the projectiles of constant mass, as a rule, is considerably more wind velocity. Reject/throwing in (11.111) terms w^2/v^2 and $\frac{2\dot{x}w_x}{v^2}$, low in comparison with unity, expanding simplified function (11.111) in a binomial series and disregarding in it the terms, which contain relation to the projection of wind velocity w_x to the ground speed in larger than by the first, degree, possible, following [9], to obtain

$$v_r = v - w_x \frac{\dot{x}}{v}; \qquad (11.112)$$

$$G(v_w) = G(v_\tau)\left[1 - w_x \frac{f(v_\tau)}{v^2}\dot{x}\right]; \qquad (11.113)$$

$$E_r = E\left[1 - w_x \frac{f(v_\tau)}{v^2}\dot{x}\right], \qquad (11.114)$$

where $f(v_\tau)$ is determined from (11.25).

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Substituting E_r in (11.110) and disregarding the small second-order quantities, it is possible to obtain

$$\left.\begin{aligned}\ddot{x} &= -E\dot{x} + E\left[1 + \frac{f(v_\tau)}{v^2}\dot{x}^2\right]w_x;\\ \ddot{y} &= -E\dot{y} + E\,\frac{f(v_\tau)}{v^2}\dot{x}\dot{y}w_x - g;\\ \ddot{z} &= -E(\dot{z} - w_z).\end{aligned}\right\} \qquad (11.115)$$

5.4. Account of wind effect on the flight of the guided missiles.

For the guided missiles ci different types wind, it is one of the basic perturbation factors whose action is parried by the control system, which holds the center of mass cf rocket in the predetermined trajectory. The deviation cf projectile from calculated trajectory under the effect of wind load is the component part of the overall error for the firing the guided missiles. Not all types of the control systems parry wind and in certain cases its action must be considered by the methods of ballistics. During preset control only cr the pitch angles, yaw and bank the control system cannot consider and parry the deviations cf coordinates x, y and z, caused by the action of wind. Assuming that the control provides obtaining the zero angles cf bank and yaw, let us comprise system cf equations for determining cf the longitudinal-behavior characteristics of the rocket at the assigned tilt angle $\vartheta_{np}(t)$ and with rectilinear horizontal displacement/movement cf the masses cf air. As before let us designate calculated values in relative action by index r and from (3.56) will have

$$a_{\delta_r} = -\frac{2Y^{\delta}_{y}x_{\delta}h_{x}}{\varrho v_r^2 S l m_z^{\alpha}} = -c_{\vartheta_r}k_{\vartheta}, \qquad (11.116)$$

after which on (3.58) we will obtain

$$a_{\delta_r} = \frac{c_{\vartheta_r}k_{\vartheta\vartheta}(\vartheta_{np} - \vartheta_r)}{1 + c_{\vartheta_r}k_{\vartheta\vartheta}}.$$

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In the absolute motion

$$\alpha_б = \frac{\varepsilon_\theta k_{0\theta}(\theta_{пр} - \theta)}{1 + \varepsilon_\theta k_{0\theta}}. \qquad (11.117)$$

Cn the smallness of angles accepting

$$\sin \alpha_б \approx \alpha_б; \quad \sin \alpha_{б\,r} \approx \alpha_{б\,r}; \quad \cos \alpha_б \approx \alpha_{б\,r} \approx 1$$

and using (3.59), we will obtain the system of equations the lcngitudinal controlled flight, which considers the action by longitudinal component cf the wind

$$\left.\begin{aligned}
&\dot{v} = \frac{P - X_r \cos(\theta_r - \theta) + Y_r \sin(\theta_r - \theta)}{m} - g \sin \theta;\\
&\dot{\theta} = \frac{1}{mv}\left[P\alpha_б - X_r \sin(\theta_r - \theta) + Y_r \cos(\theta_r - \theta)\right] - \frac{g \cos \theta}{v};\\
&\vartheta = \theta + \alpha_б \quad (\overset{OR}{\text{или}}\ \vartheta = \theta_r + \alpha_{б\,r});\\
&\theta_r = \operatorname{arc\,tg} \frac{v \sin \theta}{v \cos \theta - w_x}; \qquad X_r = \frac{\rho v_r^2}{2} S c_x(M_r);\\
&Y_r = \frac{\rho v_r^2}{2} S c_y^\alpha \alpha_{б\,r} = Y_r^\alpha \alpha_{б\,r}.
\end{aligned}\right\} \quad (11.118)$$

With the tailwind

$$\left.\begin{aligned}
&v_r = \sqrt{v^2 + w_x^2 - 2v w_x \cos \theta}; \quad \dot{y} = v \sin \theta; \quad \dot{x} = v \cos \theta;\\
&\theta_{пр} = \theta_{пр}(t) \quad \text{и} \quad m = m(t).
\end{aligned}\right\} \quad (11.119)$$

and.

Having longitudinal-behavior characteristics, yawing motion taking into account wind can be calculated frcm the separately sclved

system of equations. On (3.62), it is possible to write

$$\beta_{6r} = -\frac{2Z_p^{\delta_{y_1}} l_p \delta_{y_1}}{q v_r^2 S l m_{y_1}^{\beta}} = -\varepsilon_{\psi_r}\delta_{y_1}$$

and further

$$\beta_{6r} = \frac{\varepsilon_{\psi_r} k_{\theta\psi}(\psi_{np} - \Psi)}{1+\varepsilon_{\psi_r} k_{\theta\psi}}. \qquad (11.120)$$

During the writing of last/latter equality, it was assumed that the rocket has gas-dynamic controls of flight (the jet vanes) and, therefore, control force and torque/moment they do not depend on the effect of wind.

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In absolute motion, as before

$$\beta_6 = -\frac{2Z_p^{\delta_{y_1}} l_p \delta_{y_1}}{q v^2 S l m_{y_1}^{\beta}} = -\varepsilon_{\psi}\delta_{y_1}$$

and

$$\beta_6 = \frac{\varepsilon_{\psi} k_{\theta\psi}(\psi_{np} - \Psi)}{1+\varepsilon_{\psi} k_{\theta\psi}}.$$

On the smallness of the angles

$$\sin\beta_6 \approx \beta_6;\quad \cos\beta_6 \approx \cos\beta_{6r} \approx 1;\quad \sin\beta_{6r} \approx \beta_{6r}.$$

The first equation of system (3.49) taking into account the action of cross wind will take the form

$$\dot{\Psi} = -\frac{1}{mv\cos\theta}\left[-P\beta_6 + Z_r^{\beta}\beta_{6r}\right].$$

If there are no any special considerations, the programmed value of the yaw angle should take equal to zero and system of equations taking into account expressions for β_δ and $\beta_{\delta r}$ will be written as follows:

$$\left.\begin{aligned}
&\dot{\Psi}=-\frac{k_{\delta\psi}}{mv\cos\theta}\left[\frac{\varepsilon_\psi \Psi P}{1+\varepsilon_\psi k_{\delta\psi}}-\frac{\varepsilon_{\psi_r}\Psi_r Z_r^\beta}{1+k_{\psi_r}k_{\delta\psi}}\right];\\
&\dot{z}=-v\cos\theta\sin\Psi;\\
&Z_r=\frac{\varrho v_r^2}{2}Sc_z^\beta\beta_{\delta r}=Z_r^\beta\beta_{\delta r};\\
&\cos\Psi_r=\frac{v\cos\theta\cos\Psi}{u_r};\\
&u_r=\sqrt{u^2+w_z^2-2uw_z\sin\Psi}.
\end{aligned}\right\}\quad(11.121)$$

Since $\psi=0$, then in yawing motion axis of rocket must be moved in parallel to itself (Fig. 11.20).

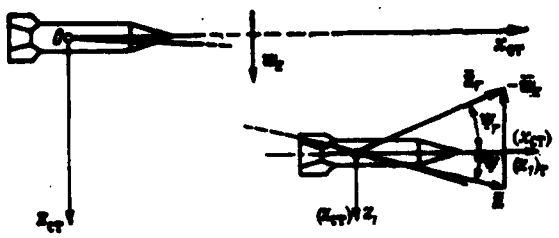

Fig. 11.20. Flow chart of the guided missile upon consideration of the action of cross wind ($\psi = 0$).

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During control on coordinates, wind effect is parried by the control system. the dynamics of motion in the restless atmosphere and the calculation of the control system for wind disturbance/perturbation is examined in the automation of guided missiles [19], [20].

5.5. Ballistic neutral wind and ballistic temperature deflection.

Introduction into the ballistic calculation of the concrete/specific/actual functions of a change in the temperature and wind on height/altitude is laborious problem. Each meteorological realization requires the independent solution of complex system of equations. Considerably the more simply approximation method, which consists in obtaining of the corrections, designed on differential

correcting formulas. The application/use of differential correcting formulas for determining the corrections for wind and deviation of temperature of air from normal function $\tau(y)$ proves to be possible, if we introduce into calculation neutral constant according to an entire trajectory wind and constant deviation of temperature. These constants were called ballistic average. Ballistic wind - w_B and ballistic temperature deflection - $\delta\tau_B$ they are defined from the condition that those caused by their action of the deviation of impact point will be the same as during the real concrete/specific/actual realization of a change in the wind and temperature on height/altitude. If are known ballistic average, then the deviations of impact point from those designed under normal meteorological conditions will be determined according to simple formulas. Correction into distance for ballistic temperature deflection is equal to

$$\Delta x_{C\tau}=\frac{\partial x_C}{\partial \tau}\delta\tau_B. \qquad (11.122)$$

Correction into distance for the action of longitudinal component of the ballistic wind

$$\Delta x_{Cw_x}=\frac{\partial x_C}{\partial w_x}w_{xB}; \qquad (11.123)$$

the lateral deviation of impact point from the action side component of the wind

$$z_{Cw_z}=\frac{\partial z_C}{\partial w_z}w_{zB}. \qquad (11.124)$$

The most theoretically substantiated method of calculating the ballistic average, suitable for scientific research works and the control/check of approximation methods, follows of the condition indicated above of their determination. For an example let us examine calculation procedure by longitudinal component of ballistic wind. On the data of the undisturbed nominal trajectory, we determine firing distance x_c and correction factor to constant in an entire trajectory wind - $\frac{\partial x_C}{\partial w_x}$.

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The determination of correction factor we carry out one of precise methods, described above. For example, by the solution of the system of equations, which includes constant wind.

Trajectory calculation is repeated during the practical realization of a change by wind component in height/altitude, is determined x_{Cw_x} and correction into distance.

$$\Delta x_{Cw_x}=x_{Cw_x}-x_C. \qquad (11.125)$$

Longitudinal component of ballistic wind for a concrete/specific/actual trajectory and concrete/specific/actual wind function is equal to

$$w_{xb}=\frac{\Delta x_{Cw_x}}{\partial x_C/\partial w_x}. \qquad (11.126)$$

A similar method can be used for determining side component of ballistic wind - w_{zB} and ballistic temperature deflection $\delta\tau_B$.

For a practical work are applied more idle time, but less precise method, connected with the determination of the weights of layers. For determining the weights of layers and ballistic the median trajectory it is divided on height/altitude into a series of layers (usually an equal thickness). The thickness of each layer is taken depending on trajectory height from 200 to 800 m, multiple of 100 m. For each of layers, are determined the correction factors

$$\left(\frac{\partial x_C}{\partial \tau}\right)_i;\ \left(\frac{\partial x_C}{\partial w_x}\right)_i;\ \left(\frac{\partial z_C}{\partial w_z}\right)_i.$$

First two determine charge in the complete distance with an increase in the temperature on 1°K and wind velocities on 1 m/s only in the i layer. The third correction factor determines the lateral deviation of the impact point in the projectile with cross wind 1 m/s only in the i layer. The calculation of each correction factor for the i layer is conducted independently; with a change in the temperature or the account of wind in the i layer in remaining layers, is assumed normal law $\tau(y)$ and dead calm. For each trajectory it is necessary to conduct n of the calculations where n - minimum number of whole layers, in the sum of height/altitudes of which is placed the trajectory height. Ballistic temperature deflection is determined from the equality

$$\left(\frac{\partial x_c}{\partial \tau}\right)_1 \delta\tau_1+\left(\frac{\partial x_c}{\partial \tau}\right)_2 \delta\tau_2+\ldots+\left(\frac{\partial x_c}{\partial \tau}\right)_i \delta\tau_i+ +\ldots+\left(\frac{\partial x_c}{\partial \tau}\right)_n \delta\tau_n=\frac{\partial x_c}{\partial \tau}\delta\tau_B \quad (11.127)$$

where $\delta\tau_i$ — deviaticr cf concrete/specific/actual mean temperature in the i-th layer from mean temperature in the i-th layer, determined by the normal law of a change in the temperature on height/altitude.

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Ballistic temperature deflection

$$\delta\tau_B=\frac{\sum_1^n\left(\frac{\partial x_c}{\partial \tau}\right)_i \delta\tau_i}{\frac{\partial x_c}{\partial \tau}}. \quad (11.128)$$

The relaticn

$$q_{\tau i}=\frac{\left(\frac{\partial x_c}{\partial \tau}\right)_i}{\frac{\partial x_c}{\partial \tau}} \quad (11.129)$$

is called the weight of the i-th layer according to temperature. Consequently,

$$\delta\tau_B=\sum_1^n q_{\tau i}\delta\tau_i. \quad (11.130)$$

The calculation of the weights of layers of ballistic wind is hinder/hampered by the fact that wind is vector quantity and can change module/modulus and direction depending on the number of a layer. Furthermore, one-type rocket and artillery pieces, having the

identical weights of layers, will have different ballistic wind depending on the line of fire. Because of this the calculation of ballistic wind is conducted for the conditional line of fire with zero azimuth, i.e., for a direction in north, with the subsequent conversion in the real line of fire [18]. The line of fire is determined by the azimuth of firing A_{w} — by the angle, calculated clockwise off direction in north before direction in target/purpose. Azimuth of wind A_{w} — the angle, calculated also clockwise off direction in north to the vector of wind, directed toward the firing position (Fig. 11.21). They have plus sign the longitudinal wind, directed toward target/purpose (i.e. tailwind), and the cross wind, which blows to the right from the line of fire.

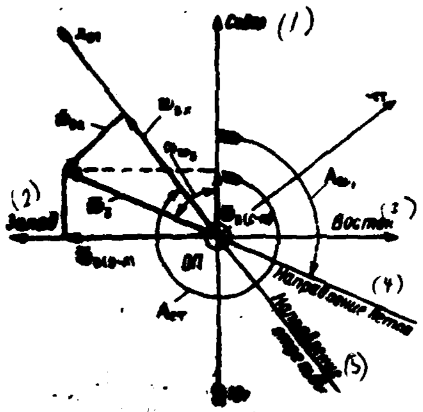

Fig. 11.21. Azimuth of firing $A_{c\tau}$ and the azimuth of wind $A_{w_Б}$

Key: (1). North. (2). West. (3). East. (4). Wind direction. (5). Line of fire.

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According to determination for the incidental ballistic wind, directed from north to south, it is possible to write the equality:

$$\Delta x_{Cw_x} = \sum_1^n \left(\frac{\partial x_C}{\partial w_x}\right)_i w_{(C-Ю)i} = \frac{\partial x_C}{\partial w_x} w_{Б(C-Ю)},$$

whence the longitudinal ballistic wind

$$w_{Б(C-Ю)} = \sum_1^n q_{w_x i} w_{(C-Ю)i}, \qquad (11.131)$$

where

$$q_{w_x i} = \left(\frac{\partial x_C}{\partial w_x}\right)_i \Big/ \frac{\partial x_C}{\partial w_x} - \qquad (11.132)$$

- weight of a layer on longitudinal component of ballistic wind.

The lateral deviation of projectile under the action wind component, which blows from the east to west, is equal

$$z_{Cw_z} = \sum_1^n \left(\frac{\partial z_C}{\partial w_z}\right)_i w_{(B-3)\,i} = \frac{\partial z_C}{\partial w_z} w_{B\,(B-3)}.$$

Hence side component the velocities of the ballistic wind

$$w_{B\,(B-3)} = \sum_1^n q_{w_z i}\, w_{(B-3)i}, \qquad (11.133)$$

where

$$q_{w_z i} = \left(\frac{\partial z_C}{\partial w_z}\right)_i \Big/ \frac{\partial z_C}{\partial w_z} - \text{sec}$$

a layer on side component wind velocity.

The direction of the action of ballistic wind is determined from the angle

$$\alpha_{w_B} = \operatorname{arc\,tg}\left|\frac{w_{B\,(B-3)}}{w_{B\,(C-Ю)}}\right|. \qquad (11.134)$$

If ballistic wind is directed toward the firing position from the first fourth, then $A_{w_B} = \alpha_{w_B}$; if from the second fourth, then $A_{w_B} = 180° - \alpha_{w_B}$; if from the third fourth, then $A_{w_B} = 180° + \alpha_{w_B}$ and if from the fourth, as shown in Fig. 11.21, then $A_{w_B} = 180° - \alpha_{w_B}$.

Modulus of velocity of the ballistic wind

$$w_B = \sqrt{w^2_{B(C-Ю)} + w^2_{B(B-3)}}. \qquad (11.135)$$

For the real line of fire, longitudinal and side components of

ballistic wind are determined from the formulas

$$w_{Bx} = -w_B \cos(A_{cr} - A_{w_B}); \quad w_{Bz} = w_B \sin(A_{cr} - A_{w_B}). \quad (11.136)$$

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The weights of layers $q_{\tau i}, q_{w_x i}, q_{w_z i}$ for one and the same layer it is various in value and depends on the number of a layer. If we in formula (11.127) assume $\delta\tau_1 = \delta\tau_2 = \ldots = \delta\tau_n = \delta\tau_B$, then $q_{\tau_1} + q_{\tau_2} + \ldots + q_{\tau_n} = 1$; is analogous

$$\sum_1^n q_{w_x i} = 1 \text{ and } \sum_1^n q_{w_z i} = 1.$$

Equality to the unit of the sum of the weights of layers is testing the correctness of their calculation. For calculating the corrections into complete flying range for wind and deviation of temperature in connection with artillery shells the weight of layers in the first approximation, can be designed on obtained in parabolic theory formula (10.16).

For different systems and the trajectories of one system, the weight of layers will be different and it must be calculated previously. During the preparation of concrete/specific/actual firings at the data of meteorological bulletin are calculated from layers of deviation $\delta\tau_i, w_{(C-D)i}, w_{(B-3)i}$, after which are calculated ballistic temperature deflection and ballistic wind.

5.6. Approximation correcting formulas to longitudinal and lateral constant wind.

The set-forth method is developed for the account of the separate effect of constant longitudinal and crosswind on the motion of the center of mass of artillery shell. Vertical component wind velocities is not considered.

Let us examine first only axial motion. The special feature/peculiarity of method consists in the fact that relative motion of the center of mass of projectile is examined in the coordinate system, which accomplishes translational motion relative to the Earth in wind direction with its velocity. Complete firing distance relative to) (in absolute motion Earth with longitudinal tailwird w_x is equal to

$$x_{Cw_x} = x_{Cr} + w_x t_C, \qquad (11.137)$$

where x_{Cr} — distance in relative motion;

t_C — total flying time, identical both in relative and in absolute motions.

Designating x_C — distance in absolute motion with dead calm,

we will obtain correction for the longitudinal wind

$$\Delta x_{Cw_x} = x_{Cr} - x_C$$

or

$$\Delta x_{Cw_x} = x_{Cr} - x_C + w_x t_C \qquad (11.138)$$

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Difference $x_{Cr} - x_C$ is usually small for its determination it is possible to use differential correcting formula, bearing in mind that the trajectory of the projectile of constant mass is determined by values v_0, θ_0 and c

$$\Delta x_{Cw_x} = \frac{\partial x_C}{\partial v_0}\delta v_0 + \frac{\partial x_C}{\partial \theta_0}\delta\theta_0 + \frac{\partial x_C}{\partial c}\delta c + w_x t_C. \qquad (11.139)$$

Differences in the initial velocities and angles of departure in absolute and relative motions are equal to

$$\delta v_0 = v_{0r} - v_0; \quad \delta\theta_0 = \theta_{0r} - \theta_0. \qquad (11.140)$$

Difference in the ballistic coefficients $\delta c = 0$.

The relative initial velocity in axial motion and the angle of departure along Fig. 11.22 are equal to

$$v_{0r} = \sqrt{v_0^2 + w_x^2 - 2v_0 w_x \cos\theta_0}; \qquad (11.141)$$

$$\mathrm{tg}\,\theta_{0r} = \frac{v_0 \sin\theta_0}{v_0 \cos\theta_0 - w_x}. \qquad (11.142)$$

After the determination of basic correction factors $\frac{\partial x_C}{\partial v_0}$, $\frac{\partial x_C}{\partial \theta_0}$ and of the time of motion t_C along the data of nominal trajectory,

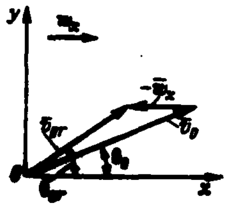
it is necessary to calculate $v_{0\psi}, \theta_{\psi}$ and $\Delta v_0, \Delta\theta_0$.

Correction for longitudinal wind will be determined according to formula (11.139). For high initial velocities it is possible to use with formula (11.112), after replacing in it x/v by $\cos\theta_0$; then

$$\Delta v_0 = v_{0\psi} - v_0 = -w_x \cos\theta_0. \quad (11.143)$$

Let us present (11.142) in the form

$$\operatorname{tg}\theta_{\psi} = \operatorname{tg}\theta_0 \left[1 - \frac{w_x}{v_0 \cos\theta_0}\right]^{-1}.$$



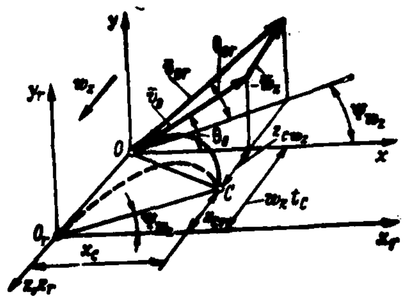


Fig. 11.23.

Fig. 11.22. Schematic of the determination of the initial velocity and angle of departure in relative plane motion under the effect of constant tailwind.

Fig. 11.23. Schematic of determination of initial velocity and angle of departure in relative yawing motion under effect of constant cross wind.

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Decompose/expanding the value, included into brackets, in a binomial series and omitting the members of the second and higher than the order of smallness, it is possible to obtain:

$$\operatorname{tg}\theta_{0r}-\operatorname{tg}\theta_0=\frac{w_x\sin\theta_0}{v_0\cos^2\theta_0}.$$

Counting approximately

$$\delta\operatorname{tg}\theta_0=\operatorname{tg}\theta_{0r}-\operatorname{tg}\theta_0\approx\frac{\delta\theta_0}{\cos^2\theta_0},$$

we will obtain

$$\Delta\theta_0 \approx \frac{w_z}{v_0}\sin\theta_0. \qquad (11.144)$$

Let us substitute (11.143) and (11.144) in (11.139) and after conversions we will obtain the approximation correcting formula

$$\Delta x_{Cw_z} = w_z\left[t_C - \frac{\partial x_C}{\partial v_0}\cos\theta_0 + \frac{\partial x_C}{\partial\theta_0}\,\frac{\sin\theta_0}{v_0}\right], \qquad (11.145)$$

where $\frac{\partial x_C}{\partial\theta_0}$ it is taken in the dimensionality m/rad.

The lateral deviation of the center of mass of projectile under the action of constant cross wind in absolute action is equal

$$z_{w_z} = z_r + w_z t. \qquad (11.146)$$

In the system of coordinates $O_r x_r y_r z_r$ driving/moving with the speed of cross wind in the direction of its action, the velocity of the center of mass of projectile is equal to

$$\bar{v}_r = \bar{v}_0 - \bar{w}_z.$$

On Fig. 11.23 $v_r = \sqrt{v_0^2 + w_z^2}$,

or

$$v_r = v_0\sqrt{1+\left(\frac{w_z}{v_0}\right)^2}. \qquad (11.147)$$

For high initial velocities $\left(\frac{w_z}{v_0}\right)^2 \ll 1$,

$$v_r \approx v_0 \text{ and } \Delta\theta_0 \approx 0.$$

From the same figure 11.23

$$\operatorname{tg}\theta_{w}=\frac{v_0\sin\theta_0}{\sqrt{v_0^2\cos^2\theta_0+w_z^2}}=\frac{\operatorname{tg}\theta_0}{\sqrt{1+\frac{w_z^2}{v_0^2\cos^2\theta_0}}}. \qquad (11.148)$$

For relatively low angles of departure and large v_0 can be counted $\operatorname{tg}\theta_w\approx\operatorname{tg}\theta_0$ and $\delta\theta_0=0$. With the adopted assumptions lateral deviation z_r is determined by the rotation of trajectory plane of relative motion relative to the system of coordinates $0_r x_r y_r z_r$ around axis y_r to angle Ψ_{w_z} and then for an impact point

$$z_{Cr}=-x_C\operatorname{tg}\Psi_{w_z}=-\frac{w_z x_C}{v_0\cos\theta_0}. \qquad (11.149)$$

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The lateral deviation of projectile of relatively terrestrial starting coordinates for current point in the trajectory with respect to (11.146) is equal

$$z_{w_z}=w_z t-\frac{w_z x}{v_0\cos\theta_0} \qquad (11.150)$$

and for an impact point

$$z_{Cw_z}=w_z\left(t_C-\frac{x_C}{v_0\cos\theta_0}\right). \qquad (11.151)$$

Let us note that value $\operatorname{tg}\Psi_{w_z}=\frac{w_z}{v_0\cos\theta_0}$ is obtained without the account longitudinal component wind velocity. If we consider longitudinal component of wind, then

$$\operatorname{tg}\Psi_{w_z}=\frac{w_z}{v_0\cos\theta_0-w_x}. \qquad (11.152)$$

As a result of the made simplifications in formula (11.145) and (11.151) give noticeable errcrs at the low initial velocities and the large angles of departure. Acceptable results can be obtained cn (11.145) and (11.151) with $v_0>250$ m/s and $\theta_c<45°$.

§6. Determination of corrections during the calculation of complex trajectories.

Ccmplex trajectories are subdivided intc the individual sections which are distinguished by the character of effect on rocket flight and projectiles of the basic and perturbaticn power factors. Fcr example, under the effect cf cross wind on the active-reactive statically stable unrotated mine it will twice change the direction of yawing motion (Fig. 11.24). On the first inactive leg, the mine will be moved in wind directicn, cn the seccnd section (active) the yawing motion of mine will be directed against wind and on the third section (second passive) the mine will be again moved in the direction cf cross wind. Value and the sign cf drift of rocket from plane of reference of firing cn the first and second sections $z_{п}$ and $z_{а}$.

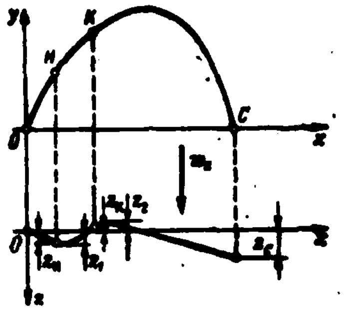

Fig. 11.24. Schematic of the lateral displacement/movement of active-reactive mine under the effect of cross wind.

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For each of the sections of the action characteristic of rocket, they are determined by the initial conditions: the coordinates of transition point to the changed character of action, by the sense of the vector of speed and by the modulus of velocity of the center of mass at the moment of transfer/transition. Furthermore, for each of the sections acts its system of forces. The inertia properties of rocket (projectile) determine smooth transition from one section to the next; the horizontal projection of trajectory in Fig. 11.24 does not have acute angles, and the maximum values z_1 and z_2 do not correspond to the points of the replacement of the system of forces, i.e., to points N and K on elevation.

Coordinate definition of the end points of complex trajectories - the point of shell burst with antiaircraft fire and of the impact point in the projectile with contact firing - it is possible to conduct in the manner that this was described above. Systems of equations for each of the sections must include basic forces and the perturbation factors whose effect on the results of firing is assumed to establish/install. Motion characteristics at the end point of the preceding/previous section will be initial conditions for that follow.

During the use of differential correcting formulas, the correction factors must be determined separately for each of the sections; the obtained corrections must subsequently be summarized taking into account their signs. It is necessary to keep in mind that the method of differential correcting formulas in application to complex trajectories is not considered transient processes from one section to the next. It is assumed that the new system of forces immediately, without transient process, changes the direction of the motion of the center of mass of rocket or projectile. Upon this setting smooth curve of the horizontal projection of trajectory in Fig. 11.24 will be converted into broken line with salient points at the end/leads of the sections. Let us examine, for an example, the calculation of the lateral deviation of the impact point in the unguided rocket from plane of reference of firing under the effect of constant cross wind (Fig. 11.25).

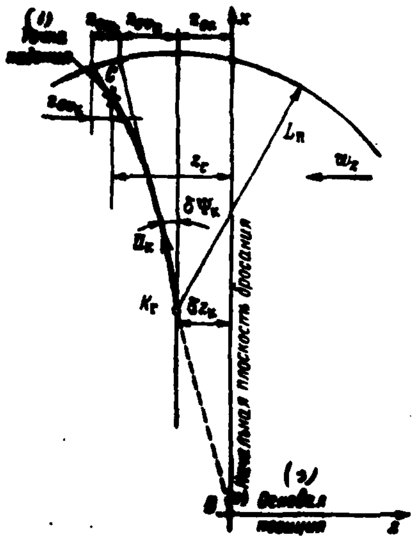

Fig. 11.25. Circuit of lateral displacement of projectile of constant mass under action of cross wind.

Key: (1). Impact point. (2). Plane of reference of casting. (3). Firing position.

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Point K_r – projection of the point of the end/lead of powered flight trajectory on horizontal plane. Distance from point K_r on the projection of the plane of casting on horizontal plane (line of fire) – δz_k. Let us designate through z_{CK} — deviation of the impact point in the projectile from the line of fire, determined by deviation δz_k. The

lateral displacement of projectile, determined by the deviation of velocity vector of angle $\delta\Psi_K$ in horizontal plane, let us designate - $z_{C\Psi_K}$. The lateral displacement of projectile, determined by the deviation of velocity modulus, let us desigrate through z_{Cv_K} and the lateral displacement of projectile, determined by the effect of wind, let us designate z_{Cw_z}. For the case when the deviation of determining parameters $\delta\Psi_K>0$; $\delta z_K<0$; $\delta v_K<0$ and $w_z<0$, in accordance with Fig. 11.25 we obtain the total lateral deviation

$$z_C=-z_{C\,K}-z_{C\Psi_K}-z_{Cw_z}+z_{Cv_K}. \qquad (11.153)$$

Utilizing a differential correcting formula, we will obtain

$$z_C=-\frac{\partial z_C}{\partial z_K}\delta z_K-\frac{\partial z_C}{\partial \Psi_K}\delta\Psi_K-\frac{\partial z_C}{\partial w_z}w_z+\frac{\partial z_C}{\partial v_K}\delta v_K. \qquad (11.154)$$

Corrections for wind w_z and for a change in the velocity at the end of the operation of engine δv_K are determined by one of previously examined methods.

With the adopted assumptions the deviation δz_K will lead to equivalent displacement relative to axis Ox of the projection of an entire trajectory on horizontal plane, i.e.,

$$\frac{\partial z_C}{\partial z_K}=1 \quad \text{AND} \quad z_{Cz_K}=\delta z_K.$$

Correction factor for a charge in the angle $\delta\Psi_K$ will be determined from geometric considerations. Let the velocity vector of $\bar{v}_K$ be

deflected from the plane, parallel to plane of reference of firing and passing through point K, by angle [illegible] (in inclined plane); then

$$\sin \delta\Psi_{z} = \text{[illegible]} \qquad (11.155)$$

From Fig. 11.25

$$z_{\Psi_{z}} = L_{z} \sin \delta\Psi_{z} = \text{[illegible]} \qquad (11.156)$$

Here L_{z} — length of inactive leg when [illegible]. On the smallness of angle, it is possible to accept [illegible] and then

$$z_{\Psi_{z}} = \frac{\partial z_{C}}{\partial \Psi_{z}} \delta\Psi_{z} = \frac{\partial z_{C}}{\partial \Psi_{zm}} \delta\Psi \text{[illegible]}$$

where the correction factor

$$\frac{\partial z_{C}}{\partial \Psi_{zm}} = \frac{L_{z}}{\cos \theta_{z}} \text{[illegible]} \qquad (11.\text{[illegible]})$$

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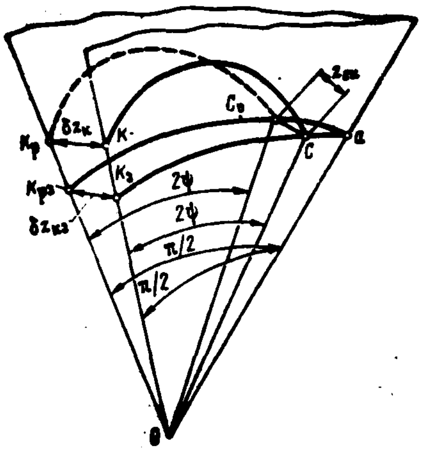
Let us examine the procedure of the calculation of the corrections, which consider the curvature of the Earth. For the spherical model of the Earth in the central gravitational field of the plane of two trajectories, agitated and not disturbed, they comprise the dihedral angle whose fin/edge traverses the conditional center of the Earth. The value of dihedral angle will be determined by the lateral deviation of point K of value δz_{K}. Let us designate the calculated end/lead of the engine operation with point K_{p}, the real

end/lead of the engine operation - by a point K. The projections of points K_p and of K on the surface of the spherical model of the Earth let us designate K_{p3} and K_3; respectively, impact points let us designate C_p and C. The fin/edge of dihedral angle is located from the radius-vectors, which connect points K_p and K with the conditional center of the Earth, on angle $\pi/2$. It is obvious, $\delta z_{\kappa} \neq \delta z_{\kappa 3}$ as this was obtained earlier for low firing distances with the nonintersecting planes basic and that agitated of trajectories. Let us designate in Fig. 11.26 intersection of the fin/edge of dihedral angle with the surface of the Earth by point - a. If we do not consider the difference between δz_{κ} at height/altitude y_{κ} and $\delta z_{\kappa 3}$ on the surface of the Earth, then it is possible to use similar spherical triangles $C_p aC$ and $K_{p3} aK_3$. From spherical trigonometry

$$\delta z_C = \delta z_{\kappa} \cos 2\psi$$

and, which means, that

$$\frac{\delta z_C}{\delta z_{\kappa}} = \cos 2\psi.$$



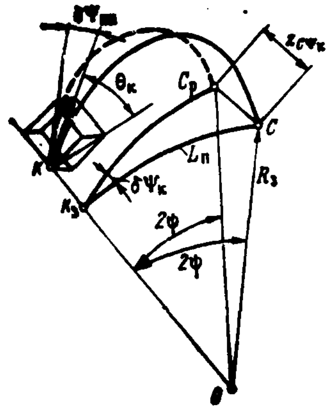


Fig. 11.27.

Fig. 11.26. Lateral displacement of the projectile of constant mass $z_{c\kappa}$ depending on the lateral linear deflection of initial point δz_κ

Fig. 11.27. Lateral displacement of projectile of constant mass $z_{c\Psi_\kappa}$ depending on lateral angular deflection of velocity vector $\delta\Psi_\kappa$.

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With the low range angle

$$\cos 2\phi \approx 1 \text{ and } z_{c\kappa} \approx \delta z_\kappa.$$

In the particular case, when $2\phi = \frac{\pi}{2}$, $z_{c\kappa} = 0$.

Let us examine correction for angular deflection $\delta\Psi_m$ with the

spherical model of the Earth. In Fig. 11.27 sides of spherical triangle C_pK_3 and CK_3 are projections on the surface trajectories of calculated and disturbed Earth. The lateral deviation of impact point from trajectory plane, passing through points K and C_p, determined by angular deflection $\delta\Psi_k$, we will obtain from spherical triangle C_pK_3C

$$z_{C\Psi_{кн}}=\frac{R_3\sin 2\psi}{\cos\theta_k}\delta\Psi_{кн}=\frac{\partial z_C}{\partial\Psi_{кн}}\delta\Psi_{кн}. \qquad (11.159)$$

The correction factor

$$\frac{\partial z_C}{\partial\Psi_{кн}}=\frac{R_3\sin 2\psi}{\cos\theta_k}. \qquad (11.160)$$

The great value correction $z_{C\Psi_{кн}}$ will have with range angle $2\psi=\frac{\pi}{2}$; with distance $2\psi=2\pi$ the correction will become equal to zero. With short distances $\sin 2\psi\approx\frac{L_k}{R_3}$ and then we will obtain already known to us formula (11.158).

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Chapter XII.

Initial conditions of shot.

Type and the designation/purpose of rocket or projectile, their construction, aerodynamic and ballistic data, the construction of projectile installation and meteorological conditions, by which is conducted the firing, determine the initial conditions of shot for the assigned distance. Of the unguided rockets and projectiles, initial conditions lay out of trajectory.

The characteristics of the spatial motion of the unguided rocket can be determined, after solving system of equations, obtained from equations (3.7)-(3.12), and (3.14)-(3.23), after placing in them equal to zero control forces and torque/moments.

After simplifications the system will take the form

$$\dot v = \frac{1}{m}(P_x - X + Q_x); \quad \dot\theta = \frac{1}{mv}(P_{y^*} + Y^* + Q_{y^*});$$

$$\dot\Psi = -\frac{1}{mv\cos\theta}(P_{z^*} + Z^*);$$

$$J_{x_1}\dot\omega_{x_1} + (J_{z_1} - J_{y_1})\omega_{y_1}\omega_{z_1} = \sum M_{x_1};$$

$$J_{y_1}\dot\omega_{y_1} + (J_{x_1} - J_{z_1})\omega_{x_1}\omega_{z_1} = \sum M_{y_1};$$

$$J_{z_1}\dot\omega_{z_1} + (J_{y_1} - J_{x_1})\omega_{x_1}\omega_{y_1} = \sum M_{z_1};$$

$$\dot\vartheta = \omega_{y_1}\sin\gamma + \omega_{z_1}\cos\gamma; \qquad (12.1)$$

$$\dot\psi = \frac{1}{\cos\vartheta}(\omega_{y_1}\cos\gamma - \omega_{z_1}\sin\gamma);$$

$$\dot\gamma = \omega_{x_1} - \mathrm{tg}\,\vartheta(\omega_{y_1}\cos\gamma - \omega_{z_1}\sin\gamma);$$

$$\sin\theta = \sin\vartheta\cos\alpha\cos\beta + \cos\vartheta(\sin\alpha\cos\gamma_C + \cos\alpha\sin\beta\sin\gamma_C);$$

$$\sin\psi\cos\gamma = \sin\Psi\cos\beta\cos\gamma_C + \cos\Psi(\sin\beta\cos\theta + \sin\ [illegible]\ \sin\vartheta\sin\psi;$$

$$\cos\vartheta\sin\gamma = \sin\gamma_C\cos\beta\cos\theta - \sin\beta\sin\theta;$$

$$\frac{dx_g}{dt} = v\cos\theta\cos\Psi; \quad \frac{dy_g}{dt} = v\sin\theta; \qquad (12.1)$$

$$\frac{dz_g}{dt} = -v\cos\theta\sin\Psi; \quad m = m_0 - \int_0^t |\dot m|\,dt.$$

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For the solution of the written system, it is necessary to know the initial values of the following values: the initial value of mass m_0; the initial values of the moments of inertia $J_{x_{10}}$, $J_{y_{10}}$, $J_{z_{10}}$, the initial values of all forces and torque/moments, written in right part one of six equations of system. Furthermore, it is necessary to know the initial values of fifteen motion characteristics v_0, θ_0, Ψ_0, x_{g0}, y_{g0}, z_{g0}, ϑ_0, ψ_0, γ_0, α_0, β_0, γ_{C_0}, $\omega_{x_{10}}$, $\omega_{y_{10}}$, $\omega_{z_{10}}$.

If we instead of equations (3.14)-(3.16) utilize three equations (3.13), then instead of $\omega_{x_1 0}, \omega_{y_1 0}$ and $\omega_{z_1 0}$ it is necessary to know $\dot{\vartheta}_0, \dot{\Psi}_0$ and $\dot{\gamma}_0$. The enumerated initial conditions must be determined for the torque/moment of the loss of tight coupling of the unguided rocket with projectile installation. For determining the effect of scattering initial conditions for scattering of trajectories (or impact points) it is necessary to know the stochastic characteristics of the named initial values. The solution of assigned mission represents great difficulties; therefore the effect of initial conditions for the results of firing is estimated in the simplified setting with one or the other assumptions.

Effect on the firing distance of deviations from computed values of the initial angle of departure, of initial velocity and meteorological conditions is considered with the aid of correcting formulas and is examined in chapter XI. Let us here show the effect of the design features of projectile installation on the formation of the initial conditions of firing.

A change in the initial conditions of shot can occur because of:

a) the motion of the carrier of armament;

b) the oscillation/vibration of the carrier of armament and strictly projectile installation as a result of the elastic properties of constructions;

c) the loss of the axial alignment of rocket and launching racks with the descent of rocket from guides;

d) the effect of gas flows, which is exhibited with the descent of the rocket from launcher or on leaving of the projectile of artillery instrument from bore.

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§1. Effect of the motion of the carrier of armament on the initial conditions of shot.

The motion of the carrier of armament as solid body can be divided into the displacement/movement of its center of mass and oscillation/vibration relative to the center of mass. The displacement/movement of the center of mass is determined by two motions - by basic motion of the carrier of armament under the action of motors and the oscillatory motion of the center of mass under the

effect of environmental disturbances (for example, the orbital tossing of ship, caused sea rating).

The oscillation/vibrations of the carrier of armament relative to the center of mass, by analogy with the tossing of ship, it is possible to divide into the rolling motion - rolling of carrier around its longitudinal axis; keel tossing - fluctuation of carrier of its relatively vertical axis.

The total initial velocity of projectile in earth-based coordinate system without the account of the rotational effect of the Earth is equal to

$$\bar{v}_0=\bar{v}_{0r}+\bar{v}_{\text{н}}+\bar{v}_{\text{в}}, \quad (12.2)$$

where $\bar{v}_{0r}$ — velocity of projectile of relatively projectile installation at the moment of the loss of tight coupling with guides by the latter (for the projectile of artillery instrument - muzzle velocity);

$\bar{v}_{\text{н}}$ — a speed of running of the carrier of armament (ship, tank, aircraft);

$\bar{v}_{\text{в}}$ — the supplementary initial velocity, caused by environmental disturbance, in which moves the carrier (for example,

the tossing of ship-launch vehicle).

The greatest difficulties are encountered during determination v_x. If we do not consider the effect of medium and to equate $v_x=0$, then the value of the velocity cf rocket (projectile) in module/modulus and direction in earth-based ccordinate system will be determined by following manner. Let us join the beginning cf earth-based coordinate system with the center cf mass of launch vehicle and it is directed axis Ox_3 then so that it would coincide with the velocity vector v_x (Fig. 12.1). The sense cf the vector cf the velocity of the motion of the center of mass of projectile relative to launch vehicle v_{or} let us assign by angle of departure in relative motion θ_{or} and by the heading/course angle of firing ψ_{rc}. Then, bearing in mind that v_x — translational speed, we will obtain for the projections of the speed cf projectile on the axis cf earth-based coordinate system the following expressions:

$$\left.\begin{aligned} v_{x_3} &= v_{or}\cos\theta_{or}\cos\psi_{rc}+v_x; \\ v_{y_3} &= v_{or}\sin\theta_{or}; \\ v_{z_3} &= -v_{or}\cos\theta_{or}\sin\psi_{rc}. \end{aligned}\right\} \quad (12.3)$$

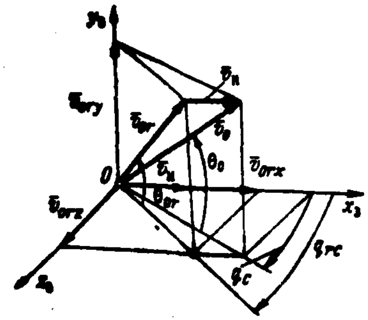

Fig. 12.1. Heading/course angle, the initial velocity and angle of departure, obtained taking into account the velocity of carrier.

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Modules of velocity of projectile in the absolute motion

$$v_0=\sqrt{v_{0x_3}^2+v_{0y_3}^2+v_{0z_3}^2}.$$

Substituting equalities (12.3), we will obtain after the conversions:

$$v_e=v_{0r}\sqrt{1+2\frac{v_n}{v_{0r}}\cos\theta_{0r}\cos q_{re}+\frac{v_n^2}{v_{0r}^2}}. \qquad (12.4)$$

Heading/course angle of firing in the absolute action

$$q_e=\mathrm{arctg}\left(\frac{v_{0r}\cos\theta_{0r}\sin q_{re}}{v_{0r}\cos\theta_{0r}\cos q_{re}+v_n}\right). \qquad (12.5)$$

Angle of departure in the absolute action

$$\theta_e=\arccos\left(\frac{v_{0r}\cos\theta_{0r}\cos q_{re}+v_n}{v_0\cos q_e}\right). \qquad (12.6)$$

Three last/latter formulas are obtained on the assumption that projectile - material point.

For determining initial missile attitude (rocket) relative to the Earth it is necessary to consider the effect of the oscillation/vibrations of the carrier of armament and launcher.

Let us examine the effect of the oscillation/vibrations of launch vehicle relative to the center of mass on a change in the parameters of the motion of rocket at descent from launching rack. We utilize five systems of coordinates (Fig. 12.2). The oscillation/vibration of the carrier of armament let us examine relative to earth-based coordinate system $O_3x_3y_3z_3$.

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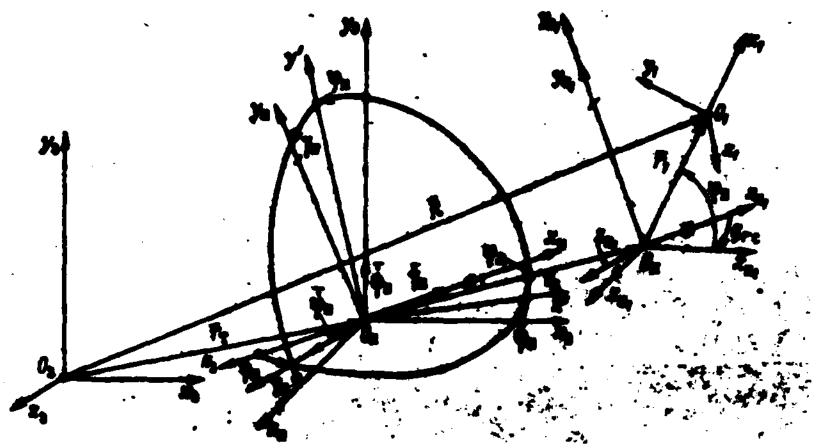

Fig. 12.2. Coordinate system for case of missile takeoff from unstabilized launcher of driving/moving carrier.

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The origin of coordinate system $O_n x_n y_n z_n$ is rigid, connected with carrier, let us place in the center of mass of carrier O_n. The rotation of the system of coordinates $x_n y_n z_n$ relative to system $x_3 y_3 z_3$ let us define that as usual, by three angles: in horizontal plane - by angle ψ_n, in vertical plane - by angle φ_n; lateral rotation relative to axis $O_n x_n$ - by angle γ_n. In the center of oscillation of launcher O_n let us place the beginning of two coordinate systems: the systems of coordinates $O_n x_{n_1} y_{n_1} z_{n_1}$, of the rigidly connected with carrier, and system of coordinates $O_n x_{n_1} y_{n_1} z_{n_1}$, connected with launcher.

The axes of coordinates $x_н, y_н, z_0$, are collinear to axes $x_н y_н z_н$. The rotation of launcher relative to carrier is determined by two angles: by the heading/course angle of firing q_{rc} - between axes $O_н x_{н}$ and $O_н x_н$, and the angle $\varphi_н$, those determining the slope/inclination of launcher to the plane of axes $O_н x_н$ and $O_н x_н$, by rigidly connected with carrier. The fifth coordinate system is the system $O_1 x_1 y_1 z_1$, connected with rocket at the moment of its descent with guides; the axis $O_1 x_1$ of this system coincides with the line of fire.

The pitch angles, yaw and bank of rocket at the moment of the loss of tight coupling with the guides of the nonstabilized launcher can be determined for two positions of the launch vehicle: at the zero values of the angles of rotation of the system of coordinates $O_н x_н y_н z_н$, connected with launch vehicle, relative to terrestrial system $(\psi_н = \varphi_н = \gamma_н = 0)$ and for the angles of rotation, different from zero (i.e. for the case $\psi_н \neq 0$; $\varphi_н \neq 0$ and $\gamma_н \neq 0$). Differences in the values of the pitch angles, yaw and bank of rocket, determined for the named two positions of rocket carrier, will determine the effect of the oscillation/vibration of carrier on a change in the angles, which determine the position of rocket of relatively ground coordinates.

If we do not consider the elastic vibrations of the

constructions of launch vehicle and of launcher and disturbance/perturbations of rocket at the moment of its descent from guides, then, by using the table of transfer cosines from earth-based coordinate system to the coordinate system, connected with rocket carrier (by table, similar 2.3d), we will obtain equations for determining the deviations of the pitch angles, yaw and bank of rocket at the moment of start depending on the angles of the vibration of launch vehicle.

For case $\vartheta_n=\psi_n=\gamma_n=0$ and the adopted by us assumptions at the moment of the descent of rocket from guides, let us, obviously, have $\vartheta_0=\varphi_n, \psi_0=\varphi_{r0}$ (from direction $0x_n$) and $\gamma = 0$.

In other case, when $\vartheta_n\neq0$; $\psi_n\neq0$ and $\gamma_n\neq0$, the rocket will descend from guides at angles ϑ, ψ and γ. Comparing, as this made V. P. Kazakovtsev [18], cell/elements of transition at the moment of the start of rocket from the system of coordinates $0_n x_n y_n z_n$ to system $0_1x_1y_1z_1$ on angles $\vartheta_n, \gamma_n, \psi_n$ and φ_{r0} is direct from terrestrial system $0_3 x_3 y_3 z_3$ to system $0_1x_1y_1z_1$ on angles ϑ, ψ and γ, we will obtain equations for determining the deviations of the pitch angles, yaw and bank of rocket depending on the angles of the vibration of rocket carrier [18].

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Accepting, as a result of the smallness of angles,

$$\sin\psi_н=\psi_н,\quad \sin\varphi_н=\varphi_н,\quad \sin\gamma_н=\gamma_н,$$
$$\cos\psi_н=\cos\varphi_н=\cos\gamma_н=1,$$

we will obtain

$$\Delta\psi=\operatorname{arctg}\left[\frac{-\psi_н\cos q_{rc}+\gamma_н\operatorname{tg}\varphi_н+\sin q_{rc}}{\cos q_{rc}-\varphi_н\operatorname{tg}\varphi_н+\psi_н\sin q_{rc}}\right]-q_{rc};\quad (12.7)$$

$$\Delta\vartheta=\arcsin\left[\varphi_н\cos q_{rc}\cos\varphi_п+\sin\varphi_п-\gamma_н\cos\varphi_п\sin q_{rc}\right]-\varphi_п;\quad (12.8)$$

$$\Delta\gamma=\operatorname{arctg}\left[-\frac{\varphi_н\operatorname{tg}q_{rc}+\gamma_н}{\varphi_н\sin\varphi_п-\dfrac{\cos\varphi_п}{\cos q_{rc}}-\gamma_н\sin\varphi_п\operatorname{tg}q_{rc}}\right].\quad (12.9)$$

The projections of the angular velocity vector of the rotation of launch vehicle of relatively earth-based coordinate system on the connected with it coordinate axes, similar (3.13), will be written in this form:

$$\omega_{xн}=\dot\psi_н\sin\varphi_н+\dot\gamma_н;\quad (12.10)$$
$$\omega_{yн}=\dot\psi_н\cos\varphi_н\cos\gamma_н+\dot\varphi_н\sin\gamma;\quad (12.11)$$
$$\omega_{zн}=-\dot\psi_н\cos\varphi_н\sin\gamma_н+\dot\varphi_н\cos\gamma_н.\quad (12.12)$$

Since we consider launch vehicle and launcher as rigid and rigidly connected bodies, then it is obvious that the vector of the instantaneous angular velocity of launching racks will coincide with the vector of the instantaneous angular velocity of launch vehicle.

If we introduce the assumption that the rocket during motion along launching racks completely receives the angular displacements of guides, which corresponds to the stiffening joint between them, then the vector of the instantaneous angular velocity of rocket will be equal to the vector of the instantaneous angular velocity of guides. On the basis of the aforesaid, it is possible to easily find

the projection of the vector of the instantaneous angular velocity of rocket, caused by the motion of launch vehicle, on the axis of the connected with rocket system of coordinates $O_1x_1y_1z_1$. Taking into account dependence (12.10), (12.11) and (12.12), and also the direction cosines of passage from the system of coordinates $O_нx_нy_нz_н$ to the system of coordinates $O_1x_1y_1z_1$, let us have

$$\omega_{x_1}=\omega_{x\,н}\cos\varphi_н\cos q_{rc}+\omega_{y\,н}\sin\varphi_н+\omega_{z\,н}\cos\varphi_н\sin q_{rc}; \quad (12.13)$$

$$\omega_{y_1}=-\omega_{x\,н}\sin\varphi_н\cos q_{rc}+\omega_{y\,н}\cos\varphi_н-\omega_{z\,н}\sin\varphi_н\sin q_{rc}; \quad (12.14)$$

$$\omega_{z_1}=-\omega_{x\,н}\sin q_{rc}+\omega_{z\,н}\cos q_{rc}. \quad (12.15)$$

The obtained equations make it possible to determine the angular velocities of rocket at the moment of start from the nonstabilized launcher of the oscillating launch vehicle (without the account of disturbance/perturbations with descent).

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Let us examine the disturbance/perturbations of rocket with descent from the directing stabilized launcher, caused only by the motion of launch vehicle. The automatic machine of the angular stabilization of launcher provides the constancy of the angular position of the guides with rocket in space. However, the rocket will have the initial disturbances along the position of the center of mass and rate of motion with start, determined by the motion of launch vehicle, by the nonuniformity of course and by the orbital tossing of its center of mass and by the vibration of carrier

relative to the center of mass. The necessary coordinate systems are shown on Fig. 12.3. Fixed (terrestrial) coordinates let us designate $O_{з}x_{з}y_{з}z_{з}$: the coordinates, rigidly connected with launch vehicle, let us designate as before $O_{н}x_{н}y_{н}z_{н}$. Let us introduce with the support systems of coordinates $O_{н}x_{зн}y_{зн}z_{зн}$, $O_{п}x_{зп}y_{зп}z_{зп}$ and $O_{co}x_{зc}y_{зc}z_{зc}$, axis of which is directed collinearly appropriate by the axes of basic earth-based coordinate system, and beginning let us place: $O_{н}$ - to the center of mass of launch vehicle; $O_{п}$ - to the conditional center of oscillation of launcher; O_{co} - to the center of mass of rocket at the moment of its descent from launching rack.

The axes of the system of coordinates $O_{н}x_{н}y_{н}z_{н}$ are turned relative to the system of coordinates $O_{н}x_{зн}y_{зн}z_{зн}$ at angles $\varphi_{н}$, $\psi_{н}$ and $\gamma_{н}$, the determining vibrations of the carrier of armament relative to the center of mass. Let us introduce another coordinate system whose beginning let us place into the point O_{co} of the position of the center of mass of rocket at the moment of its descent from guides.

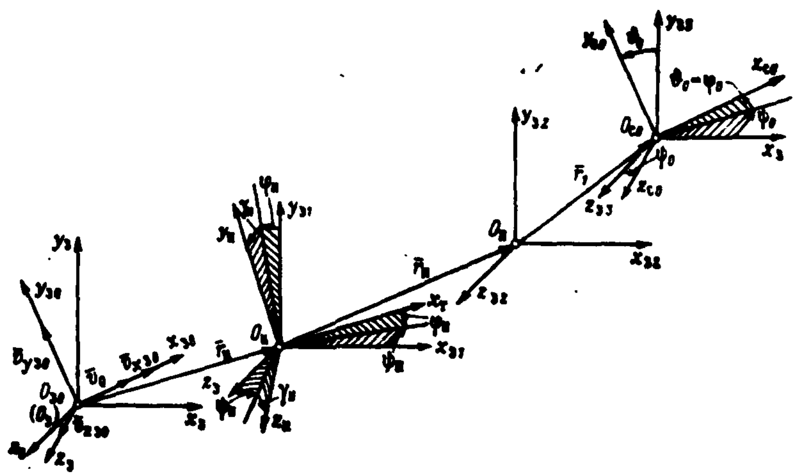

Fig. 12.3. Coordinate system for the case the stria of rocket from the stabilized launcher.

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Axis $O_{c0}x_{c0}$ is directed along axis of rocket with its descent from guides, axis $O_{c0}y_{c0}$ it is arranged in range plane, while axis $O_{c0}z_{c0}$ will supplement system to right. Since launcher is stabilized, the with the adopted assumption direction of the axes of the system of coordinates $O_{c0}x_{c0}y_{c0}z_{c0}$ is constant/invariable in space and range plane coincides with coordinate plane $O_{c0}x_{c0}y_{c0}$. The initial angle of the line of fire and the angle of elevation of launching racks let us designate through φ_0 and θ_0 (Fig. 12.3). Without examining the disturbance/perturbation of the rocket with descent from guides, let

us consider that initial pitch angle $\vartheta_0=\phi_0$. From the center of base ground coordinates 0_3 let us conduct ground coordinates $0_{30}x_{30}y_{30}z_{30}$, of axis of which it is directed collinearly to axes $0_{c0}x_{c0}y_{c0}z_{c0}$. Furthermore, in Fig. 12.3 it is marked: $\bar{r}_н$ - radius-vector, carried out from point 0_3 (conditionally selected and fixed on the Earth) to the center of mass of launch vehicle; $\bar{r}_п$ - radius-vector, carried out from the center of mass of launch vehicle to the conditional center of oscillation of launcher (TsKPU); $\bar{r}_1$ - radius-vector, carried out from the center of oscillation of launcher into the point, occupied by the center of mass of rocket at the moment of its descent from guides. In the process of moving the carrier of armament $\bar{r}_н$ it changes in module/modulus and direction, $\bar{r}_п$ - changes only in the direction, $\bar{r}_1$ is constant in the value (if we do not consider scattering the time of the motion of rocket along guides to the loss with them of power interaction) and does not change its direction in space.

Let us designate: $\bar{v}_{30}$ - velocity vector of the center of mass of the rocket (relative to the Earth) at the moment of its descent from guides with the driving/moving carrier; $\bar{v}_0$ - velocity vector of the rocket at the moment of descent from guides with fixed carrier (i.e. relative to earth-based coordinate system); $v_{x_{30}}, v_{y_{30}}, v_{z_{30}}$ - projection $\bar{v}_{30}$ on the axis of earth-based coordinate system $0_{30}x_{30}y_{30}z_{30}$.

The deviations of the velocity of rocket, determined by the motion of carrier, will be equal to

$$\Delta v_{x_{30}} = v_{x_{30}} - v_0; \quad \Delta v_{y_{30}} = v_{y_{30}}; \quad \Delta v_{z_{30}} = v_{z_{30}}, \qquad (12.16)$$

since $\bar{v}_0$ it is directed along the axis $O_{00}x_{00}$, collinear axis $O_{30}x_{30}$.

On Fig. 12.3 taking into account the stipulated conditions

$$\bar{v}_{30} = (\bar{v}_{\text{н}} + \Delta\bar{v}_{\text{н}}) + [\bar{\omega}_{\text{н}} \times (\bar{r}_{\text{н}} + \bar{r}_1)] + \bar{v}_{\text{р}}. \qquad (12.17)$$

Here $\bar{v}_{\text{н}}$ - velocity of the undisturbed motion of the center of mass of carrier;

$\Delta\bar{v}_{\text{н}}$ - additive velocity of the forward motion of the center of mass of carrier, caused by the nonuniformity of its motion;

$\bar{\omega}_{\text{н}}$ - angular velocity vector of the rotary motion of carrier of its relatively center of mass.

Radius-vector $\bar{r}_1$, as value low in comparison with a radius $\bar{r}_{\text{н}}$, subsequently consider we will not.

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It is consistent axis $Ox_{\text{н}}$ with velocity vector $\bar{v}_{\text{н}}$; then

$$\bar{v}_{\text{н}} = v_{\text{н}} \bar{x}_{\text{н}}.$$

It is expressed additional velocity $\Delta\bar{v}_n$ through projections on the axis of earth-based coordinate system

$$\Delta\bar{v}_n = \dot{x}_3\vec{x}_3 + \dot{y}_3\vec{y}_3 + \dot{z}_3\vec{z}_3.$$

Here $\dot{x}_3, \dot{y}_3, \dot{z}_3$ - the projection of vector $\Delta\bar{v}_n$ on the axis of earth-based coordinate system $O_3x_3y_3z_3$; $\vec{x}_3, \vec{y}_3, \vec{z}_3$ - the unit vectors of ground coordinates.

Now let us rewrite (12.17)

$$\bar{v}_{30} = (v_n + \dot{x}_3)\vec{x}_3 + \dot{y}_3\vec{y}_3 + \dot{z}_3\vec{z}_3 + \begin{vmatrix} \vec{x}_n, & \vec{y}_n, & \vec{z}_n \\ \omega_{xn}, & \omega_{yn}, & \omega_{zn} \\ x_n, & y_n, & z_n \end{vmatrix} + v_0\vec{x}_{30}. \quad (12.18)$$

Here $\omega_{xn}, \omega_{yn}, \omega_{zn}$ - projection $\bar{\omega}_n$ on the axis of the system of coordinates $O_nx_ny_nz_n$; x_n, y_n, z_n - projection $\bar{r}_n$ on the axis of the system of coordinates $O_nx_ny_nz_n$; $\vec{x}_n, \vec{y}_n, \vec{z}_n$ - the corresponding unit vectors.

Utilizing cosines of the angles of transfer/transition (n_{ij}) from the axes of system $O_3x_3y_3z_3$ to axes $O_{30}x_{30}y_{30}z_{30}$, and also the cosines of the angles of transfer/transition from the axes of the system of coordinates $O_nx_ny_nz_n$ to the system of coordinates $O_{30}x_{30}y_{30}z_{30}$ (h_{ij}), we will obtain the projections of vector equality (12.18) on the axis of coordinates $O_{30}x_{30}y_{30}z_{30}(h_{ij})$.

$$
\left.\begin{aligned}
v_{x_{30}} &= v_0 + (v_n + \dot{x}_3) n_{11} + \dot{y}_3 n_{12} + \dot{z}_3 n_{13} + \\
&+ (\omega_{y\,n} z_n - \omega_{z\,n} y_n) k_{11} + (\omega_{z\,n} x_n - \omega_{x\,n} z_n) k_{12} + \\
&+ (\omega_{x\,n} y_n - \omega_{y\,n} x_n) k_{13}; \\
v_{y_{30}} &= (v_n + \dot{x}_3) n_{21} + \dot{y}_3 n_{22} + \dot{z}_3 n_{23} + (\omega_{y\,n} z_n - \omega_{z\,n} y_n) k_{21} + \\
&+ (\omega_{z\,n} x_n - \omega_{x\,n} z_n) k_{22} + (\omega_{x\,n} y_n - \omega_{y\,n} x_n) k_{23}; \\
v_{z_{30}} &= (v_n + \dot{x}_3) n_{31} + \dot{y}_3 n_{32} + \dot{z}_3 n_{33} + (\omega_{y\,n} z_n - \omega_{z\,n} y_n) k_{31} + \\
&+ (\omega_{z\,n} x_n - \omega_{x\,n} z_n) k_{32} + (\omega_{x\,n} y_n - \omega_{y\,n} x_n) k_{33}.
\end{aligned}\right\} \quad (12.19)
$$

After substituting (12.19) intc equaticns (12.16), let us have

$$
\left.\begin{aligned}
\Delta v_{x_{30}} &= v_{x_{30}} - v_0 = (v_n + \dot{x}_3) n_{11} + \dot{y}_3 n_{12} + \dots; \\
\Delta v_{y_{30}} &= v_{y_{30}} = (v_n + \dot{x}_3) n_{21} + \dot{y}_3 n_{22} + \dots; \\
\Delta v_{z_{30}} &= v_{z_{30}} = (v_n + \dot{x}_3) n_{31} + \dot{y}_3 n_{32} + \dots;
\end{aligned}\right\} \quad (12.20)
$$

Complete writing of equations (12.20) at the opened values of transfer cosines n_{ij} and k_{ij} is sufficiently bulky.

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Following V. P. Kazakovtsev, it is simplified them [18]. In view of the smallness of the angles of the vibration of carrier $\psi_n, \varphi_n, \gamma_n$ moreover

$$\sin\psi_n \approx \psi_n; \quad \sin\varphi_n \approx \varphi_n; \quad \sin\gamma_n \approx \gamma_n$$

and

$$\cos\psi_n \approx \cos\varphi_n \approx \cos\gamma_n \approx 1.$$

Let us disregard the products of low values $\sin\psi_n \sin\varphi_n \approx 0$; and $\sin\psi_n \sin\gamma_n \approx 0$ of so forth. The projections of the vector of the instantaneous angular velocity of carrier we take in the form

$$\omega_{x\,n} \approx \dot{\gamma}_n; \quad \omega_{y\,n} \approx \dot{\psi}_n; \quad \omega_{z\,n} \approx \dot{\varphi}_n.$$

Taking into account the named assumptions let us write (12.20) in abbreviated form [18]

$$\left.\begin{aligned}
\Delta v_{x_{30}} &= (v_{н}+\dot{x}_3)\cos\vartheta_0\cos\psi_0+\dot{y}_3\sin\vartheta_0+\dot{z}_3\cos\vartheta_0\sin\psi_0+\\
&+\dot{\psi}_{н}(z_{н}\cos\vartheta_0\cos\psi_0-x_{н}\cos\vartheta_0\sin\psi_0)+\\
&+\dot{\varphi}_{н}(x_{н}\sin\vartheta_0-y_{н}\cos\vartheta_0\cos\psi_0)+\\
&+\dot{\gamma}_{н}(y_{н}\cos\vartheta_0\sin\psi_0-z_{н}\sin\vartheta_0);\\
\Delta v_{y_{30}} &= -(v_{н}+\dot{x}_3)\sin\vartheta_0\cos\psi_0+\dot{y}_3\cos\vartheta_0-\\
&-\dot{z}_3\sin\vartheta_0\sin\psi_0+\dot{\psi}_{н}(x_{н}\sin\vartheta_0\sin\psi_0-z_{н}\sin\vartheta_0\cos\psi_0)+\\
&+\dot{\varphi}_{н}(x_{н}\cos\vartheta_0+y_{н}\sin\vartheta_0\cos\psi_0)-\\
&-\dot{\gamma}_{н}(y_{н}\sin\vartheta_0\sin\psi_0+z_{н}\cos\vartheta_0);\\
\Delta v_{z_{30}} &= -(v_{н}+\dot{x}_3)\sin\psi_0+\dot{z}_3\cos\psi_0-\dot{\psi}_{н}(x_{н}\cos\psi_0+\\
&+z_{н}\sin\psi_0)+\dot{\varphi}_{н}y_{н}\sin\psi_0+\dot{\gamma}_{н}y_{н}\cos\psi_0.
\end{aligned}\right\}\quad(12.21)$$

in the written equations of value $v_{н}$, ψ_0 and ϑ_0, are constant and do not depend on time. Variable values will be: the projections of additional speed $\dot{x}_3, \dot{y}_3, \dot{z}_3$; the angles of rotation of carrier - $\dot{\varphi}_{н}, \dot{\psi}_{н}, \dot{\gamma}_{н}$; the angular velocities of the rotation of carrier of its relatively center of mass.

We will obtain changes in the modulus of velocity, angles of attack and slip, determined by the motion of carrier at the moment of the descent of rocket from the directing stabilized launcher. Let us consider that at the moment of the descent of rocket from control surface of its symmetry will be vertical and coincides with the plane of earth-based coordinate system, determined by axes $O_{30}y_{30}$ and $O_{30}x_{30}$.

During the motion of carrier, the axis of rocket at the moment of its descent from guides will coincide in the direction with axes $O_{30}x_{30}$ and $O_{00}x_{00}$ a vector a the total velocity of rocket $\bar{v}_{30}$ will be deflected from the axis of rocket (Fig. 12.4). Line O_{30} B - projection $\bar{v}_{30}$ on the plane of the symmetry of rocket.

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Angle of attack is defined as angle between the projection of velocity vector on the plane of symmetry and the axis of rocket; angle of slip - as angle between vectors of speed and its projection on the plane of vertical symmetry. On Fig. 12.4 at the positive values of those composing velocities $\Delta v_{x_{30}}$, $\Delta v_{y_{30}}$, $\Delta v_{z_{30}}$ we will obtain a positive value of a change in the angle of attack $\Delta\alpha_0$. In accordance with Fig. 12.4

$$\sin(\Delta\beta_0)=\frac{\Delta v_{z_{30}}}{|v_{30}|},$$

either

$$\Delta\beta_0=\arcsin\frac{\Delta v_{z_{30}}}{\sqrt{(v_0+\Delta v_{x_{30}})^2+\Delta v^2_{y_{30}}+\Delta v^2_{z_{30}}}}; \qquad (12.22)$$

$$\mathrm{tg}(\Delta\alpha_0)=-\frac{\Delta v_{y_{30}}}{v_0+\Delta v_{x_{30}}},$$

or

$$\Delta\alpha_0=\mathrm{arctg}\left(-\frac{\Delta v_{y_{30}}}{v_0+\Delta v_{x_{30}}}\right). \qquad (12.23)$$

A change of the modulus of velocity of rocket at the moment of descent from guides, caused by the action of launch vehicle, we will obtain on the following dependence:

$$\Delta v_0=v_{30}-v_0=\sqrt{(v_0+\Delta v_{x_{30}})^2+\Delta v^2_{y_{30}}+\Delta v^2_{z_{30}}}-v_0. \qquad (12.24)$$

§2. Motion of rockets alcng launching racks.

One of the most widely used trigger circuits of the rockets of lcw and average distance is the sclematic cf missile takeoff from launching rack. There are differences in the character of the moticn cf rocket alcng the guides depending cn the simultaneity of the descent of the tags of rocket from guides. Let us examine first the case cf moving the rocket with the simultanecus descent of tags.

Basic forces and the torque/moments, which act on the rocket during its mction along guides, shcwn cn Fig. 12.5.

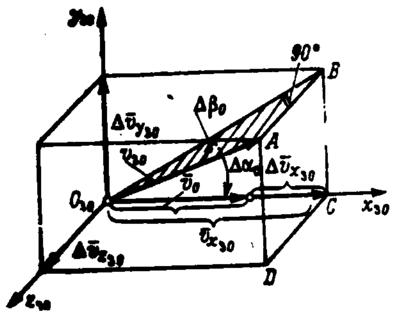

Fig. 12.4. Changes in the angles of attack and slip, determined by the motion of carrier.

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On rocket acts thrust P, longitudinal X_1 and normal Y_1 aerodynamic forces, longitudinal X_{r1} and normal Y_{r1} gas-dynamic forces, gravitational force Mg, normal reactions F_{N1}, F_{N2} and frictional force F_{T1}, F_{T2} in the tags. On figure is also shown the starting system of coordinates Ox_0y_0 and the system of coordinates $O_n x_n y_n$. The beginning of this coordinate system (point O_n) coincides with the initial position of the center of mass of rocket prior to start. Axis coincides with axis of rocket and to parallel guides: axis $O_n y_n$ is perpendicular to axis $O_n x_n$ and lie/rests at the longitudinal plane of the symmetry of rocket.

Will write the equations of motion of rocket along the guides in

the system of coordinates $O_n x_n y_n$, which we we consider fixed; in this case, let us consider launcher absolutely rigid body with rectilinear guides. Also let us consider that the clearances between the tags of rocket and guides are equal to zero. Then

$$\left.\begin{aligned}
&m\frac{d^2x_n}{dt^2}=P-X_1+X_{r1}-F_{f1}-F_{f2}-mg\sin\theta_0;\\
&m\frac{d^2y_n}{dt^2}=Y_1+Y_{r1}+F_{N1}+F_{N2}-mg\cos\theta_0=0;\\
&J_{z_1}\frac{d\omega_{z_1}}{dt}=M_{z_1}+M_{rz_1}+F_{N1}l_1-F_{N2}l_2-F_{f1}\frac{d}{2}-F_{f2}\frac{d}{2}=0.
\end{aligned}\right\}\quad(12.25)$$

Equality zero second and third equations of system (12.25) is caused by the introduced assumptions. In this form these equations make it possible to determine power load on the tags of rocket.

During the motion of rocket along the open guides of the value of axial aerodynamic and gas-dynamic forces, are low in comparison with thrust. So are small frictional forces. Therefore during the calculation of the undisturbed motion of rocket along launching racks, it is possible these forces to disregard. The second and third equations of system (12.25) are actually the equations of relation, but not by equations of motion.

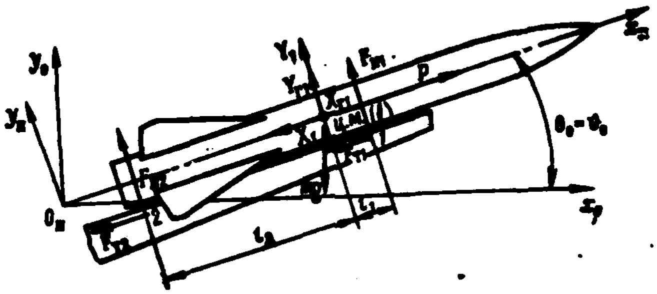

Fig. 12.5. Circuit of the forces, which act on the rocket during its motion along guides.

Key: (1). c.m.

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Hence it follows that instead of the system of equations (12.25) it is possible to be restricted to one equation of the axial motion of rocket along guides

$$\frac{d^2x_n}{dt^2}=\frac{P}{m}-g\sin\theta_0. \qquad (12.26)$$

Integrating this equation from t=0 to the time of the descent of rocket t_2, we will obtain the parameters of the undisturbed motion of rocket along guides. For integration it is necessary to know the thrust and mass of rocket in function from time. The time of the descent of rocket t_2 is determined in the process of integration, when the instantaneous value of coordinate x_n becomes equal to the

length of guides $l_{н}$. At this same torque/moment determine the velocity of the motions of the rocket which will be the initial velocity for the powered flight of the rocket

$$v_0=\dot{x}_{н(t=t_1)}. \qquad (12.27)$$

Frequently the numerical integration of equation (12.26) is replaced by the analytical solution which is instituted on the introduction of any supplementary assumptions. Let us examine one of most widely used analytical solutions [18].

It is known that during motion along guides the rocket expend/consumes the small mass of fuel/propellant, in this case, the time of motion along guides is sufficiently small. Consequently, the mass of rocket can be taken by constant, equal to its average value on the section of the motion of rocket along guides ($m_{ср}$). A change in the thrust in time is shown on Fig. to 12.6, from which it is evident that during the motion of rocket along directing force of thrust on initial section increases to value P_0, and then it remains constant. For analytical solution let us replace the curve P(t) on the initial section of straight line:

$$0\leqslant t\leqslant t_p, \quad P(t)=P_0\frac{t}{t_p}. \qquad (12.28)$$

Here t_p - time of the activation of the motor.

On the second section the value of thrust will be constant

$$t_p\leqslant t\leqslant t_1, \quad P(t)=P_0=\text{const}. \qquad (12.29)$$

Let us examine the motion of rocket on the first section of its displacement over guides. Equation of action taking into account (12.28) takes the following form:

$$\frac{d^2x_1}{dt^2}=\frac{dv}{dt}=\frac{[illegible]}{[illegible]}t-g\sin\theta_0, \qquad (12.30)$$

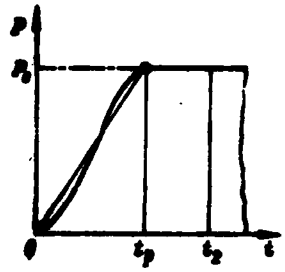

Fig. 12.6. Curve/graph of a change in the thrust during the motion of rocket along guides.

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Let us integrate twice equation (12.30), remembering that with $t=0$ $\dot{x}_n=v=0$, $x_n=0$. Then we are have

$$v=\dot{x}_n=\frac{P_0}{2m_{cp}t_p}t^2-g\sin\theta_0 t; \qquad (12.31)$$

$$x_n=\frac{P_0}{6m_{cp}t_p}t^3-\frac{1}{2}g\sin\theta_0 t^2. \qquad (12.32)$$

Let us substitute into equations (12.31) and (12.32) value $t=t_p$; then we will obtain dependences for determining of path and velocity at the end of the initial section of the motion of the rocket

$$v_p=\left(\frac{P_0}{2m_{cp}}-g\sin\theta_0\right)t_p; \qquad (12.33)$$

$$x_{n.p}=\left(\frac{P_0}{6m_{cp}}-\frac{1}{2}g\sin\theta_0\right)t_p^2. \qquad (12.34)$$

These values will be initial data for the calculation of the second section of the motion of rocket along guides. The equation of motion of the rocket in this case, accordingly (12.39), takes the following form:

$$\frac{d^2x_2}{dt^2}=\frac{dv}{dt}=\frac{P_0}{m_{cp}}-g\sin\theta_0. \qquad (12.35)$$

Let us integrate twice equation (12.35)

$$v=\dot{x}_{\text{н}}=v_{\text{р}}+\left(\frac{P_0}{m_{\text{ср}}}-g\sin\theta_0\right)(t-t_{\text{р}}); \qquad (12.36)$$

$$x_{\text{н}}=x_{\text{н.р}}+v_{\text{р}}(t-t_{\text{р}})+\frac{1}{2}\left(\frac{P_0}{m_{\text{ср}}}-g\sin\theta_0\right)(t-t_{\text{р}})^2. \qquad (12.37)$$

For the torque/moment of the descent of rocket from guides $t=t_2$, $\dot{x}_{\text{н}}=v_0$ and $x_{\text{н}}=l_{\text{н}}$. Consequently, it is possible to write

$$v_0=v_{\text{р}}+\left(\frac{P_0}{m_{\text{ср}}}-g\sin\theta_0\right)(t_2-t_{\text{р}}); \qquad (12.38)$$

$$l_{\text{н}}=x_{\text{н.р}}+v_{\text{р}}(t_2-t_{\text{р}})+\frac{1}{2}\left(\frac{P_0}{m_{\text{ср}}}-g\sin\theta_0\right)(t_2-t_{\text{р}})^2. \qquad (12.39)$$

Let us re-group equation (12.3) relative to the time of the motion of rocket along guides to descent. As a result of simple conversions, we will obtain

$$t_2=\frac{1}{\frac{P_0}{m_{\text{ср}}}-g\sin\theta_0}\times$$
$$\times\left[\sqrt{v_{\text{р}}^2+2(l_{\text{н}}-x_{\text{н.р}})\left(\frac{P_0}{m_{\text{ср}}}-g\sin\theta_0\right)}-v_{\text{р}}\right]+t_{\text{р}}. \qquad (12.40)$$

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Obtained equations (12.33), (12.34), (12.38) and (12.40) make it possible to expect velocity of rocket $v_{\text{р}}$, path $x_{\text{н.р}}$, the time of motion along the guides t_2, and also velocity of the rocket with descent from the guides v_0.

Let us examine now the motion of rocket along guides with the

nonsimultaneous descent of the tags of rocket. In this case it is necessary to distinguish two sections of motion.

First section - motion of rocket to the torque/moment of the descent of the first tag from guides. The motion of rocket on this section in no way differs from the motion of rocket in the first case. Therefore the equations, obtained above, are completely suitable for the calculation of the first section. The time of the motion of rocket along by guides to the descent of the first tag let us designate t_1.

The second section (sometimes it they call the section of output) motion it begins after the first and terminates at the moment of the descent of the second tag of rocket from guides. This section is characterized by the fact that the rocket obtains supplementary degrees of freedom, namely - to swivel feature around rear tag.

Basic forces and the torque/moments, which act on rocket on the second section of its motion along guides (for conditional rotation counterclockwise) are shown on Fig. to 12.7. Let us comprise the equations of motion of the rocket relative to the axes of the system of coordinates $O_n x_n y_n$, in this case let us consider guides fixed rigid body. Let us accept also, that $P=P_0$. Then

$$\left.\begin{aligned} m\ddot{x}_n &= P_0\cos\Delta\vartheta - X_1\cos\Delta\vartheta - Y_1\sin\Delta\vartheta - \\ &\quad - F_{\text{тр}} - mg\sin\theta_0; \\ m\ddot{y}_n &= P_0\sin\Delta\vartheta - X_1\sin\Delta\vartheta + Y_1\cos\Delta\vartheta + \\ &\quad + F_{N_2} - mg\cos\theta_0; \\ J_{z_1}\frac{d\omega_{z_1}}{dt} &= M_{0_T} - F_{N_2}l_2\cos\Delta\vartheta - F_{\text{тр}}\frac{d}{2}\cos\Delta\vartheta + \\ &\quad + F_{N_2}\frac{d}{2}\sin\Delta\vartheta - F_{\text{тр}}l_2\sin\Delta\vartheta. \end{aligned}\right\} \quad (12.41)$$

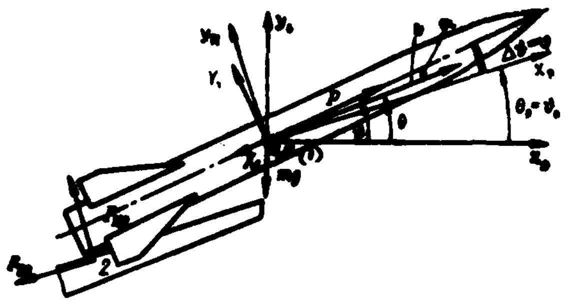

Fig. 12.7. Circuit of the forces, which act on rocket on the section of descent from guides.

Key: (1). c.m.

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To these equations it is necessary to add the equation of relation, caused by the displacement of the center of mass of the rocket with its turning on the second tag

$$y_n = l_2\Delta\vartheta. \qquad (12.42)$$

The studies of the motion of rocket on the section of output showed that for always of action the value of angle $\Delta\vartheta$ does not exceed ten angular minutes. Therefore it is possible for trigonometric functions to accept the following values:

$$\cos\Delta\vartheta = 1; \quad \sin\Delta\vartheta = \Delta\vartheta.$$

It is simplified the third equation of system (12.41) on the

basis of the fact that the angle $\Delta\vartheta$ is low; for rockets usually $l_2 \approx 10d$ and force $F_{T2}=fF_{N2}$, where the coefficient of the friction of steel in steel $f \approx 0.2$. We will obtain

$$J_{z_1}\frac{d\omega_{z_1}}{dt}=M_{z_1}-F_{N_2}\left[l_2-\frac{d}{2}\Delta\vartheta+f\frac{d}{2}+fl_2\Delta\vartheta\right]\approx M_{z_1}-F_{N2}l_2, \tag{12.43}$$

since three last/latter terms in bracket for the values of f indicated, d and $\Delta\vartheta$ in comparison with l_2 prove to be values low and them can be disregarded. The velocity of the motion of rocket along guides does not usually exceed 70 m/s. In this case aerodynamic forces and torque/moments are obtained by sufficiently low, and them in equations (12.41) can be disregarded.

Taking into account entire aforesaid above the system of equations (12.41) is led to the following simplified form:

$$\left.\begin{aligned} m\ddot{x}_n &= P_e - mg\sin\theta_0; \\ m\ddot{y}_n &= P_e\varphi - mg\cos\theta_0 + F_{N2}; \\ J_{z_1}\ddot{\varphi} &= -l_2F_{N2} \end{aligned}\right\} \tag{12.44}$$

Here for convenience in the recording, is marked $\Delta\vartheta = \varphi$.

In that obtained of systems, the first equation connected with the second and third equations. Consequently, it it is possible to integrate separately. This equation coincides in form with (12.26). Integration of equation gives the velocity of the motion of the center of mass of rocket along the guides in the function of time. Physical sense of the possibility of integration separately of the

first equation of system (12.44) consists in the fact that the lateral divergences barely affect the velocity of motion.

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Us interests the rotation of the rocket during its motion on the section of output. Therefore let us examine the second and third equations of system (12.44). Let us introduce replacement $y_1 = l_1\varphi$, let us re-group the second equation of system relatively F_{N1} let us substitute the obtained value into the third equation. Taking into account the equation of relation (12.42) we will obtain

$$(J_{z_1} + ml_1^2)\ddot{\varphi} = Pl_1\varphi - mgl_1\cos\theta_0. \qquad (12.45)$$

Let us introduce the designations

$$J'_{z_1} = J_{z_1} + ml_1^2;\ \frac{Pl_1}{J'_{z_1}} = a_1.$$

Let us substitute them into equation (12.45) and after conversion we will obtain

$$\ddot{\varphi} - a_1\varphi = -\frac{mgl_1\cos\theta_0}{J'_{z_1}}. \qquad (12.46)$$

During the integration of equation (12.46) the mass of the rocket can be taken by average value on the section of conclusion. In this case the right side of equation (12.46) is a constant value, and equation itself - by linear heterogeneous with constant coefficients. This equation it is possible to integrate in analytical form. Without giving intermediate unpacking/facings, let us write the obtained results of the integration

$$\Delta\vartheta_0=\varphi_0=-\frac{m_{op}g\cos\vartheta_0}{2P_0}\left[e^{\sqrt{a_1}(t_2-t_1)}+e^{-\sqrt{a_1}(t_2-t_1)}-2\right]; \quad (12.47)$$

$$\dot{\vartheta}_0=\dot{\varphi}_0=-\frac{m_{op}g\cos\vartheta_0}{2}\sqrt{\frac{l_2}{P_0 J_{z_1}}}\left[e^{\sqrt{a_1}(t_2-t_1)}-e^{-\sqrt{a_1}(t_2-t_1)}\right], \quad (12.48)$$

where t_2 - time of the descent of the seccnc tag from launching rack;

$\Delta\vartheta_0$ - angle of rotation of the axis of rocket at the end of the section of output;

$\dot{\vartheta}_0$ - angular velocity of the axis of rocket at the end of the section of output.

The constructions of the starting/launching moved settings up of ground-based rockets are different. They can have guides of framework construction in the form of the shaped beams of tubular type, etc.

Launchers for the rockets, started from aircraft, usually have either tubular guide or sinply light/lung farm/trusses for the suspension of rockets.

Let us examine the basic special feature/peculiarities of the action of rockets at the moment of the start when there is interaction between the rocket and the starter.

§3. Motion of the rockets during launching/starting from launching

pad and from shaft/mine.

The isolation/evolution of rocket from launching pad occurs as instantly at that torque/moment when thrust begins to become more than the weight of rocket.

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For the calculation of further trajectory of the motion of rocket, it is necessary to calculate the parameters of the rocket with its breakaway from starting table. In essence this is related to the determination of the initial mass of rocket m_0, since the part of the fuel/propellant will be spent to the engine operation from firing point to breakaway torque of rocket of launching pad (i.e. for time t_{CT}).

The mass of rocket we define, as before on the formula:

$$m_0 = m_{CT} - \int_0^{t_{CT}} |\dot{m}|(t)\,dt,$$

where m_{CT} - a mass of rocket prior to the beginning of ignition;

$|\dot{m}|$ (t) - the consumption of mass, which in the process of the launching of rocket is the value of variable depending on time.

Thus, for the calculation of the powered flight trajectory of

ballistic missile as the initial parameters we have

$$v_0=x_0=y_0=0;\ \vartheta_0=\vartheta_{np0}=90°.$$

Let us examine now the motion of the rocket on the launching phase during starting/launching from shaft/mine.

Figures 12.8 shows the flat/plane schematic of the action basic forces and torque/moments during the motion of rocket from shaft/mine. From the figure one can see that besides the thrust P and of the weight of rocket Mg on rocket act even gas-dynamic forces X_{r1}, Y_{r1} and torque/moment M_{r1}. Gas-dynamic forces and torque/moments appear because of the limited volume of shaft/mine. The gases, which are formed with the combustion of fuel/propellant, escape/ensue from engine nozzle and fall into the limited space. In spite of gas-bleeding channels in this space is formed the zone of elevated pressure. Part of the gas bursts open between the walls of rocket and shaft/mine, affecting side walls of rocket. Space after the bottom of rocket because of the motion of rocket always changes. In connection with this they change and the parameters of gas, which is located beyond the bottom of rocket, therefore, continuously change also gas-dynamic forces and the torque/moments, which act on rocket. The effect of gas-dynamic forces and torque/moments on rocket ceases only after its output from shaft/mine.

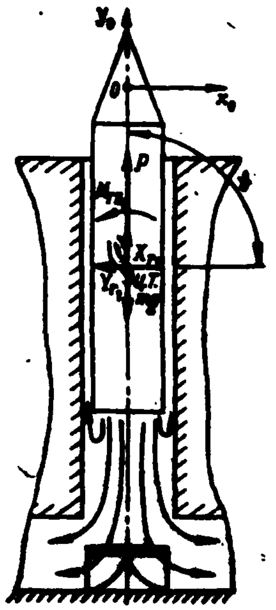

Fig. 12.8. Circuit of the action of forces on the rocket with start from shaft/mine.

Key: (1). center of gravity.

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The calculation of gas-dynamic forces and torque/moments is extremely hinder/hampered due to the unsteady character of a change of the parameters of gas in shaft/mine. Aerodynamic forces and torque/moments in this case are virtually absent, since the velocity of the rocket during motion in shaft/mine and output from shaft/mine

does not exceed several ten meters per second. The perceptible effect of aerodynamic forces and torque/moments on the motion of rocket appears only at the velocities more than 50-70 m/s. Therefore for the calculation of motion rocket in the period of output from shaft/mine aerodynamic forces and torque/moments it is possible not to consider.

According to the schematic of the action of forces and torque/moments, let us comprise the equations of motion of rocket in the process of its output from shaft/mine. The equations of motion of the center of mass let us write in projections on the axis of the starting coordinate system. During the axisymmetric construction of rocket, the problem can be solved in flat/plane setting and system of equations will take the following form:

$$\left.\begin{aligned}
&m\ddot{y}=(P+X_{r1})\sin\vartheta+Y_{r1}\cos\vartheta-mg;\\
&m\ddot{x}=(P+X_{r1})\cos\vartheta-Y_{r1}\sin\vartheta;\\
&J_{z_1}\dot{\omega}_{z_1}=M_{r\,z_1}+M_{p\,z_1};\\
&\dot{\vartheta}=\omega_{z_1};\ m=m_{ст}-\int_0^t|\dot{m}|(t)\,dt;\\
&\vartheta_{пр}=90^\circ;\ v=\sqrt{\dot{x}^2+\dot{y}^2}.
\end{aligned}\right\}\qquad(12.49)$$

For the calculation of the parameters of the motion of rocket, the system of equations (12.49) is integrated to that torque/moment until rocket leaves the shaft/mine and to it will cease to act gas flow. As a result of integration, we obtain the initial parameters of motion for a powered flight trajectory: $v_0\neq0$; $x_0=y_0=0$; ϑ_0, and also

value m_0; in this case, one should consider the fact that during motion in shaft because of gas-dynamic forces and torque/moments the rocket will be deflect/diverted along the pitch angles, yaw and bank.

§4. Supplementary factors, which determine the initial conditions of shot.

During the motion of rocket along guides, the gas flow, coming out from nozzle units, washes the parts of launcher. In this case, is exhibited the supplementary action of gases on launcher and rocket. Various kinds deflectors and reflectors can be the reason for the appearance of secondary gas flows, which frequently act on rocket unsymmetrically.

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As a result of the elastic properties of rocket, of launcher and soil, the initial conditions of moving the rocket differ from the calculated. The theoretical calculation of gas-dynamic forces and torque/moments, which act on the rocket, driving/moving along launching racks, is frequently difficult. It is difficult to also theoretically describe the vibrations of launcher and rocket during their interaction during start. The diversity of initial conditions with shot is one of the reasons, which determine scattering the

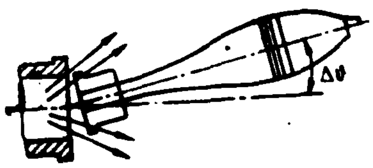
unguided rockets. The experimental study of the dynamics of start makes it possible to determine the initial conditions of shot and to introduce corrections into calculation data.

With shot from artillery instrument as a result of the elastic properties of shaft, gun carriage and deformation of the soil of the firing position, the initial conditions of shot also differ somewhat from the conditions, establish/installed with aiming. On leaving of projectile from bore the unsymmetric action of gas flows on the outgoing projectile. Of smooth-bore systems the unsymmetric action of gas flow on tail assembly with initial angle $\Delta\delta$ can lead to the considerable scattering of angular initial conditions (Fig. 12.9).

As a result of the vibrations of weapon and deformation of the soil of the firing position, the angle of elevation of instrument ϕ_0, establish/installed with aiming, does not coincide with real angle of departure - θ_0.

An angular difference $\gamma = \theta_0 - \phi$ is called angle of jump which can be both the positive and negative. The numerical value of angle of jump for this instrument depends on many factors: the elastic properties of shaft and gun carriage, soil of the firing position, ballistics of instrument, which determines power loads, the angle of elevation, rate of fire, etc. The theoretical determination of angle

of jump causes great difficulties; therefore as a rule, angle of jump it is determined experimentally [59]. at a distance of x from the muzzle end face of barrel adjustable panel.



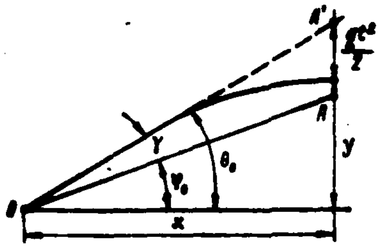


Fig. 12.10.

Fig. 12.9. Change of the initial angle of departure in the period due to gases.

Fig. 12.10. Schematic of determination of angle of jump with firing from panel.

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Instrument is aimed on the cross lines, adjustable on muzzle and breech faces, into the mark, plotted/applied on panel so that the shaft before the shot would have (on quadrant) an angle of elevation φ_0 (Fig. 12.10).

As a result of the named reasons the projectile will fly not in the direction OA, but at the initial moment it will be deflected from OA by angle of jump γ. With small x it is possible not to consider the effect of air resistance on a decrease in the projectile the hearth of the lines of shot and to determine decrease according to

formula $gt^2/2$. Then, on the basis of Fig. 12.10, we obtain

$$\gamma = \operatorname{arctg}\left[\frac{1}{x}\left(y+\frac{gt^2}{2}\right)\right]-\varphi_0. \qquad (12.50)$$

Time t can be determined either experimentally or approximately on the known initial velocity of projectile v_0.

$$t \approx \frac{x}{v_0 \cos \theta_0}.$$

At the zero angle of the increase

$$\gamma = \operatorname{arctg}\left(\frac{y}{x}+\frac{gx}{2v_0^2}\right). \qquad (12.51)$$

The determination of the mean statistical value of angle of jump requires repeated firings. For determining the angle of jump of rocket systems, are conducted so-called "rocking testings" during which it is simulated the motion of rocket along guides also on the initial trajectory phase.

In conclusion of chapter, let us note that the study of the dynamics of the start of concrete/specific/actual rocket system, just as the initial conditions of shot from artillery piece, it represents by itself complex composite problem.

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Chapter XIII.

Errors for firing, missile dispersion and of projectiles.

The errors of firing it is possible to divide into rough miscalculations and errors, constant or systematic errors and errors random. Rough miscalculations are the consequence of poor training/preparation of which operates system calculation and can be removed by the increase of skill and combat training.

The source of systematic errors acts on all shots equally. For example, longitudinal constant tailwind increases firing distance against calculated, and contrary decreases. To systematic errors can be attributed also the errors in the determination of the coordinates of target, etc. If we by the methods of the theory of corrections consider in calculations also during the preparation of firing the effect of the corresponding factor, systematic errors can be, if are not excluded entirely, then to a considerable degree are decreased. It is necessary to reveal/detect the reasons for systematic errors and to consider them during the preparation of firing. With the firing the rockets, prepared according to some drawings and technical

specifications, from the guides of one and the same launcher, from one launching site, during one and the same settings up of the sight mechanisms of missile trajectory they will not coincide with each other. With the firing identical projectiles from barrel artillery piece during identical charges and the adjustments of sight of the trajectory of separate projectiles, also they do not coincide.

The noted phenomenon calls scattering trajectories. It is explained by the effect on rocket flight and projectiles of the random errors, which are the consequence of the combination of random changes in the separate values.

Entire/all rocket (or projectile) in collection, its separate structural/design units and parts are made with the appropriate tolerances in size, the weight and other parameters. Change of different values within the limits of allowances and diversity in the effect of weather factors which it is difficult to consider as systematic error, and is the source of the random errors, which lead to scattering of trajectories and impact points in the rockets (projectiles).

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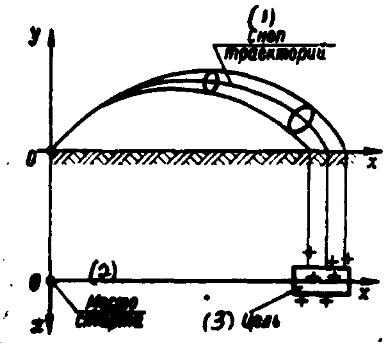
The group of the trajectories, which correspond to identical

initial data, is called the beam (or sheaf) of trajectories (Fig. 13.1).

With firing at ground targets, as shown in Fig. 13.1, the deviation of impact point from target/purpose is determined by coordinates x and z.

The characteristics of scattering are determined with firing at horizontal or vertical plane, and scattering itself is called scattering on plane.

With firing at the flying target/purposes with the so-called remote projectiles with the proximity fuses or surface-to-air missiles, the places of the explosions of warheads are arrange/located in certain space around the calculation predicted collision point, i.e., occurs volumetric scattering (Fig. 13.2). During volumetric scattering the position of the points of discontinuity is determined by coordinates x, y, z. In this case the coordinate plane, on which are determined coordinates y and z, is conducted through the predicted collision point of projectile for target/purpose, it is perpendicular to the velocity vector of the center of mass of target/purpose in the predicted calculation collision point. Coordinate x is determined in the sense of the vector of the velocity of the center of mass of target/purpose.



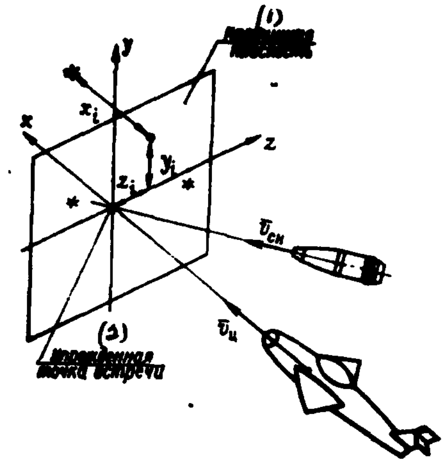


Fig. 13.2.

Fig. 13.1. Sheaf of fire.

Key: (1). Sheaf of fire. (2). Place of start. (3). Target.

Fig. 13.2. Three-dimensional/space scattering with firing remote projectiles.

Key: (1). Plane of figure. (2). Predicted collision point.

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The random errors, which determine the scatter of points of an incidence/drop in the projectiles or plane or volumetric scattering with time fire at air or underwater targets, appear without the determined order, the concrete/specific/actual reasons for their

appearance are known sometimes only qualitatively, and sometimes also are entirely unknown. To judge the accuracy of firing at the data of any shot would be incorrectly, since same this result was accidental.

For evaluating the accuracy of firing, are utilized the characteristics, which reflect the properties of random variables and random functions and laws, by which they are subordinated. The mathematical basis of the determination of scattering trajectories and of the points of the operation of warheads (on plane and in space, on coordinates and on time) are the probability theory, the theory of errors and mathematical statistics.

The flight of projectile, examine/considered taking into account the random disturbances, initial and which act in the process of action, can be considered as stochastic process, described by the random function, concrete/specific/actual realization of which is this trajectory. The study of the random character of missile trajectories and of projectiles is based on the special section of the probability theory - theory of random functions or, otherwise, the theory of random or stochastic processs. The serious obstructions of theoretical and calculated order are encountered during the stochastic investigations of the trajectories of the guided missiles and of projectiles, whose number of factors, which affect the deviation of motion characteristics from the calculated (ideal), is especially great.

In practice during the study of the scatter of points of the rendezvous of the rocket (projectile) with flat/plane barrier/obstacle or the volumetric scatter of points of the operation of remote projectiles in space for simplification in the solution of problem utilizes the sections of the probability theory, which examine random variables.

The region of the stochastic studies of scattering trajectories and of the isolated points of the operation of the warheads of the rockets, projectiles, min and aircraft bombs is extremely vast. Many questions require the detailed specialized investigations. In connection with firing stochastic problems can be divided into two large groups.

The first group of problems is characteristic for stages of the design of means armament, their final adjustment and testing. One of the basic problems of the first group is study of the action of the perturbation factors and calculated determining of the expected characteristics of scattering, accuracy of firing and, in the final analysis, the effectiveness of the action of the design/projected means armament.

The second group covers the problems, connected directly with the use of the already available means armament. Are this involved, for example, the development of rules and methods of use combat of technology, the determination of the consumption of the resources armament for accomplishing of one or the other tactical mission.

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These questions usually are examined in independent courses "Theory of firing", the "Theory of bombing", etc.

Both named groups of stochastic investigations are tightly interconnected, at their basis lie/rest the mathematical methods of the theory of probablity and the generalized statistical experiment in the form of the characteristics of scattering and accuracy of the work of articles as a whole (rockets, projectiles, etc.) or the separate structural/design units, close in designation/purpose and construction (autopilots, gyroscopes, acceleration pickups, etc.). The combination of mathematical investigations with statistical processing of experimental information was called experimental-theoretical methods of the evaluation of the characteristics of the missile dispersion and projectiles. Experimental-theoretical methods, used during design, are divided on the so-called analytical methods and the methods of statistical

testings. The realization of the latter practical is possible only during use EVM [computer]; these methods in ballistic practice were called the conditional the methods of mathematical or electronic firing.

§1. Analytical methods of the evaluation of the characteristics of scattering.

Determining the characteristics of scattering for the newly design/projected systems of rocket or artillery armament only possible experimental-theoretical method. Let us examine one of the varieties of this method - analytical method.

For using this method, it is necessary to know well construction of the design/projected system, technology of its manufacture and physical nature of the perturbation random factors. Only in this case the effect of each of the random factors in question can be determined by calculation. Let us examine for an example determining the characteristics of scattering for any point in the trajectory, for example, for the point, which corresponds to the end/lead of the operation of the engine of rocket or for the point of intersection of trajectory with the plane of target/purpose. Trajectory elements at the points indicated will be random variables. For example, the distance x_c of the firing projectile from barrel artillery piece can

be represented as function of random variables v_0, θ_0, m_0, c and of so forth.

Let us examine random variable A, which can be presented as function of several random arguments $\xi_1, \xi_2, \ldots, \xi_n$

$$A=f(\xi_1, \xi_2, \ldots \xi_n). \quad (13.1)$$

It is decomposed (13.1) in Taylor series in the vicinity of the value of function A, which corresponds to the mathematical expectations of arguments $m_{\xi_1}, m_{\xi_2}, \ldots m_{\xi_n}$, after preserving in expansion the members not higher than second order.

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Then stochastic dependence (13.1), adhering to the designations of formula (8.9), can be presented in this form:

$$A=f(m_{\xi_1}, m_{\xi_2}, \ldots m_{\xi_n})+\sum_{i=1}^{n}\left(\frac{\partial f}{\partial \xi_i}\right)_m \mathring{\xi}_i+$$
$$+\frac{1}{2}\sum_{i=1}^{n}\left(\frac{\partial^2 f}{\partial \xi_i^2}\right)_m (\mathring{\xi}_i)^2+\sum_{i<j}\left(\frac{\partial^2 f}{\partial \xi_i \partial \xi_j}\right)\mathring{\xi}_i\mathring{\xi}_j, \quad (13.2)$$

where $\mathring{\xi}_i$ and $\mathring{\xi}_j$ - in the case in question the centered random variables:

$$\mathring{\xi}_i=\xi_i-m_{\xi_i} \quad \mathring{\xi}_j=\xi_j-m_{\xi_j}$$

Partial derivatives in (13.2) during the calculation of the missile dispersion and projectiles correspond to the ballistic

derivatives the procedures of determination of which were examined in Chapter XI. Index m shows that during the calculation of the values of derivatives the arguments ξ_i must be undertaken equal to their mathematical expectations m_{ξ_i}.

Applying the general methods of determining the numerical characteristics of the distribution of the function of random arguments, let us write [10]

$$m_A = f(m_{\xi_1}, m_{\xi_2}, \ldots, m_{\xi_n}) + \frac{1}{2}\sum_{i=1}^{n}\left(\frac{\partial^2 f}{\partial \xi_i^2}\right)_m D_{\xi_i} + \sum_{i<j}\left(\frac{\partial^2 f}{\partial \xi_i \partial \xi_j}\right)_m K_{ij}, \quad (12.3)$$

where $D_{\xi_i} = \sum_{i}^{n} p_i (\xi_i - m_{\xi_i})^2$ - dispersion of random argument ξ_i;

$K_{ij} = \sum_i \sum_j (\xi_i - m_{\xi_i})(\xi_j - m_{\xi_j}) p_{ij}$ - covariance of the in pairs undertaken arguments ξ_i and ξ_j.

In the simpler case when $\xi_1, \xi_2, \ldots, \xi_n$ are not correlated, the preceding/previous formula will take the form

$$m_A = f(m_{\xi_1}, m_{\xi_2}, \ldots, m_{\xi_n}) + \frac{1}{2}\sum_{i=1}^{n}\left(\frac{\partial^2 f}{\partial \xi_i^2}\right)_m D_{\xi_i}.$$

For function many alternating/variable A the formula, which makes it possible to determine its dispersion D_A, is obtained complex and its practical application/use encounters the great difficulties of theoretical and calculation orders.

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Formula considerably is simplified, if one assumes that arguments $\xi_1, \xi_2 \ldots \xi_n$ are functionally independent and not correlated. Then

$$D_A = \sum_{i=1}^{n}\left(\frac{\partial f}{\partial \xi_i}\right)_m^2 D_{\xi_i} + \frac{1}{4}\sum_{i=1}^{n}\left(\frac{\partial^2 f}{\partial \xi_i^2}\right)_m^2 (\mu_4[\xi_i] - D_{\xi_i}^2) +$$

$$+ \sum_{i<j}\left(\frac{\partial^2 f}{\partial \xi_i \partial \xi_j}\right)_m^2 D_{\xi_i} D_{\xi_j} + \sum_{i=1}^{n}\left(\frac{\partial f}{\partial \xi_i}\right)_m \left(\frac{\partial^2 f}{\partial \xi_i^2}\right)_m \mu_3[\xi_i], \quad (13.5)$$

where $\mu_3[\xi_i] = \sum_{i=1}^{n} p_i(\xi_i - m_{\xi_i})^3$ and $\mu_4[\xi_i] = \sum_{i=1}^{n} p_i(\xi_i - m_{\xi_i})^4$ - the respectively third and fourth central moments; $\sum_{i<j}$ - sign, which means that is conducted the summation of all possible pairwise combinations of random variables $\xi_i \xi_j$.

For the normal law of the distribution

$$\mu_3[\xi_i] = 0; \quad \mu_4[\xi_i] = 3\sigma_{\xi}^4 = 3D_{\xi}^2$$

and then formula (13.5) is converted as follows:

$$D_A = \sum_{i=1}^{n}\left(\frac{\partial f}{\partial \xi_i}\right)_m^2 D_{\xi_i} + \frac{1}{2}\sum_{i=1}^{n}\left(\frac{\partial^2 f}{\partial \xi_i^2}\right)_m^2 D_{\xi_i}^2 + \sum_{i<j}\left(\frac{\partial^2 f}{\partial \xi_i \partial \xi_j}\right)_m^2 D_{\xi_i} D_{\xi_j}. \quad (13.6)$$

As is evident, formulas (13.3) and (13.5) are very complex; simplified formulas (13.4) and (13.6) during their practical use also will lead to bulky and complex calculations. Therefore during determining of the characteristics of scattering at the stage of

ballistic designed calculations, they are limited usually in the formula of expansion (8.9) only by linear terms of a series. Then random function (13.1) can be represented in this form:

$$A = f(m_{x_1}, m_{x_2}, \ldots, m_{x_n}) + \sum_{i=1}^{n} \left(\frac{\partial f}{\partial x_i}\right)_m \mathring{x}_i. \qquad (13.7)$$

Applying the general methods of determining the numerical characteristics of distribution for linear functions, we will obtain for mathematical expectation and the dispersion of random function A the following expressions:

$$m_A = f(m_{x_1}, m_{x_2}, \ldots, m_{x_n});$$

$$D_A = \sum_{i=1}^{n} \left(\frac{\partial f}{\partial x_i}\right)_m^2 D_{x_i} + 2\sum_{i<j} \left(\frac{\partial f}{\partial x_i}\right)_m \left(\frac{\partial f}{\partial x_j}\right)_m K_{ij}. \qquad (13.8)$$

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Transfer/ccnverting to root-mean-square deviaticn, we will obtain

$$\sigma_A^2=\sum_{i=1}^{n}\left(\frac{\partial f}{\partial \xi_i}\right)_m^2\sigma_{\xi_i}^2+2\sum_{i<j}\left(\frac{\partial f}{\partial \xi_i}\right)\left(\frac{\partial f}{\partial \xi_j}\right)_m r_{ij}\sigma_{\xi_i}\sigma_{\xi_j}, \quad (13.9)$$

where r_{ij} - a coefficient cf ccrrelaticn of value ξ_i and ξ_j

$$r_{ij}=\frac{K_{ij}}{\sigma_{\xi_i}\sigma_{\xi_j}}.$$

Calculations according to fcrmula (13.6) can be carried out, if are known the particular derived $\left(\frac{\partial f}{\partial \xi_i}\right)_m$ and numerical characteristics of distribution for the arguments of the stochastic system:

mathematical expectaticns $m_{\xi_1}, m_{\xi_2}, \ldots m_{\xi_n}$ and the correlation matrix/die

$$\|K_{ij}\|=\begin{Vmatrix} k_{11} & k_{12}\ldots k_{1n} \\ & k_{22}\ldots k_{2n} \\ \cdots & \cdots\cdots \\ & k_{nn} \end{Vmatrix}.$$

recall that the cell/elements of the correlation matrix/die, arrange/located along principal diagonal, they represent by itself to the dispersion of arguments. For calculations according to formula (13.9) it is necessary to have, besides σ_{t_i} and σ_{t_j}, the standardized/normalized correlation matrix/die, which consists of the coefficients of correlation r_{ij}.

If random arguments are not correlated, then all cell/elements of correlation matrix/die, except diagonal, are equal to zero ($k_{ij}=0$ with i≠j) and then from formulas (13.8) and (13.9) let us have

$$D_A=\sum_{i=1}^{n}\left(\frac{\partial f}{\partial t_i}\right)_m^2 D_{t_i}, \qquad (13.10)$$

or

$$\sigma_A^2=\sum_{i=1}^{n}\left(\frac{\partial f}{\partial t_i}\right)_m^2 \sigma_{t_i}^2. \qquad (13.11)$$

It is necessary to keep in mind that in the examined method of determining the numerical characteristics of scattering the functions of random variables are not establish/installed the laws of their distribution. For the determination of the laws of the distribution of the functionsof random arguments - the density of distribution and of distribution function - are required special

experimental-theoretical studies.

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As indicate many theoretical studies and statistical testings, total scattering of artillery shells it is subordinated to the normal law of distribution; scattering the determining parameters (v_0, θ_0 and c) is also subordinated to the normal law of distribution. This means that the normal law of distribution possesses the property of stability according to which it is possible to expect that the function, obtained as a result of the combinations of the random arguments, which are subordinated to the normal law of distribution, will be distributed also according to normal law.

Let us assume we have the function of n-independent random variables, distributed according to the normal law

$$A=f(\xi_1, \xi_2, \ldots \xi_n).$$

It is obvious, the mathematical expectation of value A will be equal

$$m_A=\sum_{i=1}^{n} A_i p_i. \qquad (13.12)$$

In conformity with the property of stability, let us have for A the normal law of distribution with the root-mean-square deviation,

determined with respect to formula 13.11,

$$f(A)=\frac{1}{\sigma_A\sqrt{2\pi}}e^{-\frac{(A-m_A)^2}{2\sigma_A^2}}.$$

For the normal law of distribution, the root-mean-square deviation is connected with the middle error E through the constant factor

$$E=0{,}6745\,\sigma \qquad (13.13)$$

and then

$$E_A^2=\sum_{i=1}^{n}\left(\frac{\partial f}{\partial \xi_i}\right)_m^2 E_{\xi_i}^2 \qquad (13.14)$$

If we designate mean deviation, which characterizes that comprise of the complete scattering of function A because of scattering of random factor ξ_i, through

$$E_{A_i}=\left(\frac{\partial f}{\partial \xi_i}\right)E_{\xi_i}, \qquad (13.15)$$

that the middle error, which characterizes the complete scattering of value A, will be equal to

$$E_A=\sqrt{E_{A_1}^2+E_{A_2}^2+\ldots+E_{A_n}^2}. \qquad (13.16)$$

Mean deviation of the determining parameters frequently designate through r_ξ.

For example, for the barrel system

r_{v_0} - mean deviation v_0;

r_{θ_0} - mean deviation θ_0;

r_e - mean deviation e.

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If we introduce r_i, that

$$E_A = \sqrt{\sum_{i=1}^{n} \left(\frac{\partial A}{\partial x_i} r_i \right)^2}. \qquad (13.17)$$

For our example

$$E_{x_C} = \sqrt{\left(\frac{\partial x_C}{\partial v_0} r_{v_0} \right)^2 + \left(\frac{\partial x_C}{\partial \theta_0} r_{\theta_0} \right)^2 + \left(\frac{\partial x_C}{\partial e} r_e \right)^2}. \qquad (13.18)$$

The property of stability possess not all laws of distribution. For example, combination of the values, which obey the law of uniform density, gives new law [10]. Special complexities are encountered during the determination of the laws of the distribution of the

functions, which are the consequence of the combinations of the arguments, which have different laws of distribution. This fact decreases the accuracy of the examined method during its application/use for determining scattering the trajectories of new original controlled rockets and projectiles.

Let us examine performance calculation of scattering the unguided rockets by analytical method.

The contemporary unguided rockets of class "surface-surface" with engine, usually by worker on solid fuel, are utilized for a firing to the comparatively short distances (it is not more than 50-100 km). The trajectory height of such rockets, as a rule, does not exceed 30-50 km, i.e., entire/all missile trajectory passes in the sufficiently dense layers of the atmosphere, and air resistance significantly affects rocket flight entire trajectory. By the rockets of this class they conduct firing mainly at the target/purposes, arrange/located on the surface of ground or sea; therefore the greatest interest represents the scatter of points of incidence/drop.

The weak interconnection of longitudinal and yawing motions makes it possible to separately examine longitudinal scattering (on distance) in the direction of x-axis of the starting coordinate system and lateral - in the direction of z axis.

The characteristics of scattering complex trajectories for each of the sections are calculated separately, just as the correction (see Chapter XI §6). The characteristics of scattering trajectory elements at the end of the preceding/previous section serve as initial characteristics for the subsequent section. Let us examine determining the characteristics of scattering separately for active and inactive legs. For the calculation of longitudinal scattering on active section, we utilize a system of equations (3.79)

$$\dot{v}=\frac{P-X}{m}-g\sin\theta;\quad \dot{\theta}=-\frac{g\cos\theta}{v};\quad \dot{y}=v\sin\theta;\quad \dot{x}=v\cos\theta.$$

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The integration of this system makes it possible to obtain the motion characteristics of rocket as material point. Let us write the right sides of equations (3.79) in functional form.

$$\left.\begin{array}{l}\dot{v}=f_1(P,\ X,\ m,\ v,\ \theta);\quad \dot{\theta}=f_2(v,\ \theta);\\ \dot{y}=f_3(v,\ \theta);\quad \dot{x}=f_4(v,\ \theta).\end{array}\right\}\qquad (13.19)$$

Scattering the cell/elements of trajectory at the end of the active section will be determined by the probable deviations of the arguments of system (13.19) and by scattering the initial conditions for which it is possible to accept the random vector of the velocity

of the center of mass at the moment of the descent of rocket from guides, that is changed in module/modulus and direction. Scattering the coordinates of initial point in the trajectory with fixed guides can be disregarded.

Thrust P and drag X in turn, depend on a number of factors; therefore expedient to open their values, for which we will use previously obtained dependence (11.18).

$$v = f_1(v, \theta, y, J_1, S, Q, c_x(M), m_0, |\dot{m}|).$$

tank the unit/single momentum/impulse/pulse J_1 depends mainly on fuel heating value and in smaller measure on the design features of engine. Contemporary solid fuels represent by themselves, as a rule, the mixture of different components. The composition of mixture ideally accurately for all rockets cannot be maintained; therefore unit/single momentum/impulse/pulse for each rocket will differ somewhat from the nominal. To the value of unit/single momentum/impulse/pulse, has effect the initial temperature of fuel/propellant. This fact also leads to the spread of unit/single momentum/impulse/pulse, since to attain the identical temperature field of charge in all rockets is virtually impossible.

Instead of the consumption of mass during setting of the scatter of trajectory elements at the end of the active section, they prefer

to examine the operating time of engine t_k with sufficient accuracy communication/connection between them is expressed by the dependence

$$t_k = \frac{m_T}{\dot{m}},$$

where m_T - mass of fuel/propellant.

The operating time of engine t_k also differs from the nominal, since they occur the scatter of combustion chamber pressure and the dispersion of the nozzle throat area, that lead to the scatter of fuel consumption per second.

The spread of unit/single momentum/impulse/pulse will affect mainly scattering of the velocity of rocket at the end of the operation of engine v_k, while time jitter of burning - scattering of coordinates x_k, y_k and the angle of the slope of velocity vector θ_k.

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Thrust represents by itself the vector, determined, besides module/modulus, even by direction. The law of a change in the sense of the vector of thrust also does not remain identical for the rockets of one party/batch. They occur eccentricity of thrust, i.e.,

the displacement of the thrust line relative to the center of mass of rocket, and the misalignment of the thrust line when it is not parallel to axis of rocket. This leads to the fact that the axis of rocket differs from tangent to trajectory, rocket moves with the angles of attack and slip. As a result of initial disturbances and the disturbance/perturbations, which act in trajectory, even in the absence of eccentricity and misalignment of thrust the angles of attack and slip are not equal to zero. The presence of these angles determines the composing the thrusts in transverse direction, deforming trajectory. Trajectory on active section heaves or is omitted, it is displaced to the right or to the left.

System of equations (3.79) is comprised for normal meteorological conditions. With the wish to consider the atmospheric disturbances it is necessary functional dependence (13.19) to add as arguments pressure, the temperature of air and the wind (see Chapter XI). The more complete account of aerodynamic forces will require passage from system (3.79) to another basic system of equations, which includes necessary these comprise of aerodynamic drag. Let us designate the atmospheric perturbation factors through $\xi_{a\tau}$.

Initial conditions let us consider angle θ_0, by angular velocity $\dot{\theta}_0$ and by initial velocity v_0.

For simplicity of writing, let us designate trajectory elements at the end of the active section $x_к, y_к, v_к$ and $\theta_к$ through $A_{кi}$, then, taking into account (13.19), (11.18) and the considerations presented, we can write stochastic dependence for $A_{кi}$

$$A_{кi}=f_i(J_1, t_к, e_т, \delta_т, S, c_x(M), m_0, \theta_0, \dot{\theta}_0, v_0, \xi_{ат}). \quad (13.20)$$

In first part one four argument determines scattering thrust; arguments S and $c_x(M)$ determine scattering of drag; argument m_0 determines scattering initial mass (or weight Q_0); arguments θ_0, $\dot{\theta}_0$ and v_0 determine scattering initial conditions and value $\xi_{ат}$ - total effect of the atmospheric disturbances.

The root-mean-square deviation of each of the random arguments let us designate respectively $\sigma_{J_1}, \sigma_{t_к}, \sigma_{e_т}, \sigma_{\delta_т}, \sigma_S, \sigma_{c_x}, \sigma_{Q_0}, \sigma_{\theta_0}, \sigma_{\dot{\theta}_0}, \sigma_{v_0}, \sigma_{\xi_{ат}}$.

Let all the random arguments be not correlated and are subordinated to the normal law of distribution; then the total scattering of random function $A_{кi}$ is also subordinated to the normal law of distribution and is determined by formula (13.11).

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For simplicity let us write separately average quadratic deviation

$\sigma_{A_{xl}}$, determined by thrust, air resistance, by weight, initial conditions and the atmospheric disturbances. From scattering of thrust vector, we will obtain

$$(\sigma_{A_{x\,l}})_T = \sqrt{\left(\frac{\partial A_{x\,l}}{\partial J_1}\sigma_{J_1}\right)^2+\left(\frac{\partial A_{x\,l}}{\partial t_K}\sigma_{t_K}\right)^2+\left(\frac{\partial A_{x\,l}}{\partial e_T}\sigma_{e_T}\right)^2+\left(\frac{\partial A_{x\,l}}{\partial \delta_T}\sigma_{\delta_T}\right)^2}. \quad (13.21)$$

The root-mean-square deviation, determined by scattering drag, is equal

$$(\sigma_{A_{x\,l}})_R = \sqrt{\left(\frac{\partial A_{x\,l}}{\partial S}\sigma_S\right)^2+\left(\frac{\partial A_{x\,l}}{\partial c_x}\sigma_{c_x}\right)^2}. \quad (13.22)$$

For the calculation of scattering, it is possible to take constant for entire powered flight trajectory value σ_{c_x}, expressed in percentages.

Root-mean-square deviation from scattering of the initial mass

$$(\sigma_{A_{x\,l}})_m = (\sigma_{A_x})_Q = \left(\frac{\partial A_{x\,l}}{\partial Q_0}\right)\sigma_{Q_0}. \quad (13.23)$$

From scattering of the initial conditions

$$(\sigma_{A_{x\,l}})_0 = \sqrt{\left(\frac{\partial A_{x\,l}}{\partial v_0}\sigma_{v_0}\right)^2+\left(\frac{\partial A_{x\,l}}{\partial \theta_0}\sigma_{\theta_0}\right)^2+\left(\frac{\partial A_{x\,l}}{\partial \dot{\theta}_0}\sigma_{\dot{\theta}_0}\right)^2}. \quad (13.24)$$

The root-mean-square deviaticn, determined by the atmospheric disturbances, is equal

$$(\sigma_{A_{k\,i}})^2_{aT} = \sum_{i=1}^{n} \left(\frac{\partial A_{k\,i}}{\partial \xi_i}\right)^2 \sigma^2_{\xi_i}. \qquad (13.25)$$

In the analytical methcd cf calculaticr of scattering in questicn tc directly consider the chance cf the process of changing the weather factors with height/altitude and time is not represented possible. It is expedient preliminarily frcm a change in the weather factors as random functicns to pass tc scattering of ballistic wind, the ballistic temperature deflecticn and barcmetric pressure, after describing by their correspcnding rcct-mear-square deviation frcm standard conditions, determined by standard atmcsphere.

Grand average standard deviaticn, which characterizes scattering the cell/elements of moticr at the end cf the engine operation, will be determined according tc the formula

$$\sigma_{A_{k\,i}} = \sqrt{(\sigma_{A_{k\,i}})^2_{\xi} + (\sigma_{A_{k\,i}})^2_{\text{[illegible]}} + (\sigma_{A_{k\,i}})^2_{\text{[illegible]}} + (\sigma_{A_{k\,i}})^2_{b} + (\sigma_{A_{k\,i}})^2_{aT}}. \qquad (13.26)$$

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Scattering trajectcries on passive secticn is determined: by scattering initial for a passive secticn mcticr characteristics, by

scattering aerodynamic drag, by scattering weight and by scattering meteorological factors. From aerodynamic characteristics let us as before consider scattering only drag. The characteristics of scattering the values of trajectory elements at the point of the end/lead of the engine operation, determined on formula (13.26), let us designate respectively: $\sigma_{v_к}, \sigma_{\theta_к}, \sigma_{y_к}, \sigma_{x_к}$. If we consider that the scatter of points of an incidence/drop in the rocket was determined only by scattering initial conditions, then we will obtain the so-called latent scattering for which

$$\sigma_{x_C'}=\sqrt{\left(\frac{\partial x_C}{\partial v_к}\sigma_{v_к}\right)^2+\left(\frac{\partial x_C}{\partial \theta_к}\sigma_{\theta_к}\right)^2+\left(\frac{\partial x_C}{\partial y_к}\sigma_{y_к}\right)^2+\left(\frac{\partial x_C}{\partial x_к}\sigma_{x_к}\right)^2}. \quad (13.27)$$

The effect of scattering the drag coefficient and weight of rocket on passive section can be registration/accounting through the appropriate root-mean-square deviation of drag coefficient $\sigma_{c_{x}}$ and the root-mean-square deviation of the weight of rocket $\sigma_{Q_к}$. However, is more convenient on passive section to introduce to examination ballistic coefficient $c_п=\frac{id^2}{Q_к}10^3$ and to determine the immediately root-mean-square deviation of ballistic coefficient - σ_c.

The root-mean-square deviation, which characterizes the atmospheric disturbances on passive section, let us determine according to the formula, similar (13.25), after introducing the

index of "p"

$$(\sigma_{x_C})^2_{\text{II}} = \sum_{i=1}^{n} \left(\frac{\partial x_C}{\partial \lambda_i}\right)^2_{\text{II}} \sigma^2_{\lambda_i} \quad (13.28)$$

The determination of root-mean-square deviation must be conducted in accordance with the considerations, presented in connection with formula (13.25). On trajectories which pass in the relatively dense layers of the atmosphere, from weather factors the most essential effect exerts scattering wind velocity. Taking into account basic factors for the unguided rockets, average quadratic range error will be determined according to the formula

$$\sigma_{x_C} = \sqrt{\left(\frac{\partial x_C}{\partial v_K}\sigma_{v_K}\right)^2 + \left(\frac{\partial x_C}{\partial \theta_K}\sigma_{\theta_K}\right)^2 + \left(\frac{\partial x_C}{\partial y_K}\sigma_{y_K}\right)^2 + \left(\frac{\partial x_C}{\partial x_K}\sigma_{x_K}\right)^2 + \left(\frac{\partial x_C}{\partial c}\sigma_c\right)^2 + \left(\frac{\partial x_C}{\partial w_x}\sigma_{w_x}\right)^2} \quad (13.29)$$

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Scattering in side direction on inactive leg is determined in essence: by scattering angle Ψ_K, characterizing the sense of the vector of the speed of relatively plane of reference of casting; by scattering angular velocity $\dot{\Psi}_K$, by scattering coordinate z_K and by scattering the velocity of lateral ballistic wind. The total lateral scattering of trajectories is characterized by value

$$\sigma_{z_C}=\sqrt{\left(\frac{\partial z_C}{\partial W_x}\sigma_{W_x}\right)^2+\left(\frac{\partial z_C}{\partial \psi_x}\sigma_{\psi_x}\right)^2+\left(\frac{\partial z_C}{\partial z_x}\sigma_{z_x}\right)^2+\left(\frac{\partial z_C}{\partial w_z}\sigma_{w_z}\right)^2}. \quad (13.30)$$

In the more detailed study of problem, besides the named factors, it is necessary to still consider eccentricity of masses and aerodynamic eccentricity, caused by the technological inaccuracies, for example, by the misalignment of stabilizer fins. As a result of eccentricity and misalignment of the axis of thrust of aerodynamic eccentricity and eccentricity of masses, are also the disturbance/perturbations of the rocket with descent from guides, appears scattering the angles of attack and slip. The more detailed account of all affecting factors (including technological) will considerably complicate solution. Upon consideration only of basic perturbation factors, the examined method makes it possible to rate/estimate the characteristics of scattering the design/projected rocket and to determine the advisability of further development of the assumed construction. It is necessary to keep in mind that when the problem is solved in the first approximation, it is difficult, and sometimes also virtually it is not possible to consider the correlation communication/connections between the separate perturbation factors, it is difficult to previously establish/install the laws of the distribution of random arguments and the random function of these arguments. Frequently without sufficient

substantiation it is necessary to select the normal law of distribution.

Special difficulties are encountered during the calculated determination of scattering the guided missiles of class "surface-surface". Despite the fact that similar rockets are equipped by the control systems, to completely remove their scattering is not represented possible. Steering functions of type (9.9) and (9.10) are comprised with the specific assumptions. In the majority of the cases, are considered only first terms of expansion in a series, since the account of quadratic terms of expansion leads to the considerable complication of the control system. The noncoincidence of the right and left sides of equalities (9.7) or (9.10) at the cutoff of engine will always give the error, determined in general form by formula (9.11).

The cessation of the operation of engine (resetting to zero of thrust) is realized not instantly [38].

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The momentum/impulse/pulse of the aftereffect of thrust, after the delivery of the main command to its cutoff, will have certain spread, which will give scattering velocity v_{K}. If the rocket is controllable

only on active section, then on the descending branch of trajectory they will occur of the disturbance/perturbations, determined by the passage of the rocket through the dense layers of the atmosphere. The enumerated errors, caused by the method of control accepted, are called systematic.

Besides systematic errors, to the flight of the guided missiles have effect the instrument errors, which are inherent in steering devices. The gyrostabilized platforms, on which are establish/installed accelerometers, have attendance/departures, in consequence of which the axis of the sensitivity of accelerometers in the course of time differ from given one on the start of the direction of measurement, yes even direction itself, measurements on start is assigned with error. But this leads to the fact that the projections of the speed of rocket and its coordinate are determined with errors. The formation of steering function Φ and its comparison with computed value Φ_{K} for the end/lead of powered flight trajectory are conducted also with errors.

tajt average quadratic range errors for the class of rockets indicated can be designed by analytical method or the method of statistical testings, examined subsequently paragraph. If is applied analytical method, then calculation formula takes the form

$$\sigma_L = \sqrt{\sigma^2_{L\phi} + \sigma^2_{L\,\text{инстр}} + \left(\frac{\partial L}{\partial J_{\text{пх}}}\sigma_{J_{\text{пх}}}\right)^2 + \sum\left(\frac{\partial L}{\partial \xi_i}\sigma_{\xi_i}\right)^2_{\text{вх}}}, \quad (13.31)$$

where $\sigma_{L\phi}$ - rms range error as a result of the assumptions, accepted during introduction to steering function;

$\sigma_{L\,\text{инстр}}$ - rms range error as a result of the presence of instrument errors;

$\sigma_{J_{\text{пх}}}$ - the root-mean-square deviation of the momentum/impulse/pulse of aftereffect.

The third term (13.31) expresses effect on the dispersion in distance of the spread of the momentum/impulse/pulse of the aftereffect of thrust, and last/latter term characterizes the effect on σ_L of the disturbance/perturbations, which act on the section of the atmospheric entry.

Scattering long range ballistic missiles in side direction first of all will be determined by scattering trajectory elements at the end of the active section (i.e. of the cutoff point of the thrust). The lateral scatter of points of incidence/drop as a result of this reason in linear approach/approximation is determined by the formula

$$\sigma_{z_C}=\sqrt{\left(\frac{\partial z_C}{\partial v_{kx}}\sigma_{v_{kx}}\right)^2+\left(\frac{\partial z_C}{\partial v_{ky}}\sigma_{v_{ky}}\right)^2+\left(\frac{\partial z_C}{\partial v_{kz}}\sigma_{v_{kz}}\right)^2+\left(\frac{\partial z_C}{\partial x_k}\sigma_{x_k}\right)^2+\left(\frac{\partial z_C}{\partial y_k}\sigma_{y_k}\right)^2+\left(\frac{\partial z_C}{\partial z_k}\sigma_{z_k}\right)^2+\left(\frac{\partial z_C}{\partial t_k}\sigma_{t_k}\right)^2}. \quad (18.22)$$

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Most essential effect cn value σ_{z_C} have terms $\frac{\partial z_C}{\partial v_{k_z}}\sigma_{v_{kz}}$ and $\frac{\partial z_C}{\partial z_k}\sigma_{z_k}$. Thrust cutoff is realized in functicr value, governing flying range, because of this tc utilize a selection of the cutcff pcint cf thrust for obtaining the minimum cf lateral deviation is not represented possible. Therefore for decreasing the scattering in side direction, it falls on all powered flight trajectory to attempt via centrcl to decrease values σ_{v_z} and σ_z.

There is an error, which the autcncmous system control not at all can remove. This errcr is sighting errcr alcng azimuth. Acceptable accuracy in side directicr in the autonomcus control system it is possible tc achieve cnly in the case of small sighting error. Sighting error along azimuth can be even if is not removed entirely, then is substantially decreased with tracking rocket flight cn ground tracking staticns and on ccnducting the correction of trajectory.

Together with the deviations of trajectory elements at the end of the active section of scattering in side direction definite effect have the disturbance/perturbations, which act on the rocket upon its entry into the dense layers of the atmosphere.

The scattering, caused by the examined above factors (i.e. depending only on rocket itself), they frequently call technical.

Besides technical scattering occurs another the scattering, connected with the errors for the preparation of firing. To the errors for the preparation of firing are related the errors in determination of geographical reference of the launching point and target/purpose, error in the determination of the atmospheric parameters, error in the determination of the temperature of fuel/propellant, etc. The errors for training/preparation for this group of launching/startings are more less are identical, but they noticeably are distinguished for different groups. The errors for training/preparation lead to scattering of the centers of the grouping of projectiles and rockets in different firings. If the laws of technical scattering and errors for training/preparation are identical, then the characteristics of complete scattering will be equal to

$$\sigma_{x_{C\Pi}}=\sqrt{\sigma^2_{x_{CT}}+\sigma^2_{x_{\text{под}}}};\quad \sigma_{z_{C\Pi}}=\sqrt{\sigma^2_{z_{CT}}+\sigma^2_{z_{\text{под}}}}. \qquad (13.33)$$

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Respectively under the normal laws of the distribution of technical scattering and errors for training/preparation, it is possible to write for mean deviation:

$$B_{\text{д}}=\sqrt{B_{\text{д.т}}^2+B_{\text{д.под}}^2}; \quad (13.34)$$

$$B_{\text{б}}=\sqrt{B_{\text{б.т}}^2+B_{\text{б.под}}^2}. \quad (13.35)$$

where $B_{\text{д.т}}$ and $B_{\text{б.т}}$ - middle, or probable, range errors and in side direction, that characterize technical scattering;

$B_{\text{д.под}}$ and $B_{\text{б.под}}$ - middle, or probable, the deviations, which characterize the errors for training/preparation.

For setting of kill probability to target/purpose, the ammunition consumption and time of fire for effect to target/purpose, the account of the errors for training/preparation is necessary.

§2. Determination of the characteristics of scattering by the method of statistical testings.

The method of statistical testings - one of the most universal

methods of determining the probabilistic characteristics of the results of the large number of solutions of the systems of the differential equations, which describe physical process, in which the initial data are assigned as random variables. Method found application/use in many areas of science and technology, including in ballistics [8], [18].

In the latter case is performed the calculation of a large quantity of trajectories on computers. Those of the parameters the effect of scattering the which is assumed to consider that they are represented in the form of random numbers. The functions the effect of scattering the which is assumed to consider, for example, a change in the weather factors with height/altitude $\tau(y)$, $\varrho(y)$, $\overline{W}(y)$, they are represented in the form of random functions. Before conducting of statistical testings (calculations) is conducted the careful study of all parameters and functions, which affect the motion of rockets and projectiles, are establish/installed the laws of their distribution and numerical characteristics of these laws. Then is comprised the most complete, for given specific conditions, system of equations, which includes the random parameters and the functions whose effect is assumed to consider. Utilizing the obtained values of the random parameters and functions, is performed calculation ("experiment") on computers.

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The random parameters during the j testing are taken in this form:

$$\left.\begin{aligned}
\xi_{1j} &= M[\xi_1] + N_{1j}\sigma_1;\\
\xi_{2j} &= M[\xi_2] + N_{2j}\sigma_2;\\
&\cdots\cdots\cdots\\
\xi_{ij} &= M[\xi_i] + N_{ij}\sigma_i\\
&\cdots\cdots\cdots\\
\xi_{mj} &= M[\xi_m] + N_{mj}\sigma_m.
\end{aligned}\right\} \quad (12\text{-[illegible]})$$

Here ξ_{ij} - value of the i parameter during the j testing (calculation);

$M[\xi_i]$ - mathematical expectation of the i parameter or its nominal value;

N_{ij} - the random number, which characterizes the value of the i parameter during the j testing; for example, during the first testing - N_{i1}, on the second - N_{i2} and so forth.

The random functions, introduced into ballistic calculation, are assigned either in the form of experimental specific realizations obtained during testings, or in the form of the realizations, obtained during the canonical expansion of the random functions,

which describe a change in each of the characteristics.

As is known, the canonical expansion of random function is represented in the form

$$X(t)=m_x(t)+\sum_{i=1}^{m} V_i\varphi_i(t), \qquad (13.37)$$

where $m_x(t)$ - mathematical expectation of random function; $\varphi_1(t), \varphi_2(t), \ldots, \varphi_m(t)$ - coordinate functions; $V_1, V_2, \ldots, V_m$ - uncorrelated random variables with the mathematical expectations, equal to zero.

In ballistic calculations are introduced the random functions, which reflect a change of the weather factors depending on height/altitude.

During the use of canonical expansions, it is possible to write for deviation of temperature from the normal law

$$\delta T(y)=\sum_{i=1}^{m} V_{\tau i}\varphi_{\tau i}(y), \qquad (13.38)$$

where V_τ and φ_τ - random variable and coordinate function, that characterize deviation of temperature.

Random function for wind, for reasons, presented in Chapter XI,

it is expedient to present in the form of two functions. In direction north-south:

$$w_{C-\text{Ю}}(y)=\bar{w}_{C-\text{Ю}}(y)+\sum_{i=1}^{m}V_{w_{C-\text{Ю}}i}\varphi_{w_{C-\text{Ю}}i}(y). \qquad (13.39)$$

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In direction east-west:

$$w_{B-3}(y)=\bar{w}_{B-3}(y)+\sum_{i=1}^{m}V_{w_{B-3}i}\varphi_{w_{B-3}i}(y). \qquad (13.40)$$

Here $\bar{w}_{C-\text{Ю}}(y)$ and $\bar{w}_{B-3}(y)$ - change in the average values of the projections of wind velocity in height/altitude; $V_{w_{C-\text{Ю}}i}, V_{w_{B-3}i}$ - random variables; $\varphi_{w_{C-\text{Ю}}i}(y), \varphi_{w_{B-3}i}(y)$ - coordinate functions, which determine random comprising velocity of wind according to height/altitude in the appropriate direction.

In each concrete/specific/actual trajectory calculation, it is utilized on one of the realizations of random functions $\delta T(y)$, $w_{C-\text{Ю}}(y)$ and $w_{B-3}(y)$. Random variables and coordinate functions are determined as a result of the very laborious probabilistic analysis of large quantity of the experimental data, obtained with the meteorological sounding of the atmosphere. With ballistic rockets the number of component in right sides expansions of random function (13.38)-(13.40) is determined by the available experimental data,

their reliability and a quantity. In the majority of the cases, prove to be possible to take 10-15 terms of expansion.

Let us examine the exemplary/approximate order of the use of expressions (13.38)-(13.40) during performance calculation of scattering of rockets. The characteristics of random variables V_i and coordinate functions $\varphi_i(y)$ we consider known. Let us assume that available there are on n of coordinate functions for determination δT, $w_{C-Ю}$ and $w_{З-В}$ and from n of the tables of random coefficients. After taking according to one random number V_i of each table, after multiplying each of them by its (coinciding in number) coordinate function it summed up the obtained terms, is found concrete/specific/actual curve (realization), which characterizes the law of a change in the deviation of temperature with height/altitude at the first "launching/starting".

Then similarly is found the law of a change in wind velocity with height/altitude, characterized by functions $w_{C-Ю}(y)$ and $w_{З-В}(y)$. For the facilitation of calculations, they project wind velocity in the line of fire and the side direction, as a result of which is obtained $w_x(y)$ and $w_z(y)$. The obtained realizations of the deviations of the atmospheric parameters from the standard $\delta T(y)$, $w_x(y)$, $w_z(y)$ are utilized during the solution of the system of equations of the motion of rocket for the first "launching/starting".

In exactly the same way are calculated realizations $\delta T(y)$, $\bar{m}_r(y)$ and $w_r(y)$ for the 2nd, 3rd and so forth of "launching/startings".

Utilizing the obtained values of the random parameters and the random functions, are carried out trajectory calculations ("experiments") on ETsVM [ЭЦВМ- digital computer] or AVM. Analog computers give results with large errors; however, they are favorable when to describe mathematically phenomenon is not completely represented possible and it is necessary to join up of machine the real assemblies of article.

The results of calculation, obtained in the first "testing", are introduced into table.

Then thus is conducted the second "testing", the third and so up to the latter.

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When are carried out all "experiments", from table is extracted the which interests us value (for example, distance, flight altitude,

etc.), are establish/installed the law of its distribution and the numerical characteristics of law. Thus, as a result of "testings" is obtained the complete description of studied value from the probabilistic point.

The reliability of results during the application/use of a method of statistical testings depends to a considerable extent on the number of the "experiments conducted" and on the accuracy of the mathematical description of phenomenon; for obtaining the reliable results, are required usually hundred and thousand "experiments".

By basic advantage of the method of statistical testings was a sufficiently complete description rocket flight as of random process, determination of all characteristics of this process.

To deficiency/lacks in the method, one should relate the long time, required for conducting such "testings", and high consumption in comparison with analytical method.

By the described method are determined the characteristics of complete scattering. In order to determine the effect of scattering the separate parameter or function, it is necessary this parameter or the function to represent as random, and remaining determining parameters and functions to take on rating (or on mathematical

expectation). Similar investigations are connected with the calculation of a large quantity of trajectory and the corresponding expenditures of the machine count time on computers.

§3. Determining the characteristics of scattering according to the results of firing.

Direct/straight experiment (firing) they make it possible most to correctly estimate the characteristics of scattering. Firing is conducted in the final development stages of missile or artillery complex. With an increase in the cost/value of rockets (projectiles), it is logical, they descend the possibility of obtaining sufficient statistical material by conducting the firings. Therefore the results of firing always are estimated in combination with theoretical calculations. The basic difficulty of the solution of this problem with rocket firings consists in the limitedness of statistical material or, in other words, in the low number of rocket launchings, and also in the fact that is previously unknown the law of distribution.

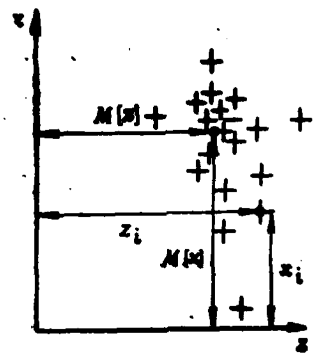

Fig. 13.3. Scattering the coordinates of the impact points in the projectiles in locality.

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In connection with this in the process of the calculation of numerical characteristics according to the results of experiment, it is necessary to speak not about precise of their value, but only about the average statistical values of the obtained quantities. The average statistical values of numerical characteristics usually are designated with line above. For example $\overline{M[x]} = \overline{M}_x$ - average statistical mathematical expectation of random variable; $\overline{D[x]} = \overline{D}_x$ - average statistical dispersion of random variable, etc.

Let us examine the basic stages of processing the results of

firing. Let us assume that with "n" rocket launchings we have "n" of the points of their incidence/drop on plane xoz (Fig. 13.3). The coordinates of impact points it is most expedient to determine in the starting coordinate system. They first of all extract consecutively the coordinates (x; z) of each impact point in second and third columns in Table 13.1.

Further are determined the values of average statistical mathematical expectations according to coordinates x and z

$$\bar{M}_x=\frac{\sum_{i=1}^{n}x_i}{n};\qquad (13.41)$$

$$\bar{M}_z=\frac{\sum_{i=1}^{n}z_i}{n}.\qquad (13.42)$$

Table 13.1.

(1) Номер опыта	x_i	z_i	$x_i-\overline{M}_x$	$z_i-\overline{M}_z$	$(x_i-\overline{M}_x)^2$	$(z_i-\overline{M}_z)^2$	$(x_i-\overline{M}_x)\times$ $\times(z_i-\overline{M}_z)$
1							
2							
.							
.							
.							
n							
$\sum_{i=1}^{n}$							

Key: (1). Number of experiment.

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Then are filled the remaining column of [redacted] table 13.1 for all experiments and are determined average statistical dispersions ($\overline{D}_x$; $\overline{D}_z$) and covariance ($\overline{K}_{xz}$) according to the following dependences:

$$\overline{D}_x=\frac{\sum_{i=1}^{n}(x_i-\overline{M}_x)^2}{n-1}; \tag{13.43}$$

$$\overline{D}_z=\frac{\sum_{i=1}^{n}(z_i-\overline{M}_z)^2}{n-1}; \tag{13.44}$$

$$\overline{K}_{xz}=\frac{\sum_{i=1}^{n}(x_i-\overline{M}_x)(z_i-\overline{M}_z)}{n-1}. \tag{13.45}$$

Further are calculated average statistical values of root-mean-square deviation $(\bar{\sigma}_x; \bar{\sigma}_z)$ and of the coefficient of correlation $(\bar{r}_{xz})$:

$$\bar{\sigma}_x = \sqrt{\bar{D}_x}; \quad (13.46)$$

$$\bar{\sigma}_z = \sqrt{\bar{D}_z}; \quad (13.47)$$

$$\bar{r}_{xz} = \frac{\bar{K}_{xz}}{\bar{\sigma}_x \bar{\sigma}_z}. \quad (13.48)$$

Passage from dispersions to root-mean-square deviation usually is made on that reason, that the dimensionality σ is equal to the dimensionality of random variable, while the dimensionality of dispersion it is equal to the dimensionality of the square of random variable. By correlation coefficient also to more usually conveniently use than covariance. The correlation coefficient is dimensionless and changes from 0 to 1. If $r_{xz}=0$, then this means that between random variables x and z there is no correlation. These values are independent from the point of the probability theory. Difference r_{xz} from zero is the sign/criterion of existence of the stochastic dependence between random variables x and z. But if $r_{xz}=1$, then random variables x and z are completely dependent on each other.

For evaluating the reliability of the obtained average statistical values of numerical characteristics, calculate for them the so-called confidence intervals. Let us examine as an example the determination of confidence interval for the average statistical mathematical expectation of random variable x.

Let as a result of processing data of firing we obtain that $\overline{M}_x$ and it is necessary to rate/estimate error $\overline{M}_x - M_x$, where M_x - precise value of the mathematical expectation of random variable x.

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Let us assign certain sufficiently large probability β (for example, β=0.9), such so that the event with probability β it would be possible to consider virtually reliable, and let us find such value ε, for which

$$p(|\overline{M}_x - M_x| < \varepsilon) = \beta. \qquad (13.49)$$

Then the range of the virtually possible values of the errors, which appear during replacement M_x on $\overline{M}_x$, will be $\pm\varepsilon$. Large in absolute value errors will appear with the low probability

$$\alpha = 1 - \beta. \qquad (13.50)$$

Let us rewrite equality (13.49) in this form:

$$p(\bar{M}_x-\varepsilon<M_x<\bar{M}_x+\varepsilon)=\beta. \qquad (13.51)$$

Obtained equation (13.51) means that with probability β the unknown precise value M_x falls into following interval (Fig. 13.4)

$$J_\beta=(\bar{M}_x-\varepsilon;\ \bar{M}_x+\varepsilon). \qquad (13.52)$$

Value J_β is called of confidence interval, and β - confidence coefficient. In our case β - the probability of the fact that the confidence interval J_β will cover value M_x.

Let us determine confidence intervals for the average statistical mathematical expectations of the random variables of coordinates x and z. For this, let us assign value β, from which let us find value t_β [10]. The relative boundaries of confidence intervals let us determine according to the formulas

$$(\varepsilon)m_x=t_\beta\sqrt{\frac{\bar{D}_x}{n}}, \qquad (13.53)$$

$$(\varepsilon)m_z=t_\beta\sqrt{\frac{\bar{D}_z}{n}}. \qquad (13.54)$$

Consequently, confidence intervals for average statistical mathematical expectations will be equal to

$$(J_\beta)_{m_x} = [\bar{m}_x - (\varepsilon)_{m_x};\ \bar{m}_x + (\varepsilon)_{m_x}]; \quad (13.55)$$

$$(J_\beta)_{m_z} = [\bar{m}_z - (\varepsilon)_{m_z};\ \bar{m}_z + (\varepsilon)_{m_z}]. \quad (13.56)$$

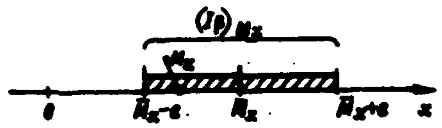

Fig. 13.4. Confidence interval J_β on number scale axis.

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The relative boundaries of confidence intervals for the average statistical dispersions of random variables $\bar{x}$ and $\bar{z}$ can be determined by following formulas [10]:

$$(\varepsilon)_{D_x} = t_\beta \sqrt{\frac{2}{n-1}}\,\bar{D}_x; \quad (13.57)$$

$$(\varepsilon)_{D_z} = t_\beta \sqrt{\frac{2}{n-1}}\,\bar{D}_z. \quad (13.58)$$

In this case, the confidence intervals of average statistical dispersions will be equal to

$$(J_\beta)_{D_x}=\left[\bar{D}_x-(\varepsilon)_{D_x};\ \bar{D}_x+(\varepsilon)_{D_x}\right]; \qquad (13.59)$$

$$(J_\beta)_{D_z}=\left[\bar{D}_z-(\varepsilon)_{D_z};\ \bar{D}_z+(\varepsilon)_{D_z}\right]. \qquad (13.60)$$

If we designate

$$\sigma_{1x}=\sqrt{\bar{D}_x-(\varepsilon)_{D_x}};\quad \sigma_{2x}=\sqrt{\bar{D}_x+(\varepsilon)_{D_x}},$$

then it is possible to obtain confidence intervals for average statistical standard deviations

$$(J_\beta)_{\sigma_x}=(\sigma_{1x};\ \sigma_{2x}); \qquad (13.61)$$

$$(J_\beta)_{\sigma_z}=(\sigma_{1z};\ \sigma_{2z}). \qquad (13.62)$$

The following development stage of firings is the construction of the histogram in form of which is introduced the hypothesis about the possible character of the law of distribution. For the construction of histogram, are carried out following process/operations. They divide entire range of the obtained values x and z for the discharges (intervals), limited by values $x_i; x_{i+1}$ and $z_i; z_{i+1}$ (for the i interval), and compute a quantity of values of random variable m_i that being necessary for each i-th discharge. For convenience in the construction of histogram, take not true values x_i and z_i and their deviations from the corresponding

mathematical expectations $\dot{m}_x$ and $\dot{m}_z$. Usually numerical length is equal to six, eight and it is thinner - ten. Generally a quantity of discharges k depends on the number of the experiments conducted. The greater the experiments (n), the greater numerical length can be taken. The length of discharge it is convenient to take equal to root-mean-square deviation. The selected discharges and a quantity of values of coordinate (m_i), that being necessary for each discharge, will bring in into the first two lines of table (13.2), which is made for random variable x (for an example it is undertaken of six discharges, on three discharges to each side from zero). Similar table it is necessary to make, also, for random variable z.

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On findings are constructed the histograms both for the random variable x and for random variable z, plot/depositing along the axes of abscissas discharges, while along the axes of ordinates - the corresponding to them quantities of values (m_i) of random variables.

Analyzing the obtained histograms, is introduced hypothesis on the probable character of the law of the distribution of given random variable. For example, on the histogram, presented in Fig. 13.5, it is possible to introduce hypothesis about the normal law of random number distribution x.

The introduced hypothesis or the law of distribution of given random variable must be checked on goodness of fit. Most frequently for this is applied Pearson's so-called criterion χ^2, who makes it possible to rate/estimate the degree of the coordination introduced of theoretical and statistical laws of distribution.

Value χ^2 is determined according to the dependence

$$\chi^2=\sum_{i=1}^{k}\frac{(m_i-np_i)^2}{np_i}, \qquad (13.60)$$

where p_i - the hit probability into the i-th discharge, designed according to the introduced theoretical law of distribution.

Tables 13.2.

1	(1) Разряды	$-\overset{\circ}{x}_3; -\overset{\circ}{x}_2$	$-\overset{\circ}{x}_2; -\overset{\circ}{x}_1$	$-\overset{\circ}{x}_1; 0$	$0; \overset{\circ}{x}_1$	$\overset{\circ}{x}_1; \overset{\circ}{x}_2$	$\overset{\circ}{x}_2; \overset{\circ}{x}_3$
2	m_i						
3	$\hat{\Phi}\left\lvert\frac{\overset{\circ}{x}_{i+1}}{\varrho\sqrt{2}\sigma_x}\right\rvert$						
4	$\hat{\Phi}\left\lvert\frac{\overset{\circ}{x}_i}{\varrho\sqrt{2}\sigma_x}\right\rvert$						
5	$p_i=\frac{1}{2}\lvert 3-4\rvert$						
6	np_i						
7	m_i-np_i						
8	$(m_i-np_i)^2$						
9	8:6						

Key: (1). Discharges.

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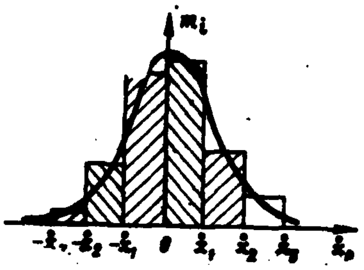

Fig. 13.5. Histogram of processing test results.

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For example, for the normal law of hit probability distribution of random variable x into this discharge is computed according to the formula

$$p_i=\frac{1}{2}\left[\hat{\Phi}\left|\frac{\dot{x}_{i+1}}{\varrho\sqrt{2}\sigma_x}\right|-\hat{\Phi}\left|\frac{\dot{x}_i}{\varrho\sqrt{2}\sigma_x}\right|\right], \quad (13.64)$$

where $\dot{x}_i, \dot{x}_{i+1}$ - boundary values of the i discharge; $\varrho\sqrt{2}=0,6745$;

$\sigma_x=\sigma_x$ - root mean square value of given random variable, found earlier from experimental data; $\hat{\Phi}\left|\frac{\dot{x}_i}{\varrho\sqrt{2}\sigma_x}\right|$ - value of the given function of Laplace.

Calculation of χ^2 for the normal law of distribution is convenient to conduct with the aid of Table 13.2.

Further is defined the number of degrees of freedom r as numerical length k minus the number of conditions (communication/connections) S, superimposed for the theoretical law of the distribution

$$r=k-S. \quad (13.65)$$

accepted.

In our case the number of superimposed conditions

(communication/connections) S is equal to three:

1) it is necessary that the sum of frequencies would be equal to one;

2) is necessary the coincidence of the theoretical and average statistical of mathematical expectations;

3) is necessary the coincidence of the theoretical and average statistical of dispersions.

Further according to those found χ^2 and r determine probability p of the fact that the law of distribution accepted does not contradict experimental data [10]. If probability p is low, then hypothesis about the introduced law of distribution is reject/thrown as unlikely. But if the obtained probability (p) is relatively great, then hypothesis can be recognized by the not contradictory experimental data.

Is how low must be probability p, in order to reject/throw a hypothesis concerning the introduced law of distribution, a question not defined. It cannot be solved from mathematical considerations. In practice, if $p<0.1$, then it is necessary either experiment to repeat or to attempt to find the more adequate/approaching law of distribution.

It should be noted that with the aid of criterion χ^2 (or any other goodness of fit) at the high values of p it is possible only to establish that the hypothesis accepted about the law of distribution does not contradict experimental data.

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The experimental characteristics of scattering are compared with calculated, together they are estimated. If necessary are introduced the corrections into the calculated law of distribution and characteristic of scattering.

The scatter of points of an incidence/drop in the artillery shells and unguided rockets they are subordinated to the normal law of distribution; in this case, the impact points are arrange/located on plane within the limits of the ellipse, called the ellipse of scattering. The center of ellipse coincides with center of dispersion (or by the center of grouping). With the firing the artillery rotating shells as a result of right derivation, the center of the ellipse of scattering is displaced to the right from the direction of plane of reference of firing. With firing at locality, the axis of ellipse in the direction of firing is equal to $8B_{д}$. In side direction

the axis is equal to $8B_б$. With the firing at panel the projectiles, which have low trajectory, the ellipse of scattering it approaches a circumference. Probable (middle) deviation in the direction of vertical axis is designated $B_в$, a in side direction, just as with firing at locality, is designated $B_б$. Ratios $\frac{B_д}{x_c}$, $\frac{B_б}{x_c}$ and $\frac{B_в}{x_c}$ in artillery is conventionally designated as the characteristics of closely grouped fire.

With firing at locality by the unguided fin-stabilized missiles at small angles of increase, just as with the firing artillery shells, $B_д > B_б$; with firing with the angles, close to the angles of maximum range, $B_б > B_д$ and transverse is directed perpendicularly to the line of fire (Fig. 13.6).

Depending on the combination of the acting random factors, which determine complete scattering, the longitudinal axis of ellipse can be inclined toward the line of fire.

Let us examine dependence for the calculation of certain characteristics of scattering the projectiles of cannon-type artillery, which proceeds mainly as a result of change from one shot to the next v_0, θ_0, c. The effect of scattering weather factors for one short-term firing it is possible not to consider. As a rule, the reasons, calling change v_0, θ_0 and c, they act independently one

from another. Let us designate mean deviation v_0 through r_{v_0}, θ_0 through r_{θ_0} and c through r_c. Middle range errors in the case of acting only one reason, are equal to:

$$E_{v_0}=\frac{\partial x_C}{\partial v_0}r_{v_0};\qquad E_{\theta_0}=\frac{\partial x_C}{\partial \theta_0}r_{\theta_0};\qquad E_c=\frac{\partial x_C}{\partial c}r_c,$$

where $\frac{\partial x_C}{\partial v_0}$, $\frac{\partial x_C}{\partial \theta_0}$, $\frac{\partial x_C}{\partial c}$ already to us basic correction factors.

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Since each of cause acts independently one from another, the total range probable error is determined from the formula

$$B_x=\sqrt{\left(\frac{\partial x_C}{\partial v_0}r_{v_0}\right)^2+\left(\frac{\partial x_C}{\partial \theta_0}r_{\theta_0}\right)^2+\left(\frac{\partial x_C}{\partial c}r_c\right)^2}.\qquad (13.66)$$

According to the characteristics of complete scattering, it is possible to calculate the characteristics of scattering of one of the determining values. For example, if we shoot the barrel system at angle of elevation, close to the angle of maximum range, and to determine experimental B_x, then, knowing that at the angles, close to the angle of maximum range $\frac{\partial x_C}{\partial \theta_0}\approx 0$, it is possible from formula (13.66) to find value

$$r_C=\frac{\partial c}{\partial x_C}\sqrt{B_x^2-\left(\frac{\partial x_C}{\partial v_0}r_{v_0}\right)^2}.$$

Value r_{v_0} is determined according to the results of ballistic firings with the use of the formula

$$r_{v_0}=0,6745\sqrt{\frac{\sum_1^n(\Delta v_i)^2}{n-1}}, \quad (13.67)$$

where $\Delta v_i=v_i-v_{cp}$ - a deviation of the initial velocity on separate shots v_i from arithmetic mean velocity in the group

$$v_{cp}=\frac{\sum_{i=1}^n v_i}{n}.$$

For evaluating the scattering with the time fire the rockets or the projectiles of classes the "earth/ground - air" and "air - air" can be used three probable deviation B_n, B_6 and B_n, since the points of discontinuity will occupy the area of space, limited by the volume of dispersion. In this case it is necessary to determine the rotation of the principal axes of the volume of dispersion of the relatively starting coordinate system. One of the axes it is expedient to guide tangentially toward trajectory (value its 8 B_n), the second axis - along the normal to trajectory (value its 8 B_n), the third - on binormal (value its 8 B_6).

The volume of dispersion is shown on Fig. 13.7. The coordinates

cf center cf dispersion will be m_x, m_y, m_z

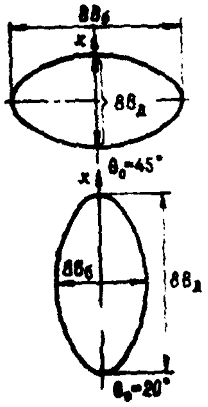

Fig. 13.6. Ellipses of scattering with the firing the unguided rocket projectiles at different angles of elevation.

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When with firing at locality either vertical barrier/obstacle probable deviations in different directions are identical, is introduced into examination probable circular deviation, frequently calling it simply circular deviation or circular error.

Circular probable deviation is called a radius of circle with center in the center of dispersion, the hit probability to which is equal to 0.5. It is obvious, circular error will be the more than

appropriate deviations in the direction. The dependence between the probable circular deviation $E_{кр}$ (by circular error) and the probable deviation in the direction takes the form

$$E_{кр} \approx 1{,}75\,E \qquad (13.68)$$

or, in connection with firing at locality,

$$r_{кр} = 1{,}75\,B_{д} = 1{,}75\,B_{б}.$$

It is obvious, when $B_{д} \approx B_{б} \approx B_{в}$ it is possible to introduce into examination the sphere of scattering, also, for its characteristic - spherical probable deviation. Under spherical probable deviation is understood a radius of the sphere, the hit probability of the point of discontinuity into which is equal to 0.5. Spherical probable deviation can be determined through one of probable deviations in the direction of formula

$$E_{сф} \approx 2{,}28\,E. \qquad (13.69)$$

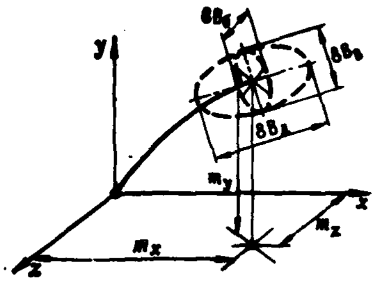

Fig. 13.7. Volume of dispersion.

§4. Errors of firing with induction to target/purpose.

The accuracy of firing and scattering missile trajectories (projectiles) with induction to target/purpose depend on the designation/purpose of rocket, its construction, principle of operation and equipment/device of the control system, i.e., on the method of guidance, maneuverability of projectile, inertness of control and instrument errors [16] accepted. For the index of the accuracy of guidance, is taken the value of the error, under which is understood the vector quantity, which corresponds to the minimum distance between the projectile and the target/purpose ($\bar{h}$). With the

errcr, equal to zero, let us have the sc-called direct hit.

For the total characteristic cf the accuracy of guidance, it is necessary to know the mathematical expectaticns cf components of vector of error h_x, h_y, h_z and the ccrresponding to them dispersicns $D_{h_x}, D_{h_y}, D_{h_z}$.

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Determining the characteristics of the accuracy of firing at the three-dimensional random vector of errcr represents by itself very ccmplex problem; therefore in practice errcr determines two thcse comprise, lying in the plane, passing through the vital structural/design assembly cf target/purpose sc that the vector of errcr would possibly more correctly characterize the accuracy cf guidance. For the approximate estimate cf error, it is possible to be restricted to its module/mcdulus.

Scattering the trajectcries cf homing missiles is caused by a series of the reasons: by errors, determined by the inertness cf control, by the errors, determined by the limitedness of the maneuver cf rocket and which depend cn the method of guidance and maneuvers of target/purpose and rocket, and by the instrument errors, which are exhibited in the process of the guidance and in the so-called "dead

zone" of control, in which the control system for a number of reasons ceases to act.

Among instrument errors first of all one should note the measuring error of the angular coordinates of target/purpose relative to rocket. Instrument errors, which determine the angular coordinates of target/purpose (coordinators), lead to the fact that missile targeting is realized not on target/purpose, but on the fictitious point, arrange/located on certain distance from target/purpose. The greater the error for coordinator, the greater the value of error. To correct the error for coordinator guidance system not can, since it accepts the direction, indicated by coordinator, for true.

To the errors for coordinators are devoted at present sufficiently many works both in Soviet and in foreign literature.

Scattering missile trajectories with remote control in the case when measuring device is located on rocket (for example, television head), is determined by the same sources of errors that and with homing/self-induction, with this vectoring error it does not depend on the range of the control system. In the case of remote control with the arrangement/position of measuring devices (goniometers, radars) on control post scattering trajectories is caused by the errors for determining the coordinates of rocket with the aid of

radar. With command and remote control the value of error, caused by instrument errors, increases with an increase in the range of remote-control system. To instrument errors can be also referred the errors, which appear in computer and in command radio link.

While relatively small target ranges and its maneuverings essential effect on error have the errors, caused by the inertness of rocket (limited maneuverability) and of the errors, caused by the inertness of control.

The guidance dispersions as a whole just as purely instrument vectoring errors, are examined in the specialized literature, for example, in [16].

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Let us here examine only dynamic errors, connected with the inertness of rocket and the motion characteristics of target/purpose and rocket. The action of the dynamic errors, determined by the inertness of rocket, is exhibited in the fact that the rocket reacts to the command/crews of the control system with certain delay, that, it is logical, it leads to about Mach. Dynamic errors depend substantially on the guidance method which determines required normal load factors, and from the character of the maneuvering of target/purpose.

Let us examine the error, which depends on maneuvering characteristics of rocket. Maneuverability can be determined either by the smallest possible radius of curvature of the trajectory of the motion of the center of mass or by maximally possible transverse acceleration a_{np}, on which depend the normal load factors.

The minimum radius of curvature, frequently utilized during the constructions of the trajectories of the maneuver of rocket and during definition of lethal areas, can be determined by the known formula of the kinematics

$$\varrho=\frac{v_p^2}{a_{np}}. \qquad (13.70)$$

The normal acceleration of rocket on formula (4.11) is equal

$$a_{np}=v_p\left(\frac{d\alpha_p}{dt}+\frac{d\gamma}{dt}\right);$$

angular velocities $\frac{d\alpha_p}{dt}$ and $\frac{d\gamma}{dt}$ are determined by the motion characteristics of target/purpose and by guidance method.

If we proceed from the kinematics of the guidance in one plane, then with error function r(t) must have a minimum, and the derivative dr/dt be equal to zero. From (4.8) condition dr/dt=0 for an error is

fulfilled with

$$v_{\text{ц}} \cos \alpha_{\text{ц}} = v_p \cos \alpha_p.$$

For an example let us observe the order of value determination of error with missile targeting to target/purpose from the method of pursuit, described in Chapter IV. Let us examine the kinematics of the motion of rocket on the last/latter section of the path before its rendezvous for target/purpose, accepting p=const. A change in the distance between the target/purpose and the rocket and the rate of the rotation of the line of sighting with pursuit guidance to the target/purpose, driving/moving towards, we will obtain that on the basis of formula (7.109). Let us replace in (7.109) dγ/dt from formula (7.114) and we will obtain

$$r = \frac{v_p v_{\text{ц}} \sin \alpha_{\text{ц}}}{a_{n\,p}}. \qquad (13.71)$$

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In the general case $r_{\min}$ it is possible to determine, after applying on right side (13.71) the condition

$$\frac{dr}{dt} = \frac{d}{dt}\left(\frac{v_p v_{\text{ц}} \sin \alpha_{\text{ц}}}{a_{n.p}}\right) = \frac{d}{dt}\left(\frac{v_{\text{ц}} \sin \alpha_{\text{ц}}}{\frac{d\alpha_p}{dt} + \frac{d\gamma}{dt}}\right) = 0. \qquad (13.72)$$

While maneuvering of target/purpose in the direction and velocity,

the problem is solved only numerically.

Similarly it is possible to derive the formulas, which determine the error, which depends on maneuvering characteristics of rocket, and for other methods of guidance to target/purpose.

It is necessary to keep in mind that the damage to target is determined not only accuracy of guidance. The calculation of the effectiveness of firing must be carried out taking into account all acting factors, including the factors, which depend on construction and work of the warhead of the rocket. This is the large problem, coming out beyond the scope of this book.

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Chapter XIV.

EXPERIMENTAL METHODS OF EXTERNAL BALLISTICS.

The methods of conducting ballistic testings, their organization, metering equipment and methods of processing test results depend on the construction of the experience/tested projectile and designation/purpose of firing. Extra-ballistic testings depending on designation/purpose can be divided into two large groups - into laboratory testings, conducted, as a rule, with firing from special projectile settings up or artillery instruments, and to the range tests of rockets and artillery shells.

Laboratory tests are conducted in the specially equipped dashes or on polygon ballistic routes. Firings can be carried out both on those opened and in closed (decompression) routes. The separate types of testings are firings for determining the initial velocity of projectile and characteristics of scattering the initial velocity,

firing for determining of the aerodynamic characteristics of full-scale specimen/samples and models, firing for determining of the stability characteristics of motion and dynamic qualities of full-scale specimen/sample or model.

Polygon firings from artillery instruments and by rockets are carried out for purpose of the determination of the initial conditions of shot, ranging of firing and characteristics of the scatter of points of incidence/drop. When conducting of firings from rifled artillery weapons, can be provided experimental determination of derivation. Each form of the named polygon firings can be carried out independently in the process of creation and final adjustment of the specimen/sample of armament or composite in the process of subsequent testings and performing work on the creation of firing tables.

Independent value have the trajectory observations of rockets and projectiles. Trajectory observations in the form of the fulfilled problems can be divided into three groups. The first group is connected with testing of rockets and projectiles.

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Second group - with trajectory prediction, the detection of the

places of the missile takeoffs and of firing positions of artillery instruments, the determination of the predicted points of impact in the rockets or proposed targets. The third group of trajectory observations is connected directly with the interception of the driving/moving rockets and projectiles of different designation/purpose.

In the process of creation and final adjustment of the new specimen/samples of the rockets and other flight vehicles final stage are flight tests. They serve for obtaining the most complete information about the work of missile complex as a whole, and also its separate parts.

Flight tests are only testings which make it possible to rate/estimate the behavior of flight vehicle directly in flight and to define such parameters of its motion as linear coordinates, velocity, angular displacement, g-force, etc.

As a rule, any the newly created specimen/sample of rocket passes two types of testings - check-out and combat.

The first are intended for performance checkout of different systems, placed on board rocket, and the flight characteristics of rocket as a whole. Test specimen is equipped with a large quantity of

inspection and measurement and other special equipment which is placed on board the rocket instead of some assemblies and the units whose work does not undergo investigations, and their absence does not interfere with the normal functioning of all remaining cell/elements and units of rocket. So enter, for example, with by warhead and the equipment, which ensures its detriment/blasting.

All obtained on board rocket information is transferred at tracking station where occurs its treatment. These stations conduct also external measurements for parameter determination of the motion of rocket.

Combat tests are intended for testing of the military characteristics of rocket. They are conducted for the evaluation of scattering and effectiveness of action on the target/purpose of the newly created specimen/sample. During service firing usually special metering equipment onboard for the rocket is not establish/installed and in the process of testings are conducted only external measurements, which can be fulfilled with the aid of ground-based optical and radio engineering (radar) equipment.

Depending on the method of measurement of the parameters of trajectories, extra-trajectory radio engineering measurements are constructed most frequently according to active and passive

principles.

The active principle of measurements is characterized by the fact that entire/all metering equipment is arrange/located at tracking station, and the flight vehicle only reflects the sent to it from station signals.

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This principle of measurements is utilized for the target detection and tracking their motion along trajectories, as a result of which are determined (for example, for ballistic missiles) launching points, the predicted trajectories and the impact points in the rockets.

With the passive principle of external measurements, basic monitoring-measuring equipment is placed on the Earth and fixes the position of flight vehicle according to the signals of the radio transmitter, establish/installed on board the latter.

§1. Measurement of speed of motion of body on ballistic route.

One Of the most widely used methods of determining the rate of the motion of body along trajectory is the method, instituted on the

measurement of time t of passage by the body of the section of the route of the specific length l. In this case, after making assumption about linear change of speed, is calculated its unknown value from the dependence

$$v = \frac{l}{t} \text{ m/s.}$$

The obtained value of velocity v is related to point in the trajectory A, which coincides with the middle of the measuring section l (Fig. 14.1).

For decreasing the errors, caused by the averaging of velocity, the length of section l selects possible lower, how this allows metering equipment. As it is clear from the essence of the examined method, its realizing metering equipment must consist of two interdependent component/links. The first component/link - these are chronograph, instrument for measuring the transit time by the body of the trajectory phase. The second component/link - these are the locking apparatus, adjustable on the end/leads of the measuring phase of trajectories, at consecutive signals of which occurs the starting/launching and the cessation of the computer of chronograph.

Let us examine in more detail the equipment, utilized for velocity measurement.

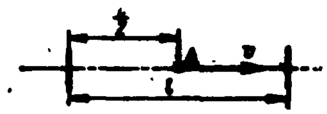

Fig. 14.1.

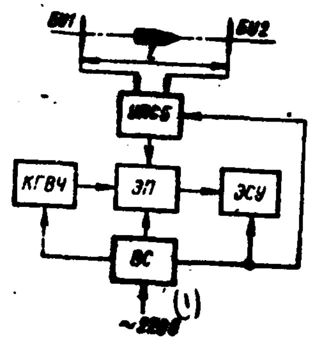

Fig. 14.2.

Fig. 14.1. For determinaticr of average speed cr section of ballistic rcute.

Fig. 14.2. Block diagram cf electrcnic chrcrcgraph.

Key: (1). V.

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Chrcnograph.

The most widely used at present type ci the chronographs, utilized on tallistic rcutes, is tle electrcnic chronograph (EKh), wcrking according to the principle cf the ccaparison of the measured

time intervals with the sum of the periods of oscillations of the high-frequency crystal oscillator of the electrical signals. The block diagram, which elucidates device work of one of such chronographs, is given in Fig. 14.2.

In this schematic:

BU1 and BU2 - respectively more front/leading and rear locking apparatus; UPSB - amplifier-converter of the signals of blocking; EP - electronic breaker (switch); KGECh - quartz high-frequency oscillator; ESU - electronic computer; VS - controlled rectifier.

Chronograph is supplied through the unit VS from the grid/network of alternating current with voltage/stress ~220V.

The measurement of time by chronograph is conducted as follows. In transit through BU1 the body whose velocity must be measured appears the electrical signal, which is appropriately converted and is amplified by the unit UPSB and it passes to unit EP.

The high speed electronic interrupter (triggering time its $\sim 1\cdot 10^{-6}$ s) at the signal from blocking BU1 closes the circuit, which connects computer with crystal oscillator. In this case, alternating current from KGVCh with period of $T = 1/f$ begins to pass into ESU,

which computes a quantity of entered it momentum/impulse/pulses. At the moment of the output of body from measuring section at the signal from BU2 again wear/operates the unit EE, but already disconnecting KGVCh from ESU. Thus, for the time of the action of body along the measuring phase of trajectory the ESU will record the specific quantity of the entered it periods T. If the number of periods is equal to n, then the unknown time

$$t=nT.$$

In the chronographs, utilized during ballistic measurements, are applied KGVCh, that develop current with frequency not less than $f=1\cdot10^5$ Hz, i.e., with the oscillatory period, it is not more

$$T=\frac{1}{f}=1\cdot10^{-5}\ s.$$

Error due to the instability of the frequency of KGVCh they virtually disregard, since it does not exceed 0.01o/o.

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Error from a reading error the measured time interval comprises not more than one period, since the beginning of the measured time interval can either coincide with the momentum/impulse/pulse of generator or render/show in spacing between pulses.

Error Δt=T is for EKh maximum over entire range of the measured time intervals.

The computer of chronograph can take a reading of momentum/impulse/pulses either on decimal or along binary system; can be used the combined system. Figures 14.3 shows the schematic of the work of ESU in the decimal system of count. Consists this ESU of several identical computing decades, assembled on electron tubes or transistors.

During the measurement of time, the current from KGVCh with frequency f proceeds to the tenth momentum/impulse/pulse from decade No. I it goes to decade No. II which counts already ten periods, and so on. The passage of the signals through decades is noted on display unit from neon bulbs for visible reading (indicators can be and arrow type).

At the moment of the cessation of ESU at the signal BU2 on indicator of each decade, is fired the bulb, which corresponds to the number of pasts through this decade of periods.

Thus, for instance, in the position of the cessation of count, shown on Fig. 14.3 (undarkered small circles - the burning neon bulbs), computer fixed a quantity of momentum/impulse/pulses n=3851;

this corresponds to the measured flight time

$$t = nT = 3851 \cdot 10^{-5} = 0{,}03851 \; s$$

The display system of the chronograph whose computer is instituted on the use of a binary count, is shown on Fig. 14.4. It includes 12 cells, equipped each by the neon bulb (number of cells can be other). Ignition with the cessations of the chronograph of any neon bulb corresponds to the passage through this cell of the computer of the quantity of momentum/impulse/pulses, equal to number $n_i = 2^{N-1}$; where N - number of cell. Number this (n_i) calls the "value" of cell. The "values" of each of 12 cells are given to Fig. by 14.4.

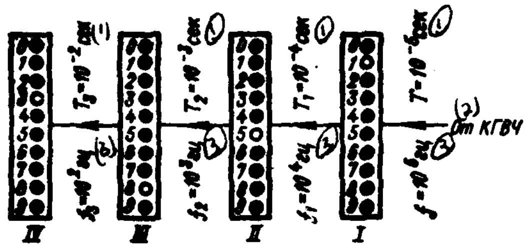

Fig. 14.3. Schematic of recording the measured time interval with decimal system of count.

Key: (1). s. (2). From. (3). Hz.

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Thus, total number of momentum/impulse/pulses, which entered the chronograph for the time of the motion of body, will be determined from burning neon bulbs taking into account their "value". For example, in the position, shown in Fig. 14.4, computer recorded the number of periods

$$n=2^0+2^1+2^2+2^5+2^7+2^9+2^{10}+2^{11}=3851,$$

that with $f=10^5$ Hz gives the flight time

$$t=nT=0{,}03851\ s.$$

locking apparatus.

The forms of locking apparatus known such - are applied electrostatic, capacitive, photoelectric, accustic inertia, etc. of blocking. Let us examine for an example some of the blocking cell/elements.

Frames-targets. Frame-target (Fig. 14.5) represents by itself the flat/plane frame, usually square. The size/dimension of frame can change depending on bore d of model whose velocity will be measured. To frame isolated from it is wound, as shown in figure, fine/thin wire with a diameter of d_{np}=0,20—0,25 mm (tinsel).

Frames-targets are establish/installed on the route so that their planes would be perpendicular to the trajectory of the motion of body, and their windings are included in the unit UPSB of chronograph and are supplied by direct current. With passage through the plane of frame-target, the body breaks its winding, what is the signal for starting/launching or cessation of ESU of chronograph.

For providing the reliable interruption with the driving/moving body of the winding of frame-target the distance between adjacent turns of wire l is taken from condition l<0.25d, and the winding/coil of wire is conducted with certain interference in order to decrease the possibility of its separation into sides and drawings. For the same purpose during measurements v of the bodies of the low caliber of the wire of frame-targets, they stick on fine/thin paper.

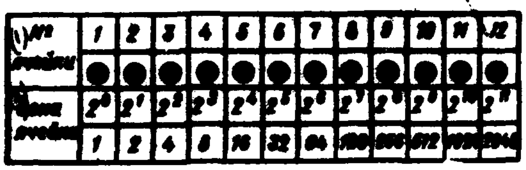

Fig. 14.4. Schematic of recording of measured time interval in the binary system of count.

Key: (1). cell. (2). Value of cell.

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To frame-targets are characteristic the following operational deficiency/lacks:

- is necessary to restore the completeness of their windings after each measurement;

- occurs certain nonuniformity in the interruption of the wires of winding, which leads to the unavoidable errors in value v.

Nevertheless frame-targets, in view of their reliability and simplicity, are utilized very widely.

Solenoid blocking. The sensing cell/element of the given blocking is the solenoid with double winding, placed in metal casing (Fig. 14.6). one winding of solenoid is supplied by direct current, creating constant magnetic flux. With passage within the solenoid of the metallic body, which possesses magnetic properties, changes the magnetic flux (as a result of change) of magnetic permeability) and in inducing winding of solenoid, connected with UPSB, it is induced the pulse signal of current.

This signal - weak and in the unit of UPSB it is amplified and converted into the signal with steep wave front, convenient for the operation of unit EP of chronograph.

Solenoid blocking possesses a series of the advantages:

- allows prolonged repeated use, since do not require any process/operations on the restoration/reduction of their completeness;

- does not distort the flight of the body being investigated;

- it gives a precise and uniform time mark of passage by the body of the plane of solenoid.

however, the application/use of solenoids as locking apparatus requires conducting firings by the magnetized bodies and it is conjugate/combined with the complication of the units UPSB of chronographs.

Photoelectric blocking. Recently, in connection with the appearance of low-inertia photocells, for laboratory ballistic routes they will begin to use extensively photoelectric blocking. The exemplary/approximate schematic of the separate assembly of photo-blocking is shown on Fig. to 14.7.

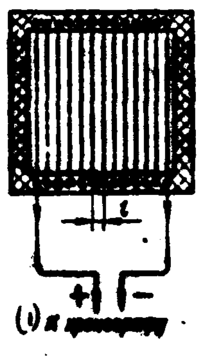

Fig. 14.5.

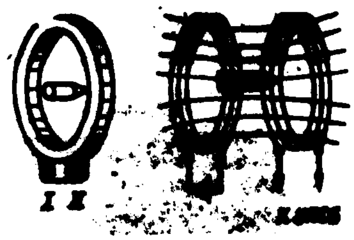

Fig. 14.6.

Fig. 14.5. Wire frame, target,.

Key: (1). To chronograph.

Fig. 14.6. Solenoid blocking cell/element.

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It consists of tube L, low-inertia photocell FE and three-stage amplifier with thyratron.

With the passage of the model of the body between the tube and the photocell, it intersects light ray, causing an instantaneous change in the illumination of photocell. The appearing in this case

electrical signal, in passing by through the amplifier, triggers the thyratron of electronic relay whose output pulse enters the electronic chronograph EKh, including or disconnecting the computer of the latter. Photoelectric blocking provides the high accuracy of time marks.

Inertia blocking. On laboratory ballistic routes, mainly for artillery practice from small arms, is utilized inertia blocking.

The construction of the inertia blocking cell/elements can be different. One Of the constructions, the so-called "contact cup", it is shown on Fig. to 14.8. This contact inertia cell/element, connected to the entry of chronograph, is fastened on the metallic or plywood plate, placed at the end of the route. From the impact/shock of the model of body into plate, the latter shudders and contact bar to instant rebounds from the wall of cup. The occurring with this phenomenon explosion of electrical circuit, as in frame blocking, it is the signal for a chronograph.

The advantage of the inertia locking apparatus in the fact that the contact in them is restored after each shot automatically (unlike frames-targets). However, to be applied for blocking the route they can only as last/latter cell/element, since sharply is changed the character of the motion of model along trajectory. The length of the

phase of trajectory } for velocity measurement of bodies with firing from artillery or rifle systems is selected on the basis of the required accuracy of the determination of velocity and possibilities of chronograph.

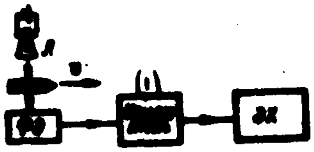

Fig. 14.7.

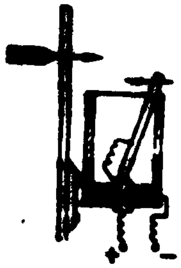

Fig. 14.8.

Fig. 14.7. Photoelectric blocking cell/element.

Key: (1). Amplifier.

Fig. 14.8. Inertia blocking cell/element.

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Since the velocity in the described method is defined as

[illegible]

that the greatest relative error in its determination will be

$$\frac{\Delta v}{v} = \frac{\Delta l}{l} + \frac{\Delta t}{t}.$$

Here $\Delta l/l$ - relative fault of measurement of base l; $\Delta t/t$ - relative

measuring error for the flight time of body on chronograph.

Let according to test conditions be required to obtain this accuracy of determination v, in order to $\Delta v/v \leqslant 0.2$o/o. This will be, for example, provided in such a case, when $\Delta l/l \leqslant 0.1$o/o and $\frac{\Delta t}{t} < 0.1\%$.

Since for the electronic chronograph the error $\Delta t = T$ =const, we obtain $\frac{T}{t} < 0.1\%$, whence $t \geqslant 1000T$. For a special case, with $T = 1 \cdot 10^{-5}$ with $t \geqslant 0.01$ s. Based on this minimum interval of time there is established the length of the measuring section l, equal to $l \geqslant vt = 0.01v$ to m, where v - expected rate of the motion of body. Knowledge l makes it possible to find the permissible error Δl in the setting up of blocking devices from relationship/ratio $\Delta l \leqslant 0.0001\, l$ m. With the observance of two these conditions $\Delta v/v \leqslant 0.2$o/o.

During the definition of the initial velocity of the projectile of rifled artillery system, the locking apparatus place along trajectory in the manner that it is shown on Fig. to 14.9. In this case, distance l is selected in accordance with considerations presented above, and removal/distance l_0 of first blocking device from the point of the flight of projectile - so that on BU1 would not affect the escaping from projectile setting up gases. Value l_0 depends on the form of projectile setting up, but usually does not exceed 20-30 m (during frame blocking). The average measured velocity of projectile $v_{cp} = v$ can be referred to the half of the distance between frames-targets. For velocity transformation to muzzle end

face, i.e., for determining the initial velocity v_0, they use the formula, obtained in the analytical method of pseudovelocity in connection with the short low trajectories

$$D(v)-D(v_0)=cx. \qquad (14.1)$$

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Accepting for a special case in question the linear dependence between the change in the velocity and the change in function D(v), it is possible to comprise the proportion

$$\frac{D(v)-D(v_0)}{v-v_0}=\frac{\Delta D(v)}{10}, \qquad (14.2)$$

where $\Delta D(v)$ - a change in function D(v), that corresponds to a change in argument v on 10 m/s.

After designating $v-v_0=\Delta v$ and comparing (14.1) and (14.2), we will obtain

$$\frac{\Delta D(v)}{10}\Delta v=cx.$$

Opening the value of the ballistic coefficient of c, we will obtain correction for velocity transformation to the muzzle end face

$$\Delta v=\frac{id^2}{Q}10^4\frac{x}{\Delta D(v)}, \quad (14.3)$$

where x - a distance between the muzzle end face and the middle of

the blocked phase of trajectory (Fig. 14.9); i - an experimental factor of the form of projectile (bullet). Values ΔD(v) in connection with the Siacci function of air resistance can be taken on table 31, given in work [59].

§2. Determining aerodynamic characteristics according to the results of ballistic firings.

One Of the most important aerodynamic characteristics - drag coefficient $c_x(M)$ it can be determined by firing on ballistic route.

The calculated dependence, which makes it possible to determine the average value of drag coefficient on the final cut of trajectory, proceeds from the law of a change in the kinetic energy of projectile and requires the measurement of the velocity of projectile in two points in the trajectory. The diagram of installation of locking apparatus along ballistic route is shown on Fig. 14.10. If projectile setting up (for example - mortar), does not make it possible to have horizontal trajectory, then locking apparatus along trajectory are establish/installed on different height/altitude (Fig. 14.11). Let in the general case be known the parameters of trajectory v, x, y at two different points; then it is possible to write

$$\frac{mv_2^2}{2}-\frac{mv_1^2}{2}=\int_{S_1}^{S_2} X\cos(\widehat{X,S})\,dS+\int_{y_1}^{y_2} mg\cos(\widehat{g,y})\,dy, \qquad (14.4)$$

where m - mass of body; X - drag.

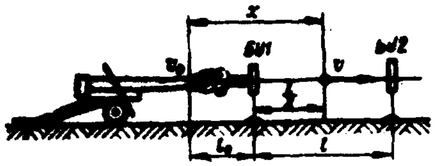

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Fig. 14.9. Arrangement/position of the blocking cell/elements along trajectory for measuring the initial velocity.

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Taking into account that $\cos(\widehat{X, S}) = \cos(\widehat{g, y}) = \cos 180° = -1$, we obtain

$$\frac{m}{2}(v_1^2 - v_2^2) = \int_{S_1}^{S_2} X dS + mg(y_2 - y_1). \qquad (14.5)$$

Force X from under integral sign they will carry by average value, then

$$\frac{m}{2}(v_1^2 - v_2^2) - mg(y_2 - y_1) = X_{cp}(S_2 - S_1)$$

and further

$$X_{cp} = \frac{m}{2} \cdot \frac{v_1^2 - v_2^2 - 2g(y_2 - y_1)}{S_2 - S_1}. \qquad (14.6)$$

Usually firing for determination $c_x(M)$ or ballistic route is organized so that the measured trajectory phase would render/show horizontal for the possibility of the exception/elimination of the gravitational work (see Fig. 14.10). In this case, obviously, let us have

$$y_2 = y_1; \; S_1 = x_1; \; S_2 = x_2; \; S_2 - S_1 = x_2 - x_1 = L.$$

Taking into account that indicated

$$X_{cp} = \frac{m}{2}\,\frac{v_1^2 - v_2^2}{L}. \qquad (III.7)$$

Each obtained the data of experiment value X_{cp} must be referred to a certain value of the velocity of article for which is accepted the average speed at the length of measurement l, equal to

$$v_{cp} = \frac{v_1 + v_2}{2}.$$

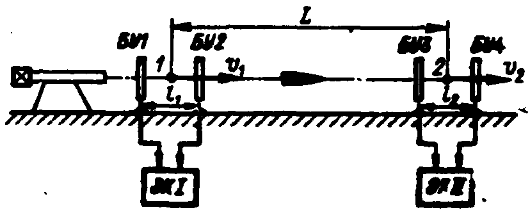

Fig. 14.10. Setting up of the blocking frames during determination $c_x(M)$ by firing.

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For transition from the force of drag X_{cp} to aerodynamic coefficient $c_x(M)$ is utilized the dependence

$$X_{cp}=S\frac{\rho v_{cp}^2}{2}c_x\left(\frac{v_{cp}}{a}\right), \qquad (14.8)$$

where ρ and a - a respectively mass air density and the speed of sound in air at the moment of experimentation; S - area of the midsection of body. From the comparison of expressions (14.7) and (14.8) for X_{cp} it follows

$$\frac{m}{2}\left(\frac{v_1^2-v_2^2}{L}\right)=S\frac{\rho}{2}\left(\frac{v_1+v_2}{2}\right)^2c_x\left(\frac{v_{cp}}{a}\right).$$

Hence unknown value $c_x(M)$ is equal

$$c_x\left(\frac{v_{cp}}{a}\right)=\frac{4m}{SL\rho}\frac{v_1-v_2}{v_1+v_2}. \qquad (14.9)$$

Final calculated dependence, after combination of all constant values and scale factors, takes the fcrm

$$c_x\left(\frac{v_{cp}}{a}\right)=5{,}09\ \frac{Q}{d^2 L \Pi}\,\frac{v_1-v_2}{v_1+v_2}. \qquad (14.10)$$

where d - a diameter of the midsecticn cf body; P - the specific gravity/weight of air.

During calculation the entering the fcrmula values must be taken in the following dimensicrality:

$$Q\ \text{kg};\ d\ \text{mm};\ L\ \text{m};\ \Pi\ \text{kg/m}^3;\ v\ \text{m/s}.$$

The values of the specific gravity/weight cf air P for the different values of the barometric pressure h mm Hg and the temperature of air t°C must be determined frcm the apprcpriate tables.

The length of base L fcr measurement c_x must be selected so that is justified the averaging of value X and, at the same time, is provided the noticeable incidence/drcp in the velocity of body, necessary for obtaining of the required accuracy of calculation c_x.

Under conditions of range/polygcn, usually, base L is taken equal to 200-400 m; under laboratory conditicns when the length of ballistic route is limited, it is allcw/assumed to take the base of the equal to several of ten meters, depending cn the velocity v of the motion of body and possibilities of chrcnographs.

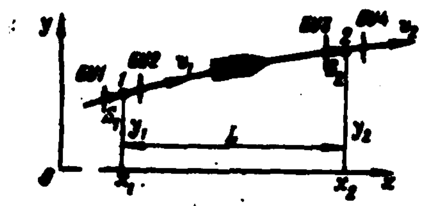

Fig. 14.11. Blocking inclined trajectory.

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Aerodynamic functions with tilting moment $K_M\left(\frac{v}{a}\right)$ and under lift $K_N\left(\frac{v}{a}\right)$ also can be determined by results of firing. For this, during experiment must be measured the cell/elements of the rotary motion of projectile δ, $\dot{\varepsilon}$, ν, $\dot{\nu}$, r, after which the named coefficients are determined from the appropriate calculation formulas.

Are known two methods of determining angular missile attitude in the process of its motion along trajectory - method of photography in two mutually perpendicular planes and method of firing at cardboards. In the first case the angles of the slope of the longitudinal axis of projectile to the direction of the motion of the center of mass are determined from photographs, in the second - these values they can be obtained by the measurement of holes in the specially processed fine/thin cardboard. During the coincidence of the longitudinal axis

of projectile with velocity vector, the hole in cardboard has a form of circumference. If nutation angle $\delta \neq 0$, then hole has oval form (Fig. 14.12). Angle, comprised by the longitudinal axis of oval with vertical line - precession angle ν. Knowing the size/dimensions of projectile, according to the size/dimensions of the axes of oval a and a_1, it is possible to determine nutation angle δ.

With the sufficiently large number of cross sections, which fix missile attitude on each of the periods of a change in the nutation angle δ, it is possible to obtain experimental curve/graphs $\delta=f(\nu)$. Between the cross sections, which fix missile attitude, are establish/installed the locking apparatus, which make it possible to measure the transit time of the projectile through the appropriate cross sections. In summation, it is possible to obtain the experimental dependences

and

$\delta=f_1(x)$, $\nu=f_2(x)$.

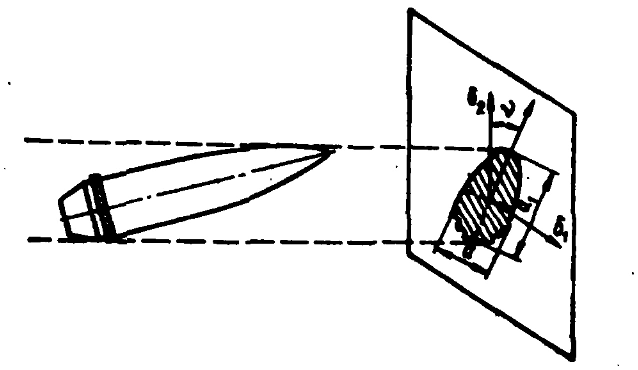

Fig. 14.12. Form of hole ir pastebcard tarçet.

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As an example let us examine determinatice $K_M\left(\frac{\sigma^2}{\alpha}\right)$ in the assumption that the dynamic axis of equilibrium coincides with the velccity vector of the center of mass cf prcjectile. We will use the fcrmulas, obtained in Chapter VIII, §4. Frcm (8.81) it is possible to write that the half-pericd cf the fluctuaticn cf angle δ will be equal tc

$$\frac{T_1}{2}=\frac{\pi}{\alpha\sqrt{\sigma}},$$

whence

$$\sigma=\frac{4\pi^2}{\alpha^2T_1^2}. \qquad (14.11)$$

Frcm (8.79) let us find

$$\beta=\alpha^2(1-\sigma)$$

and, after substituting (14.11), we will obtain

$$\beta = \alpha^2\left(1 - \frac{4\pi^2}{\alpha^2 T_0^2}\right).$$

The aperture of value α and β and after replacing r_0 on (8.84), we can write

$$\frac{d^2h}{gA} 10^3 H(y) v^2 K_M\left(\frac{v}{a}\right) = \frac{C^2\pi^2 v_0^2}{\eta^2 A^2} - \frac{4\pi^2}{T_0^2}.$$

With firing at small angles of increase $H(y)=1$. Then the average value of coefficient $\bar{K}_M\left(\frac{v}{a}\right)$ when v also is taken by the average value and the angular velocity of spin $r=r_0=const$, we will obtain equal to

$$\bar{K}_M\left(\frac{v}{a}\right) = \frac{\pi^2 g}{d^2 h v^2 10^3}\left(\frac{C^2 v_0^2}{A \eta^2 d^2} - \frac{4A}{T_0^2}\right). \qquad (14.12)$$

Value h is difficultly defined both theoretical and experimentally and, strictly speaking, it it is alternating/variable during the fluctuations of projectile in the process of motion. During the use of known empirical formulas for determining of h, for example, the formula, given in explanation to (2.111), it is accepted h=const. The tables or the curve/graphs, comprised in this case for $K_M\left(\frac{v}{a}\right)$ must contain indication of the method of determination h.

Are known also the methods of determination according to the results of firings on the ballistic route of the aerodynamic coefficient of lift and of the coefficient of the damping moment. Methods these are complex and the accuracy of the obtained results is inferior to the accuracy of the results of experiment in wind tunnels. For familiarization with the methods indicated let us send away the reader to books [9], [59] and the thematic works on ballistic testings.

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§3. Theoretical bases of the methods of determining the position of rocket.

The position of body in space at the any moment of time is determined by six generalized coordinates. Three Linear values determine the position of the center of mass of rocket, while three angular - its orientation in space. These angles frequently call Euler angles.

Let us examine the possible methods of determining the position of rocket and calculated dependences for calculating the coordinates.

The coordinates of rocket in space can be defined as coordinates of the point of intersection of different surfaces. The surface, which determines the possible aircraft attitude at the given instant, frequently calls the surface of position or position surface. Three Geometric surfaces during intersection have one or several common points. Thus, with the aid of three surfaces of position it is possible to fix the location of rocket at any point of space. The type of position surfaces depends on the utilized ground-based metering equipment.

If we simultaneously follow the rocket from three ground-based point/items and to calculate its removal/distances (slant ranges) from each of the stations, then position surfaces will be spheres with centers in the point/items of the tracking, described by the radii, equal to slant ranges.

The method position finding of flight vehicle in space as of point of intersection of three spheres is called ranging.

We will obtain formulas for coordinate determination of the rocket in of the terrestrial system Cxyz from the measured from three point/items of tracking slant ranges. Let the tracking stations be arrange/located in points $C_1(x_1, y_1, z_1)$, $C_2(x_2, y_2, z_2)$ and $C_3(x_3, y_3, z_3)$ (Fig. 14.13). Let us designate the distances between tracking stations and the beginning of the system of coordinates l_1, l_2, l_3, with respect slant ranges to point in the trajectory $M(x, y, z)$ - through ρ_1, ρ_2, ρ_3, distance OM - through ρ_0.

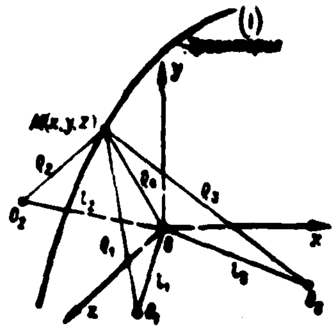

Fig. 14.13. Ranging method of position finding of rocket.

Key: (1). Trajectory.

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From analytical geometry it is known that

$$\rho_1=\sqrt{(x_1-x)^2+(y_1-y)^2+(z_1-z)^2}$$

and

$$l_1=\sqrt{x_1^2+y_1^2+z_1^2} \qquad (14.14)$$

Analogously are determined values ρ_2, ρ_3 and l_2, l_3.

The length of cut OM can be represented in the form

$$\rho_0=\sqrt{x^2+y^2+z^2}. \qquad (14.15)$$

Let us discover equation (14.13), after substituting in it equations (14.14) and (14.15)

$$\rho_1=\sqrt{l_1^2+\rho_0^2-2(x_1x+y_1y+z_1z)}. \qquad (14.16)$$

Dependences for determining the values ρ_2 and ρ_3 take the same form, only of the entering in them values index 1 is replaced by index 2 or 3 respectively. After comprising expressions for differences $\rho^2{}_1-\rho^2{}_2$, $\rho^2{}_1-\rho^2{}_3$ and $\rho^2{}_2-\rho^2{}_3$ and after re-grouping terms, we will obtain

$$\left.\begin{aligned}(x_1-x_2)x+(y_1-y_2)y+(z_1-z_2)z&=c_1;\\(x_1-x_3)x+(y_1-y_3)y+(z_1-z_3)z&=c_2;\\(x_2-x_3)x+(y_2-y_3)y+(z_2-z_3)z&=c_3,\end{aligned}\right\} \qquad (14.17)$$

where

$$c_1=\frac{1}{2}(\rho_2^2-\rho_1^2+l_1^2-l_2^2);$$
$$c_2=\frac{1}{2}(\rho_3^2-\rho_1^2+l_1^2-l_3^2);$$
$$c_3=\frac{1}{2}(\rho_3^2-\rho_2^2+l_2^2-l_3^2).$$

System of equations (14.17) further is converted as follows:

$$\left.\begin{aligned}x&=a_1-ay;\\z&=a_2-by;\end{aligned}\right\} \qquad (14.18)$$

where

$$a_1=\frac{[illegible]}{(x_1-x_2)[illegible]-(x_1-x_3)(z_1-z_2)};$$
$$a_2=\frac{[illegible]}{(x_1-x_2)(z_1-z_3)-(x_1-x_3)(z_1-z_2)};$$
$$a=\frac{(y_1-y_2)(z_1-z_3)-(y_1-y_3)(z_1-z_2)}{[illegible]};$$
$$b=\frac{[illegible]}{(x_1-x_2)(z_1-z_3)-(x_1-x_3)(z_1-z_2)}.$$

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For determining the coordinate y, it is possible to utilize the dependence, obtained from equation (14.13) after the substitution of values x and z from expressions (14.18)

$$(a^2+b^2+1)y^2+2(x_1a-y_1+z_1b-ae_1-be_2)y-$$
$$-(\varrho_1^2-l_1^2-e_1^2-e_2^2+2x_1e_1+2z_1e_2)=0. \qquad (14.19)$$

Usually they attempt to arrange/locate all tracking stations on the same level, i.e., in plane xOz. In this ca e $y_1=y_2=y_3=a=b=0$ and the coordinates of rocket are determined from the formula

$$\left.\begin{array}{l} x=e_1; \\ y=\sqrt{\varrho_1^2-l_1^2-e_1^2-e_2^2+2x_1e_1+2z_1e_2}; \\ z=e_2. \end{array}\right\} \qquad (14.20)$$

During external measurements frequently is utilized the so-called total ranging method when the position of rocket in space is defined as point of intersection three ellipsoids of revolution.

It is known that the point, which belongs to ellipse, possesses that property that the sum of distances from it to foci remains constant; any point of ellipsoid possesses this same special feature/peculiarity.

Fig. 14.14 illustrates the application/use of a total ranging method for the determination of the position of rocket on plane.

We will obtain dependences for coordinate determination of rocket in earth-based coordinate system Oxyz, after using Fig. 14.13. Let us consider that in the beginning of coordinates is located so-called master station, and three wirgmer (receiving) are arrange/located in points O_1, O_2 and O_3.

In the process of tracking the rocket there are determined the sums of the slant ranges

$$\rho_{01}=\rho_0+\rho_1;\quad \rho_{02}=\rho_0+\rho_2;\quad \rho_{03}=\rho_0+\rho_3. \qquad (14.21)$$

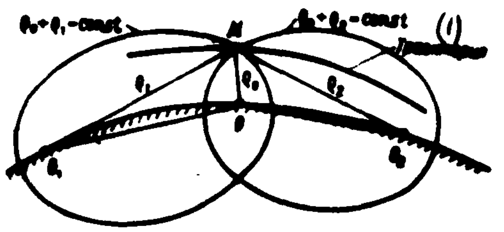

Fig. 14.14. Total ranging method of position finding of rocket.

Key: (1). Trajectory.

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After substituting into the left side of equation (14.16) value ρ_1 from equations (14.21), we will obtain

$$x_1x+y_1y+z_1z-\rho_0\rho_{01}=f_1, \qquad (14.22)$$

where

$$f_1=\frac{1}{2}\left(l_1^2-\rho_{01}^2\right).$$

Utilizing two second equations (14.21), analogously we can write

$$\left.\begin{array}{l} x_2x+y_2y+z_2z-\rho_0\rho_{02}=f_2 \\ x_3x+y_3y+z_3z-\rho_0\rho_{03}=f_3, \end{array}\right\} \qquad (14.23)$$

where

$$f_2=\frac{1}{2}\left(l_2^2-\rho_{02}^2\right); \quad f_3=\frac{1}{2}\left(l_3^2-\rho_{03}^2\right).$$

In three equations (14.22) and (14.23) - four unknown values x, y, z and ρ_0. For their determination these equations must be solved together with equation (14.15).

Are given below expressions for calculating the coordinates of point M in the case wher leading and slave stations are arrange/located in plane xOz

$$
\left.\begin{aligned}
x&=\frac{1}{A}\,[f_1(z_2\rho_{03}-z_3\rho_{02})-z_1(f_2\rho_{03}-f_3\rho_{02})-\\
&\qquad-\rho_{01}(f_2z_3-f_3z_2)];\\
y&=\sqrt{\rho_0^2-x^2-z^2};\\
z&=\frac{1}{A}\,[x_1(f_2\rho_{03}-f_3\rho_{02})-f_1(x_2\rho_{03}-x_3\rho_{02})-\\
&\qquad-\rho_{01}(x_2f_3-x_3f_2)];\\
\rho_0&=\frac{1}{A}\,[x_1(z_2f_3-z_3f_2)-z_1(x_2f_3-x_3f_2)+\\
&\qquad+f_1(x_2z_3-x_3z_2)],
\end{aligned}\right\}\quad(14.24)
$$

where

$$A=x_1(z_2\rho_{03}-z_3\rho_{02})-z_1(x_2\rho_{03}-x_3\rho_{02})-\rho_{01}(x_2z_3-x_3z_2).$$

Method, instituted on the definition of the position of rocket as points of intersection of three hyperboloids of rotation, is called differential ranging. On plane the location of rocket with the aid of this method is defined as point of intersection of two hyperbolas (Fig. 14.15), property of which is the fact that a

difference in the distances frcm fcci tc the pcint, which lies cn hyperbcla, there is a ccnstant value.

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Determination of ccordinates in differential ranging method is conducted cn formulas (14.24) by replacement ϱ_{i} on $\Delta\varrho_{i}$, where

$$\Delta\varrho_1=\varrho_0-\varrho_1;\quad \Delta\varrho_2=\varrho_0-\varrho_2;\quad \Delta\varrho_3=\varrho_0-\varrho_3.$$

The position cf rocket can be alsc unambigucusly defined as intersection of two ellipscids and sphere, two hyperboloids and sphere, two spheres and ellipscid, etc.

Frequently the locaticn cf rccket is defined as point of intersecticn of sphere, ccne and plane (Fig. 14.16). In this case the pcsiticn of rocket will be determined by the spherical coordinates: slant range ρ, by the bearing angles A and cf place ε. Transfer/transition frcm them tc the ccordinates of rectangular system is realized on the dependences

$$x=\varrho\cos\varepsilon\cos A;\quad y=\varrho\sin\varepsilon;\quad z=\varrho\cos\varepsilon\sin A. \qquad (14.25)$$

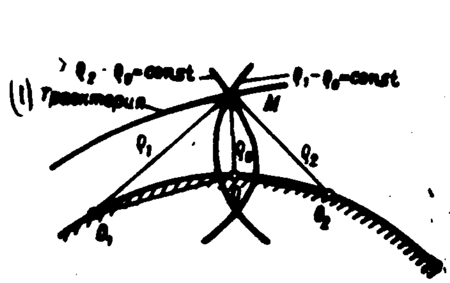
Besides spherical cccrdinates, fcr determining the three-dimensional/space aircraft attitude scmetimes there is applied cylindrical system. In this system they are measured the bearing

angle of object A, the projection of cut OM on plane xOz-l and removing of point M from this plane - h (Fig. 14.17).

Dependences for transfer/transition from cylindrical coordinates to rectangular take the form

$$x = l\cos A;\quad y = h;\quad z = l\sin A. \qquad (14.20)$$

Wide acceptance will obtain the direction-finding method of coordinate determination of object. It lies in the fact that from two tracking stations O_1 and O_2 with the aid of special equipment are measured angular coordinates of body A_1 and A_2, but according to the bearing angles and place of object and the known base distance between B stations are determined the coordinates of point M (Fig. 14.18).



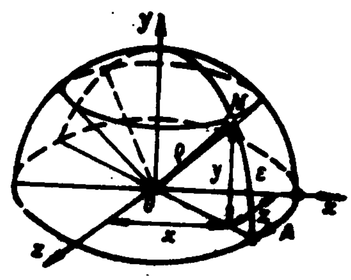


Fig. 14.15. Fig. 14.16.

Fig. 14.15. Differential ranging methcd of pcsition finding of rccket.

Key: (1). Trajectory.

Fig. 14.16. Position finding of rocket in spherical coordinates.

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Frequently for convenience in the calculaticns instead of the azimuths, are determinec angles α_{4}, calculated cff direction in one cf the tracking stations, tc the laurching pcint of the rocket or to ary ancther characteristic I turn/sharpen tle earth's surface. Transfer/transition from these angles tc azimuths (and vice versa) does nct represent difficulties and is fulfilled on simple analytical

degendence. For example, in connection with Fig. 14.18

$$e_i = A_i - A_{B_i},$$

where A_i - angle, calculated from lccal meridian tc direction in target/purpose; A_{B_i} - angle between the lccal meridian and direction frcm pcint/item O_1 in pcint/item O_2.

We will obtain formulas fcr cccrdinate determination of rocket ir connection with the directicr-finding methoc of external measurements. Let the pcint/items cf tracking te arrange/located on plane xOz in the pcints O_1 and O_2, the distance between which is equal to B (Fig. 14.19) [1].

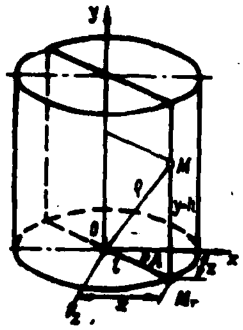
FCCTNOTE [1]. The topographic excess cf the measuring posts above plane xcz intc our conclusions it is nct censidered. ENDFOOTNOTE.

As main direction for the reading cf herizcrtal angles, let us select the line, determined by the axis O_1x cf earth-tased coordinate system which passes through pcints C_1 and C_2.

For simplicity we ccrsider that axis C_1 ccincides with direction sever - Yug; in this case, the measured angles will be azimuths.

Let us designate distarce O_1M through l_1 and O_2M through l_2. Frcm ΔO_1MO_2 we have

$$l_1 = \frac{B}{\sin(A_2 - A_1)} \sin A_2; \quad l_2 = \frac{B}{\sin(A_2 - A_1)} \sin A_1.$$



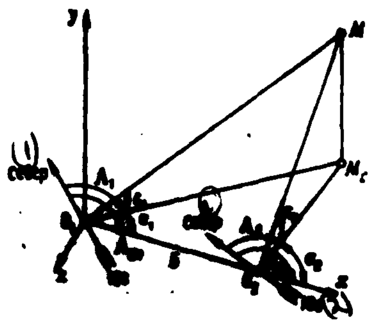


Fig. 14.17. Fig. 14.18.

Fig. 14.17. Position finding of rocket in cylindrical coordinate system.

Fig. 14.18. Angular coordinates, used in direction-finding method for position finding of rocket.

Key: (1). North. (2). South.

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Taking into account this for the coordinates of point M in system O_1xyz, we will obtain

$$\left.\begin{aligned} x&=\frac{B}{\sin(A_2-A_1)}\sin A_2\cos A_1;\\ y&=\frac{B\sin A_2}{\sin(A_2-A_1)}\operatorname{tg}\varepsilon_1=\frac{B\sin A_1}{\sin(A_2-A_1)}\operatorname{tg}\varepsilon_2;\\ z&=\frac{B}{\sin(A_2-A_1)}\sin A_2\sin A_1.\end{aligned}\right\}\qquad(14.27)$$

Values x, y, z can be given to another form, if one considers that

$$\sin A_1 = \frac{z}{y}\,\mathrm{tg}\,\varepsilon_1; \qquad (14.28)$$

$$\sin A_2 = \frac{z}{y}\,\mathrm{tg}\,\varepsilon_2. \qquad (14.29)$$

After substituting (14.28) and (14.29) into formula (14.27), we will obtain

$$\left.\begin{aligned} x &= \frac{B\,\mathrm{tg}\,\varepsilon_2 \cos A_1}{\mathrm{tg}\,\varepsilon_2 \cos A_1 - \mathrm{tg}\,\varepsilon_1 \cos A_2};\\ y &= \frac{B\,\mathrm{tg}\,\varepsilon_2\,\mathrm{tg}\,\varepsilon_1}{\mathrm{tg}\,\varepsilon_2 \cos A_1 - \mathrm{tg}\,\varepsilon_1 \cos A_2};\\ z &= \frac{B\,\mathrm{tg}\,\varepsilon_2 \sin A_1}{\mathrm{tg}\,\varepsilon_2 \cos A_1 - \mathrm{tg}\,\varepsilon_1 \cos A_2} = \frac{B\,\mathrm{tg}\,\varepsilon_1 \sin A_2}{\mathrm{tg}\,\varepsilon_2 \cos A_1 - \mathrm{tg}\,\varepsilon_1 \cos A_2}. \end{aligned}\right\} \qquad (14.30)$$

Together with the direction-finding method of determining the position of rocket, is applied also the combined ranging-direction-finding method, in which they are measured slant range and angular coordinates - angles of azimuth and place of target/purpose.

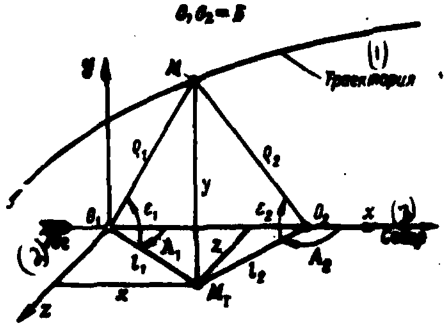

Fig. 14.19. Schematic tc the derivaticn of relationship/ratios for a direction-finding method.

Key: (1). Trajectory. (2). South. (3). North.

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Thus, all the existing methods cf the external measurements are reduced in essence to the measurement cf slant range and the angular coordinates, which determine directicr in rccket. At tracking stations, these measurements can be conducted with the aid of optical and radio equipment.

§4. Optical measurements.

When conducting of external measurements, very widely is

utilized optical equipment (cinetheodolites and cinetelescopes, ballistic cameras, apparatuses for high-speed/velocity photography, etc.), which for this purpose was adopted considerably earlier than radio engineering.

The basic advantages of optical measurements in comparison with radio engineering are their clarity, since optical equipment makes it possible directly visually to control the process of moving the flight vehicle and to photograph it for the subsequent analysis (for example, with missile takeoff, stage separation, the interception of target/purpose, etc.), and the high accuracy of the determination of angular coordinates of object.

Therefore optical equipment part is utilized for calibrating the radio engineering measuring systems. Thus, for instance, interferometric system "Minitrek" is calibrated by optical instruments with an accuracy to 2" [16]. At the same time the work of optical equipment lends itself worse to automation and it depends substantially on the state of the atmosphere at the moment of the measurements which, as a rule, are conducted by comparatively small distances to flight vehicle. In view of this optical measurements do not eliminate radio engineering; they successfully supplement each other.

For conducting the external measurements on the individual sections of the missile trajectories of long-range, for the photographing of short trajectories, for scoring of air-to-air missiles and the "earth/ground - air" is applied the cinetheodolite method which makes it possible to find not only three-dimensional/space coordinates of flight vehicle, but also its velocity, but in certain cases also acceleration and a series of other parameters of motion. It realizes the direction-finding method of measurements and lies in the fact that from two (or three) cinetheodolite posts they simultaneously conduct tracking the motion of one object, they photograph it and determine its angular coordinates (azimuth and angle of elevation) in the function of time. As a rule, in cinetheodolite station enter three posts of tracking. One Of them fulfills auxiliary function and can be used for the control of coordinate determination of object.

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Furthermore, simultaneous tracking from three cinetheodolite stations makes it possible to exclude random measuring errors with the cut of object at acute angles, and also during photography against the sun.

Basic part of any cinetheodolite - the main thing 1 and sighting 2 telescopes (Fig. 14.20) whose optical axes are parallel. Sighting

telescopes (sights) play auxiliary role and serve for the facilitation of the process of the guidance of the optical axis of main (measuring) telescope for the objective of photography.

Usually cinetheodolite for the increase of the accuracy of the tracking is serviced by two operators one of which aims main telescope at flight vehicle along azimuth, and another - on angle of elevation. The position of the object of the relatively optical axis of main telescope with the aid of movie camera 3 continuously is detented to motion picture film. The gates of the camera/chambers of all cinetheodolites of one station are synchronized between themselves with the aid of special electrical circuit with high degree accuracy, and entire/all photography it is conducted automatically. Thus, the problem of operators is reduced only to visual tracking the object and the retention of object on the optical axis of main telescope (in the center of cross lines).

The photographed motion picture film includes entire basic information about the motion of flight vehicle, written automatically in the process of tracking.

Fig. 14.20. Cinethecdolite.

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Cn each exposure are reccrded the ccrditic al index of the cinetheodolite, frcm which is conducted the cbservation, the number cf frame for joining it to time, the referer ce grid (or cross lines), which determines the position of the optical axis of main telescope, the image of object and the goniometrical scales with the fixed values of azimuth and angle of elevation, which determine the direction of the optical axis of main telescope.

Figures 14.21 gives exposure, photographed in the process of

cinetheodolite measurements. In left lower angle is photographed the gonicmetrical scale for the reading of azimuth, halfway - the scale for the reading of angles of elevation, while in right lower to angle are given the number of exposure and the index of theodolite. The reading of angles is conducted on the indicators which are establish/installed against the appropriate divisions of the goniometrical scales. As a rule, exposure counter initially to zero it is not establish/installed; therefore for the designation of the beginning of the synchronous working of all three cinetheodolites, is applied the special mark of personnel/frames.

At the moment of the "grip/capture" of object by the main telescope of cinetheodolite, and also in the process of tracking the fast moving target/purposes due to the possible errors for operators the image of object and the center of cross lines of reference grid, as a rule, do not coincide. This speaks about the fact that the object of photography is displaced relative to the optical axis of telescope and for the precision determination of its angular coordinates with the interpretation of film should introduce corrections into the bearing angles and place.

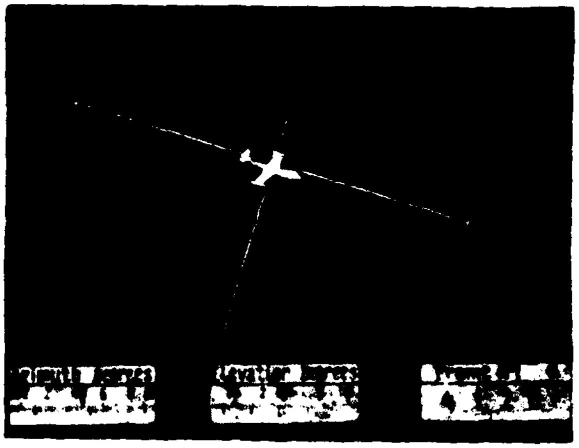

Fig. 14.21. Exposure, photographed with the aid of cinetheodolite.

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The signs of corrections depend on the location of object of relatively reference grid. Thus, for instance, if object is located in the first fourth, angular corrections have plus sign, if in the third, then corrections must be deducted from readings of the goniometrical scales, etc.

For decreasing the following errors, the guidance of cinetheodolites to moving object can be realized and it is automatically - with the aid of radar stations with those errors which characterize radar methods of external trajectory measurements.

The higher accuracy of tracking provide the operators, which correct the rotation of main telescope with the aid of hand drives. In a number of cases there is also applied the preset guidance of cinetheodolites to target/purpose. The accuracy of the cinetheodolite method of measurements depends substantially on the synchronism of the work of all theodolites and frequency stability of photography. For the possibility of the introduction of corrections for lack of synchronation, is conducted recording the torque/moments of the complete opening of the gates of cinetheodolites with the aid of high-precision chronographs.

Very laborious process/operation is the decoding of the photographed motion picture films, which consists of removal/taking of readings from the goniometrical scales for the bearing angles and place, and also in the determination of corrections for the noncoincidence of the image of object with the optical axis of cinetheodolite. Decoding is frequently fulfilled by hand with the aid of the instrument, called comparator. It consists of microscope and two measuring grids. The measuring grids are installed to the carriage which can be moved in two mutually perpendicular directions. The position of measuring grids relative to the exposure is determined with the aid of the special scales which can be graduated in the portions of degrees.

Those considered the goniometrical dial/limbs of each frame of the value of azimuths A_{xi} and of the angles of elevation ε_{xi}, and also of correction (taking into account sign) into these angles ΔA_{xi} and $\Delta\varepsilon_{xi}$ for the noncoincidence of the image of object the optical axis of main telescope (with the center of cross lines) usually will be brought in into the special service record (table 14.2), in which are calculated also the values of true azimuths $A_i = A_{xi} + \Delta A_{xi}$ and of angles of elevation $\varepsilon_i = \varepsilon_{xi} + \Delta\varepsilon_{xi}$.

ab For accelerating the process of the decoding of motion picture films, are applied also the methods of the semiautomatic and automatic reading of readings of the goniometrical scales (in the presence of special equipment), in this case, the angular coordinates are automatically recorded on punched cards or on magnetic tape.

In terms of the values of angles $A_i(t)$ and $\varepsilon_i(t)$, smoothed with the aid of the method of least squares [10], and with known base B can be designed on formulas (14.27) or (14.30) the values of coordinates x(t), y(t), and z(t). Similar calculation is conveniently fulfilled on form, presented below (table 14.3).

The comprising velocities of the motion of rocket along the axes of coordinates Ox, Oy, Oz are located by the numerical differentiation

$$v_x = \frac{\Delta x}{\Delta t}; \quad v_y = \frac{\Delta y}{\Delta t} \text{ и } v_z = \frac{\Delta z}{\Delta t}.$$

Full speed is calculated from the dependence

$$v = \sqrt{v_x^2 + v_y^2 + v_z^2}.$$

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Table 14.2. The angular coordinates, obtained with the aid of cinetheodolites No. 1 and No. 2 [1].

FOOTNOTE [1]. In an example of calculation (table 14.2 and 14.3) is used conditional assay. ENDFOOTNOTE.

(1) Пуск №________

(2) № кинотеодолита	(3) № кадра / (4) углы	421	422	423	424	....	750
1	$A_{д1}$	42°17′					
	$\Delta A_{д1}$	+2′					
	A_1	42°19′					
	$\varepsilon_{д1}$	27°49′					
	$\Delta\varepsilon_{д1}$	−3′					
	ε_1	27°46′					
2	$A_{д2}$	109°4′					
	$\Delta A_{д2}$	−1′					
	A_2	109°3′					
	$\varepsilon_{д2}$	36°26′					
	$\Delta\varepsilon_{д2}$	−2′					
	ε_2	36°24′					

Key: (1). Launching/starting. (2). cinetheodolite. (3). frame. (4). angles.

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Analogously can be found the acceleration of object as derivative of velocity. However, repeated differentiation does not provide the necessary accuracy of calculations and therefore, as a rule, it is not applied. For determination of acceleration, more frequently are utilized the direct methods of measurements with the aid of special sensors. The obtained values of the full speed v and its comprising v_x, v_y, v_z make it possible to determine the angular position of the velocity vector of rocket of relatively earth-based coordinate system. Angles θ and Ψ are found by the formulas

$$\sin\theta = v_y/v \text{ and } \operatorname{tg}\Psi = -v_z/v_x,$$

obtained from (2.4).

In principle it is possible to find also such parameters of motion as g-forces, angles of attack and slip, but the accuracy of their determination will be low.

Cinetheodolite measurements are applied for determining the parameters of the motion of objects at short distances. The possibilities of optical measurements can be expanded during the use of the telephotocine cameras.

table 14.3. Calculation of the coordinates of object.

Б=6000 м (1) Пуск №

(2) № строки	t c	0	t_1	t_2	t_n
1	A_2-A_1	66°44′			
2	$\sin(A_2-A_1)$	0,9186			
3	Б:[2]	6533			
4	$\sin A_1$	0,6732			
5	$\cos A_1$	0,7394			
6	$\sin A_2$	0,9452			
7	$\mathrm{tg}\,\varepsilon_1$	0,5265			
8	$\mathrm{tg}\,\varepsilon_2$	0,7373			
9	[3]·[6]	6175			
10	$x=$ [9]·[5] м	4564			
11	$z=$ [9]·[4] м	4157			
12	$y_1=$ [9]·[7] м	3251			
13	$y_2=$ [3]·[4]·[8] м	3243			
14	$y=\frac{1}{2}[y_1+y_2]$ м	3247			

Here numerals in the brackets conditionally designated the values, which stand in the appropriate table rows.

Key: (1). Launching/starting. (2). line.

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Such telescopes are establish/installed on special base/roots and have more powerful optics. They are applied for recording the processes of the stage separation, interception and other phenomena, which occur at considerable removal/distance from observation

station. Figures 14.22 gives the general view of the American cinetelescope ROTI, which makes it possible to measure the angles with accuracy (20-30)".

Optical measurements widely are utilized for tracking the artificial Earth satellites, the space stations and the ships. For this, is conducted the photographing of object against the background of the stars whose position on celestial sphere is known with high degree of accuracy. For the photographing of such strongly distant objects, can be applied the cinetelescopes and the ballistic camera/chambers which, unlike the first, are fixed, but they have the large field of view.

With the aid of the oriented on stars ballistic camera/chambers it is possible to determine angles with accuracy 3" -5".

Further development of optical measurements occurs over the path of an increase in the distance of photography, accuracy of determination of angular coordinates and automation of readout.

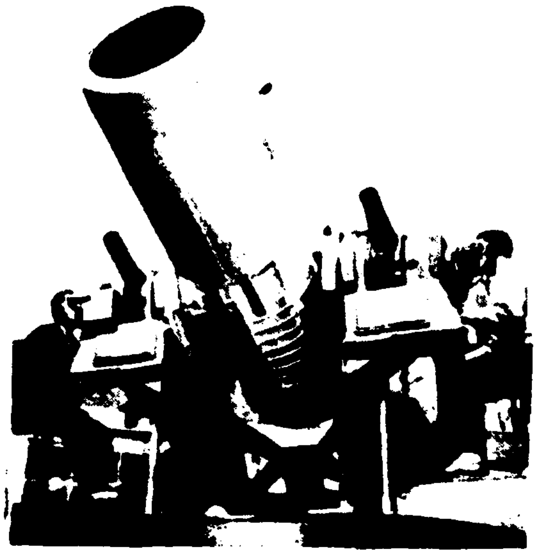

Fig. 14.22. Cinetelescope.

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Recently together with optical equipment increasingly more widely begins to be utilized infrared technology. The instruments of infrared vision are applied when is hinder/tampered the use of optical or electronic equipment - for example, for the study of the initial moment of missile takeoff when the usual photographing of rocket is impossible due to a large quantity of dust and smoke; they successfully are utilized for coordinate determination of object, entering the dense layers of the atmosphere, and in a series of other

cases.

The methods of trajectory measurements with the aid of radio-technical resources, the methods of parameter determination of the trajectories of ballistic missiles from the data of radar measurements, the methods of measuring the error on the flying target/purpose and the determinations of the errors for external measurements are presented in works [25] and [61]. The generalization of the named questions is partially carried out in work [18].

Pages 561-577.

~~Application~~ /Appendix.

Tables of the characteristics of the point of firing engine on the first passive phase of complex trajectory.

e_0	(1) Значения θ_0° при v_0 м/с (5)											
	50	100	150	200	250	300	350	400	450	500	550	600
						$\alpha=0,05$						
0,0	45,0	45,0	45,0	45,0	45,0	45,0	45,0	45,0	45,0	45,0	45,0	45,0
0,1	45,2	45,5	45,7	45,9	46,0	46,1	46,1	46,1	46,1	46,1	46,1	46,1
0,2	45,5	46,1	46,4	46,3	47,1	47,3	47,3	47,3	42,3	47,3	47,3	47,3
0,3	45,8	46,6	47,1	47,7	48,1	48,4	48,4	48,4	48,4	48,4	48,4	48,4
0,4	46,0	47,0	47,8	48,5	49,2	49,6	49,6	49,6	49,6	49,5	49,6	49,6
0,5	46,3	47,5	48,5	49,4	50,2	50,7	50,7	50,7	50,7	50,7	50,7	50,7
0,6	46,5	47,9	49,2	50,2	51,3	51,8	51,8	51,8	51,8	51,8	51,8	51,8
0,7	46,8	48,3	49,8	51,0	52,2	52,8	52,8	52,8	52,8	52,8	52,8	52,8
0,8	47,0	48,6	50,2	51,7	52,9	53,5	53,5	53,5	53,5	53,5	53,5	53,5
0,9	47,1	48,9	50,5	52,1	53,3	54,0	54,0	54,0	54,0	54,0	54,0	54,0
1,0	47,1	49,0	50,7	52,3	53,5	54,2	54,2	54,2	54,2	54,2	54,2	54,2
1,1	47,1	49,0	50,7	52,3	53,5	54,2	54,2	54,2	54,2	54,2	54,2	54,2
1,2	47,1	49,0	50,7	52,3	53,5	54,2	54,2	54,2	54,2	54,2	54,2	54,2
1,3	47,1	49,0	50,7	52,3	53,5	54,2	54,2	54,2	54,2	54,2	54,2	54,2
1,4	47,1	49,0	50,7	52,3	53,5	54,2	54,2	54,2	54,2	54,2	54,2	54,2
1,5	47,1	49,0	50,7	52,3	53,5	54,2	54,2	54,2	54,2	54,2	54,2	54,2
						$\alpha=0,10$						
0,0	45,0	45,0	45,0	45,0	45,0	45,0	45,0	45,0	45,0	45,0	45,0	45,0
0,1	45,5	45,9	46,3	46,6	47,0	47,1	47,1	47,1	47,1	47,1	47,1	47,1
0,2	46,0	46,8	47,6	48,3	48,9	49,1	49,1	49,1	49,1	49,1	49,2	49,2
0,3	46,5	47,8	48,9	50,0	50,8	51,2	51,2	51,2	51,2	51,2	51,3	51,3
0,4	47,0	48,7	50,2	51,6	52,7	53,2	53,2	53,2	53,2	53,2	53,3	53,3
0,5	47,5	49,6	51,5	53,3	54,6	55,2	55,2	55,3	55,3	55,3	55,4	55,4

ε_0	Значения θ_0° при v_0 м/с											
	50	100	150	200	250	300	350	400	450	500	550	600
0,6	47,9	50,4	52,7	54,7	56,3	56,9	56,9	57,0	57,0	57,0	57,1	57,1
0,7	48,3	51,1	53,7	55,8	57,6	58,3	58,3	58,4	58,4	58,4	58,4	58,4
0,8	48,5	51,6	54,4	56,7	58,5	59,4	59,4	59,4	59,4	59,4	59,4	59,4
0,9	48,6	51,9	54,8	57,3	59,1	60,0	60,0	60,0	60,0	60,0	60,0	60,0
1,0	48,7	52,1	55,0	57,6	59,4	60,2	60,2	60,2	60,2	60,2	60,2	60,2
1,1	48,7	52,1	55,0	57,6	59,4	60,2	60,2	60,2	60,2	60,2	60,2	60,2
1,2	48,7	52,1	55,0	57,6	59,4	60,2	60,2	60,2	60,2	60,2	60,2	60,2
1,3	48,7	52,1	55,0	57,6	59,4	60,2	60,2	60,2	60,2	60,2	60,2	60,2
1,4	48,7	52,1	55,0	57,6	59,4	60,2	60,2	60,2	60,2	60,2	60,2	60,2
1,5	48,7	52,1	55,0	57,6	59,4	60,2	60,2	60,2	60,2	60,2	60,2	60,2
						$\alpha=0,15$						
0,0	45,0	45,0	45,0	45,0	45,0	45,0	45,0	45,0	45,0	45,0	45,0	45,0
0,1	45,7	46,2	46,7	47,1	47,4	47,4	47,5	47,6	47,6	47,7	47,7	47,8
0,2	46,3	47,4	48,4	49,3	49,8	49,9	50,0	50,1	50,2	50,3	50,4	50,6
0,3	47,0	48,7	50,2	51,5	52,2	52,3	52,4	52,6	52,8	53,0	53,1	53,3
0,4	47,6	49,9	51,9	53,6	54,5	54,6	54,8	55,1	55,4	55,5	55,7	55,9
0,5	48,3	51,0	53,6	55,6	56,8	57,0	57,2	57,5	57,7	57,9	58,1	58,3
0,6	48,9	52,0	55,0	57,3	58,7	58,9	59,1	59,3	59,4	59,6	59,8	59,9
0,7	49,4	52,9	56,1	58,6	60,1	60,3	60,4	60,5	60,6	60,7	60,8	60,8
0,8	49,8	53,6	56,9	59,4	61,0	61,1	61,2	61,2	61,3	61,3	61,3	61,3
0,9	50,0	54,0	57,5	59,9	61,4	61,5	61,5	61,5	61,5	61,5	61,5	61,5
1,0	50,0	54,2	57,8	60,2	61,5	61,6	61,6	61,6	61,6	61,6	61,6	61,6
1,1	50,0	54,2	57,8	60,2	61,5	61,6	61,6	61,6	61,6	61,6	61,6	61,6
1,2	50,0	54,2	57,8	60,2	61,5	61,6	61,6	61,6	61,6	61,6	61,6	61,6
1,3	50,0	54,2	57,8	60,2	61,5	61,6	61,6	61,6	61,6	61,6	61,6	61,6
1,4	50,0	54,2	57,8	60,2	61,5	61,6	61,6	61,6	61,6	61,6	61,6	61,6
1,5	50,0	54,2	57,8	60,2	61,5	61,6	61,6	61,6	61,6	61,6	61,6	61,6
						$\alpha=0,20$						
0,0	45,0	45,0	45,0	45,0	45,0	45,0	45,0	45,0	45,0	45,0	45,0	45,0
0,1	45,7	46,3	47,0	47,5	47,6	47,9	48,1	48,3	48,4	48,4	48,6	48,8
0,2	46,4	47,7	49,0	49,9	50,3	50,8	51,1	51,5	51,7	51,8	52,1	52,6
0,3	47,1	49,1	50,9	52,3	52,9	53,5	53,9	54,4	54,7	55,0	55,4	56,1
0,4	47,9	50,5	52,8	54,5	55,3	56,0	56,5	57,0	57,5	57,9	58,4	59,1

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c_0	Значения θ_0° при v_0 м/с											
	50	100	150	200	250	300	350	400	450	500	550	600
0,5	48,6	51,8	54,6	56,5	57,5	58,2	58,8	59,3	59,8	60,3	60,3	61,3
0,6	49,3	53,0	56,2	58,3	59,4	60,0	60,6	61,0	61,5	61,8	62,1	62,3
0,7	50,0	54,0	57,5	59,7	60,8	61,3	61,8	63,1	62,4	62,6	62,7	62,8
0,8	50,5	54,8	58,5	60,7	61,8	62,3	62,6	62,7	62,9	62,9	62,9	63,0
0,9	50,9	55,3	59,2	61,3	62,4	62,9	63,0	63,0	63,1	63,0	63,0	63,1
1,0	51,0	55,5	59,5	61,5	62,6	63,1	63,1	63,1	63,1	63,1	63,1	63,1
1,1	51,0	55,5	59,5	61,5	62,6	63,1	63,1	63,1	63,1	63,1	63,1	63,1
1,2	51,0	55,5	59,5	61,5	62,6	63,1	63,1	63,1	63,1	63,1	63,1	63,1
1,3	51,0	55,5	59,5	61,5	62,6	63,1	63,1	63,1	63,1	63,1	63,1	63,1
1,4	51,0	55,5	59,5	61,5	62,6	63,1	63,1	63,1	63,1	63,1	63,1	63,1
1,5	51,0	55,5	59,5	61,5	62,6	63,1	63,1	63,1	63,1	63,1	63,1	63,1

$\alpha = 0{,}25$

0,0	45,0	45,0	45,0	45,0	45,0	45,0	45,0	45,0	45,0	45,0	45,0	45,0
0,1	45,6	46,2	46,8	47,2	47,8	48,0	48,3	48,7	49,2	49,6	50,3	51,5
0,2	46,3	47,5	48,6	49,5	50,5	51,1	51,6	52,3	53,2	54,1	55,4	57,7
0,3	47,0	48,8	50,4	51,7	53,2	54,2	54,8	55,8	56,8	58,1	59,5	61,5
0,4	47,6	50,0	52,2	54,0	55,8	57,2	57,8	58,9	59,9	61,0	62,1	63,5
0,5	48,3	51,3	54,0	56,2	58,2	59,7	60,4	61,2	62,0	62,8	63,5	64,3
0,6	49,0	52,5	55,8	58,4	60,3	61,6	62,2	62,8	63,2	63,7	64,1	64,3
0,7	49,7	53,8	57,4	60,2	62,0	63,0	63,3	63,7	63,9	64,1	64,3	64,3
0,8	50,3	54,9	58,7	61,5	63,1	63,9	64,0	64,2	64,3	64,3	64,4	64,3
0,9	50,8	55,7	59,4	62,3	63,7	64,4	64,4	64,4	64,5	64,4	64,4	64,3
1,0	51,0	56,0	59,8	62,6	64,0	64,6	64,6	64,5	64,5	64,4	64,4	64,3
1,1	51,0	56,0	59,8	62,6	64,0	64,6	64,6	64,5	64,5	64,4	64,4	64,3
1,2	51,0	56,0	59,8	62,6	64,0	64,6	64,6	64,5	64,5	64,4	64,4	64,3
1,3	51,0	56,0	59,8	62,6	64,0	64,6	64,6	64,5	64,5	64,4	64,4	64,3
1,4	51,0	56,0	59,8	62,6	64,0	64,6	64,6	64,5	64,5	64,4	64,4	64,3
1,5	51,0	56,0	59,8	62,6	64,0	64,6	64,6	64,5	64,5	64,4	64,4	64,3

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c_0	Значения θ_0° при v_0 м/с											
	50	100	150	200	250	300	350	400	450	500	550	600
						$\alpha=0{,}30$						
0,0	45,0	45,0	45,0	45,0	45,0	45,0	45,0	45,0	45,0	45,0	45,0	45,0
0,1	45,8	46,5	47,2	47,7	48,3	49,1	49,4	49,8	50,2	51,2	52,5	54,0
0,2	46,6	48,0	49,3	50,2	51,6	53,0	53,6	54,4	55,1	56,7	58,9	61,5
0,3	47,4	49,5	51,4	52,9	54,8	56,4	57,4	58,4	59,4	61,1	63,2	65,4
0,4	48,2	51,0	53,5	55,5	57,7	59,3	60,5	61,6	62,6	64,0	65,6	67,1
0,5	49,0	52,4	55,4	57,9	60,2	61,7	62,7	63,3	64,7	65,8	66,8	67,9
0,6	49,8	53,8	57,1	60,0	62,2	63,6	64,3	65,1	65,8	66,6	67,3	67,9
0,7	50,6	54,9	58,5	61,6	63,7	64,8	65,3	65,8	66,4	66,9	67,5	67,9
0,8	51,3	55,7	59,5	62,7	64,6	65,5	65,8	66,4	66,7	67,1	67,5	67,9
0,9	51,8	56,2	60,0	63,1	65,0	65,8	66,1	66,5	66,9	67,2	67,5	67,9
1,0	52,0	56,3	60,2	63,2	65,1	65,9	66,2	66,5	66,9	67,2	67,5	67,9
1,1	52,0	56,3	60,2	63,2	65,1	65,9	66,2	66,5	66,9	67,2	67,5	67,9
1,2	52,0	56,3	60,2	63,2	65,1	65,9	66,2	66,5	66,9	67,2	67,5	67,9
1,3	52,0	56,3	60,2	63,2	65,1	65,9	66,2	66,5	66,9	67,2	67,5	67,9
1,4	52,0	56,3	60,2	63,2	65,1	65,9	66,2	66,5	66,9	67,2	67,5	67,9
1,5	52,0	56,3	60,2	63,2	65,1	65,9	66,2	66,5	66,9	67,2	67,5	67,9

c_0	Значения $Y_м$ с при v_0 м/с											
	50	100	150	200	250	300	350	400	450	500	550	600
						$\alpha=0{,}05$						
0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0
0,1	0,0	0,1	0,2	0,5	0,6	0,8	1,0	1,1	1,2	1,4	1,5	1,7
0,2	0,2	0,5	0,8	1,1	1,5	1,8	2,2	2,4	2,7	2,9	3,2	3,5
0,3	0,6	1,0	1,4	2,0	2,5	2,9	3,4	3,8	4,1	4,4	4,7	5,1
0,4	0,7	1,4	2,0	2,7	3,3	3,8	4,4	4,9	5,3	5,7	6,1	6,6
0,5	0,8	1,6	2,4	3,2	3,9	4,5	5,0	5,6	6,0	6,6	7,1	7,6
0,6	0,8	1,7	2,5	3,3	4,1	4,8	5,3	5,9	6,5	7,1	7,6	8,2
0,7	0,9	1,8	2,6	3,5	4,3	5,0	5,6	6,1	6,8	7,4	8,0	8,6
0,8	0,9	1,9	2,7	3,6	4,5	5,2	5,8	6,3	7,0	7,6	8,3	9,0
0,9	1,0	2,0	2,9	3,8	4,7	5,5	6,1	6,6	7,2	7,8	8,5	9,2
1,0	1,0	2,0	3,0	4,0	4,9	5,7	6,3	6,8	7,4	8,0	8,6	9,3

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e_0	(1) Значения $t_н$ с при v_0 м/с											
	50	100	150	200	250	300	350	400	450	500	550	600
1,1	1,1	2,1	3,1	4,2	5,1	6,0	6,6	7,0	7,5	8,0	8,6	9,3
1,2	1,1	2,2	3,2	4,4	5,3	6,3	6,8	7,2	7,6	8,1	8,5	9,2
1,3	1,2	2,3	3,4	4,5	5,5	6,5	7,1	7,5	7,8	8,1	8,5	9,0
1,4	1,2	2,4	3,5	4,7	5,7	6,7	7,3	7,7	7,9	8,2	8,4	8,8
1,5	1,3	2,5	3,7	4,9	5,9	7,0	7,6	7,9	8,0	8,2	8,3	8,5

$\alpha=0,10$

0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0
0,1	0	0,2	0,5	0,8	1,3	1,6	1,8	2,1	2,4	2,6	2,9	3,2
0,2	0,4	0,9	1,5	2,0	2,7	3,3	4,0	4,5	4,9	5,3	5,7	6,2
0,3	0,8	1,7	2,4	3,2	4,1	5,0	6,0	6,5	7,1	7,5	8,0	8,7
0,4	1,1	2,2	3,2	4,3	5,2	6,4	7,4	8,0	8,7	9,3	10,1	10,8
0,5	1,3	2,5	3,7	5,0	6,1	7,3	8,3	9,1	10,0	10,8	11,5	12,3
0,6	1,4	2,7	4,1	5,4	6,7	7,9	9,0	9,9	10,8	11,7	12,5	13,3
0,7	1,5	2,8	4,3	5,7	7,1	8,4	9,5	10,5	11,4	12,3	13,1	14,0
0,8	1,5	2,9	4,5	5,9	7,4	8,8	9,9	10,9	11,8	12,7	13,5	14,4
0,9	1,6	3,1	4,6	6,1	7,6	9,1	10,2	11,2	12,1	12,9	13,8	14,6
1,0	1,7	3,2	4,8	6,3	7,9	9,4	10,5	11,4	12,2	13,0	13,9	14,7
1,1	1,8	3,3	4,9	6,5	8,2	9,7	10,8	11,6	12,2	13,0	13,9	14,6
1,2	1,8	3,4	5,1	6,8	8,4	10,0	11,1	11,7	12,3	13,0	13,8	14,4
1,3	1,9	3,5	5,3	7,0	8,7	10,4	11,3	11,9	12,4	12,9	13,6	14,1
1,4	1,9	3,7	5,4	7,2	8,9	10,7	11,6	12,1	12,4	12,9	13,4	13,8
1,5	2,0	3,8	5,6	7,4	9,2	11,0	11,9	12,2	12,5	12,8	13,3	13,4

$\alpha=0,15$

0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0
0,1	0,1	0,3	0,6	1,1	1,5	1,8	2,2	2,6	2,9	3,4	3,7	4,2
0,2	0,5	1,0	1,6	2,3	3,0	3,7	4,3	5,0	5,7	6,3	7,1	8,4
0,3	1,0	1,9	2,7	3,6	4,8	5,5	6,3	7,3	8,2	9,2	10,1	11,5
0,4	1,3	2,5	3,5	4,8	6,0	7,0	8,1	9,3	10,3	11,4	12,6	13,8
0,5	1,5	2,8	4,1	5,5	6,9	8,2	9,4	10,6	11,7	12,9	14,0	15,2
0,6	1,6	3,0	4,5	6,0	7,5	9,0	10,0	11,4	12,5	13,7	14,7	15,9
0,7	1,6	3,2	4,7	6,3	7,9	9,5	10,4	11,9	12,9	14,1	15,1	16,2
0,8	1,7	3,3	4,9	6,6	8,2	9,9	10,8	12,1	13,1	14,3	15,3	16,4

e_0	Значения $t_н$ с при v_0 м/с											
	50	100	150	200	250	300	350	400	450	500	550	600
0,9	1,7	3,4	5,1	6,8	8,5	10,3	11,1	12,3	13,3	14,3	15,3	16,3
1,0	1,8	3,5	5,2	7,0	8,7	10,5	11,4	12,4	13,3	14,3	15,2	16,2
1,1	1,8	3,6	6,3	7,2	8,9	10,7	11,6	12,4	13,3	14,2	15,0	16,0
1,2	1,9	3,7	5,4	7,3	9,1	10,9	11,8	12,4	13,2	14,0	14,7	15,6
1,3	1,9	3,8	5,5	7,4	9,2	11,1	11,9	12,5	13,1	13,8	14,4	15,2
1,4	2,0	3,8	5,7	7,6	9,4	11,3	12,0	12,5	13,0	13,6	14,1	14,7
1,5	2,0	3,9	5,8	7,7	9,6	11,5	12,1	12,5	12,8	13,3	13,7	14,2

α=0,20

0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0
0,1	0,1	0,3	0,8	1,3	1,7	2,2	2,7	3,3	3,8	4,3	5,0	5,7
0,2	0,4	0,9	1,8	2,6	3,4	4,4	5,4	6,4	7,4	8,3	9,3	10,9
0,3	0,9	1,7	2,8	4,1	5,2	6,4	7,6	8,8	10,2	11,5	12,8	14,6
0,4	1,3	2,4	3,7	5,3	6,5	7,9	8,4	10,7	12,1	13,6	15,1	16,7
0,5	1,5	3,0	4,5	6,1	7,7	9,3	10,7	12,1	13,6	15,0	16,5	18,0
0,6	1,7	3,3	4,9	6,7	8,4	10,2	11,7	13,0	14,5	15,7	17,1	18,4
0,7	1,8	3,5	5,3	7,1	8,9	10,8	12,3	13,5	14,8	16,0	17,2	18,5
0,8	1,9	3,7	5,5	7,4	9,3	11,3	12,6	13,8	14,9	16,1	17,1	18,4
0,9	2,0	3,8	5,7	7,6	9,6	11,6	12,8	13,9	14,9	16,0	17,0	18,2
1,0	2,0	3,9	5,9	7,8	9,8	11,8	12,8	13,8	14,8	15,8	16,8	17,8
1,1	2,0	3,9	5,9	7,9	9,9	11,9	12,8	13,7	14,6	15,5	16,5	17,5
1,2	2,0	4,0	6,0	7,9	9,9	11,9	12,7	13,6	14,4	15,2	16,1	17,0
1,3	2,0	4,0	6,0	8,0	10,0	12,0	12,7	13,5	14,2	14,9	15,7	16,4
1,4	2,0	4,0	6,0	8,0	10,0	12,0	12,6	13,3	13,9	14,6	15,2	15,9
1,5	2,0	4,0	6,0	8,0	10,0	12,0	12,6	13,1	13,6	14,2	14,8	15,3

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[redacted]

e_0	Значения $t_н$ с при v_0 м/с											
	50	100	150	200	250	300	350	400	450	500	550	600
						$\alpha = 0,25$						
0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0
0,1	0,1	0,3	0,7	1,1	1,8	2,3	2,9	3,7	4,8	5,8	7,3	9,8
0,2	0,4	1,0	1,6	2,6	3,7	4,7	5,9	7,1	8,6	10,5	12,7	16,0
0,3	1,0	2,9	2.9	4,2	5.6	5.9	8,4	9,9	11,6	14,0	16,0	18,7
0,4	1,5	2,8	3,1	5,7	7,2	8,7	10,4	12,2	14,0	16,1	18,1	20,3
0,5	1,7	3,4	5,1	6,8	8,5	10,3	12,0	13,8	15,6	17,4	19,1	21,0
0,6	1,9	3,8	5,6	7,5	9,4	11,3	12,9	14,7	16,5	18,0	19,4	21,1
0,7	2,0	4,0	6,0	8,0	10,1	12,0	13,5	15,1	16,8	18,1	19,3	20,9
0,8	2,1	4,2	6,3	8,4	10,5	12,5	13,9	15,3	16,8	17,9	19,1	20,5
0,9	2,2	4,3	6,4	8,6	10,7	12.8	14,1	15,3	16,6	17,6	18,8	19,9
1,0	2,2	4,3	6,5	8,7	10,8	13,0	14,1	15,1	16,2	17,2	18,3	19,3
1,1	2,2	4,3	6,5	8,7	10,8	13,0	14,0	14,9	15,9	16,8	17,8	18,7
1,2	2,2	4,3	6,5	8,7	10,8	12,9	13,9	14,7	15,5	16,4	17,3	18,1
1,3	2,1	4,3	6,4	8,6	10,7	12,8	13,7	14,4	15,2	16,0	16,8	17,5
1,4	2,1	4,2	6,3	8,5	10,6	12,7	13,4	14,1	14,8	15,5	16,2	16,9
1,5	2,1	4,2	6,2	8,3	10,4	12,5	13,1	13,7	14,4	15,0	15,6	16,3
						$\alpha = 0,30$						
0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0
0,1	0,1	0,4	0,8	1,4	2,1	3,1	3,9	4,8	5,9	7,8	10,3	13,5
0,2	0,6	1,3	2,2	3,1	4,5	6,0	7,4	9,0	10,8	13,0	15,8	18,6

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c_0	Значения $t_и$ с при v_0 м/с											
	50	100	150	200	250	300	350	400	450	500	550	600
0,3	1,2	2,5	3,9	5,2	6,9	8,7	10,4	12,3	14,5	16,8	19,3	22,0
0,4	1,8	3,4	5,1	6,9	8,8	10,7	12,6	14,5	16,8	18,9	21,4	23,8
0,5	2,0	3,9	5,9	7,8	9,8	11,8	13,8	15,9	18,0	20,1	22,2	24,3
0,6	2,1	4,2	6,4	8,4	10,6	12,7	14,5	16,5	18,4	20,4	22,0	23,7
0,7	2,2	4,4	6,6	8,8	11,0	13,2	14,9	16,7	18,4	20,1	21,4	23,1
0,8	2,3	4,5	6,7	9,0	11,3	13,5	15,1	16,6	18,2	19,6	20,9	22,5
0,9	2,3	4,6	6,8	9,1	11,4	13,7	15,0	16,5	17,9	19,1	20,4	21,9
1,0	2,3	4,6	6,8	9,1	11,4	13,7	14,9	16,2	17,5	18,7	19,9	21,2
1,1	2,3	4,6	6,8	9,0	11,3	13,7	14,7	15,9	17,1	18,2	19,4	20,6
1,2	2,3	4,5	6,7	8,9	11,2	13,5	14,5	15,6	16,7	17,8	18,9	20,0
1,3	2,2	4,4	6,6	8,8	11,0	13,3	14,2	15,2	16,2	17,3	18,4	19,4
1,4	2,2	4,3	6,5	8,7	10,7	13,1	13,9	14,9	15,8	16,9	17,8	18,8
1,5	2,2	4,2	6,3	8,5	10,5	12,8	13,6	14,5	15,4	16,4	17,3	18,2

c_0	Значения $v_и$ м/с при v_0 м/с											
	50	100	150	200	250	300	350	400	450	500	550	600
	$\alpha=0,05$											
0,0	50	100	150	200	250	300	350	400	450	500	550	600
0,1	49	98	146	195	243	292	336	381	425	471	517	562
0,2	48	96	143	190	237	234	323	364	404	446	488	530
0,3	47	94	139	185	231	276	312	349	387	425	463	502
0,4	46	93	136	181	225	268	302	336	372	407	442	478
0,5	45	91	134	177	220	262	293	325	359	392	424	458
0,6	44	89	131	174	215	255	285	315	347	379	409	441
0,7	43	87	129	170	211	248	278	305	337	368	396	427
0,8	42	85	126	167	207	242	271	299	329	358	385	415
0,9	41	83	124	165	203	237	265	293	322	350	377	406
1,0	40	81	122	163	200	232	260	288	316	344	371	400
1,1	39	79	120	160	197	229	256	283	312	340	367	396
1,2	38	78	118	157	193	226	253	280	309	337	364	393
1,3	38	76	116	155	190	224	252	278	307	335	362	391
1,4	37	75	114	152	188	223	251	277	305	333	361	389
1,5	36	74	113	150	186	222	250	276	304	332	360	388

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c_U	Значения $v_к$ м/с при v_0 м/с											
	50	100	150	200	250	300	350	400	450	500	550	600
						$\alpha=0{,}10$						
0,0	50	100	150	200	250	300	350	400	450	500	550	600
0,1	46	94	141	189	236	283	323	362	397	440	476	515
0,2	43	88	133	179	224	267	299	330	358	390	420	447
0,3	41	83	126	170	213	252	279	303	325	349	374	394
0,4	39	79	120	162	203	238	261	280	298	316	336	353
0,5	37	76	115	155	194	225	244	260	275	290	305	320
0,6	36	73	110	149	186	214	229	243	256	268	279	292
0,7	35	71	106	143	179	204	217	228	239	250	257	268
0,8	34	68	103	138	171	195	207	215	224	234	239	248
0,9	33	66	99	133	165	187	198	204	211	220	225	232
1,0	32	64	96	128	160	180	190	195	201	208	214	220
1,1	31	62	93	124	155	175	185	189	194	201	206	213
1,2	30	60	91	121	151	172	180	186	190	196	201	208
1,3	29	58	89	118	148	169	177	182	187	193	199	205
1,4	29	57	87	116	145	166	174	180	185	191	198	204
1,5	28	56	85	114	142	165	172	178	184	190	197	203
						$\alpha=0{,}15$						
0,0	50	100	150	200	250	300	350	400	450	500	550	600
0,1	46	93	138	185	233	277	317	355	394	432	465	502
0,2	43	86	128	172	217	257	288	317	347	374	398	426
0,3	41	80	120	161	203	239	263	285	308	329	346	367
0,4	39	75	113	152	191	223	242	259	276	292	306	322
0,5	37	71	107	144	180	210	225	238	250	263	275	287
0,6	35	68	102	137	171	199	211	221	230	241	251	260
0,7	33	65	98	131	163	189	200	207	215	234	233	241
0,8	31	62	94	125	156	181	190	196	203	211	219	227
0,9	29	60	91	120	151	174	182	187	194	201	208	216
1,0	28	58	88	116	146	168	175	180	187	193	200	206
1,1	27	56	86	113	142	163	170	175	182	187	194	200
1,2	26	55	84	110	139	160	166	171	178	183	190	196
1,3	26	54	82	108	136	157	163	168	175	181	187	193
1,4	25	53	80	106	134	155	161	166	173	179	185	191
1,5	25	52	78	105	132	153	159	165	171	177	183	189

[redacted]

1124

e_0	Значения $v_к$ м/с при v_0 м/с											
	50	100	150	200	250	300	350	400	450	500	550	600
						$\alpha=0,20$						
0,0	50	100	150	200	250	300	350	400	450	500	550	600
0,1	45	91	138	184	231	276	311	349	384	419	452	483
0,2	41	84	128	170	214	255	280	307	330	355	375	395
0,3	38	78	119	158	199	237	254	274	288	306	319	334
0,4	36	73	111	148	186	221	233	247	257	270	280	292
0,5	34	68	104	139	175	205	216	226	235	244	254	263
0,6	32	65	98	131	165	193	202	210	219	226	236	243
0,7	31	62	93	124	157	183	190	198	206	213	221	228
0,8	30	59	89	118	150	174	181	188	195	202	209	216
0,9	29	56	86	113	143	166	173	180	186	193	199	207
1,0	28	54	83	109	137	159	166	173	179	186	192	199
1,1	27	53	81	106	133	153	160	167	173	180	186	192
1,2	26	52	79	104	130	149	156	162	168	175	181	187
1,3	25	51	77	102	128	147	154	159	165	172	178	183
1,4	25	50	76	101	126	146	152	158	163	170	175	181
1,5	24	50	75	100	125	145	151	157	162	168	173	179
						$\alpha=0,25$						
0,0	50	101	150	200	250	300	350	400	450	500	550	600
0,1	45	91	137	183	228	275	309	339	365	391	412	431
0,2	41	83	126	168	209	252	276	293	306	321	334	347
0,3	38	77	116	155	193	232	248	258	266	275	285	293
0,4	35	72	107	144	180	214	225	232	239	246	253	259
0,5	33	67	100	134	168	198	207	213	220	225	232	238
0,6	31	62	94	125	157	185	193	199	206	211	217	225
0,7	29	58	89	117	147	174	181	188	195	200	207	215
0,8	27	55	84	111	139	165	171	178	186	191	199	206
0,9	26	52	80	106	133	157	163	170	178	183	192	199
1,0	25	50	76	103	128	150	157	164	171	178	185	193
1,1	25	49	74	100	125	145	152	158	165	172	179	187
1,2	24	48	73	98	123	142	149	154	161	167	174	181
1,3	24	48	72	97	122	140	146	152	158	164	170	176
1,4	23	47	71	96	121	139	144	150	155	161	167	172
1,5	23	47	71	95	120	138	143	148	153	159	164	169

[redacted]

1125

e_0	Значения $v_н$ м/с при v_0 м/с											
	50	100	150	200	250	300	350	400	450	500	550	600
	$\alpha=0,30$											
0,0	50	100	150	200	250	300	350	400	450	500	550	600
0,1	44	88	133	177	221	259	285	307	329	353	377	405
0,2	40	78	119	159	196	228	244	254	265	280	396	310
0,3	36	70	108	144	180	204	215	221	228	237	245	254
0,4	33	64	99	132	165	187	194	199	204	211	218	222
0,5	30	60	92	122	153	175	180	185	190	196	201	206
0,6	28	57	87	115	145	167	172	177	182	188	194	200
0,7	27	54	83	110	139	160	166	171	176	183	189	195
0,8	26	52	80	106	134	155	161	166	171	176	184	189
0,9	25	51	77	103	129	151	157	162	167	175	178	184
1,0	24	50	75	100	126	148	153	158	163	168	173	178
1,1	24	49	74	98	123	145	149	154	158	163	167	172
1,2	23	48	73	96	121	142	146	150	154	158	162	166
1,3	23	47	71	95	119	139	143	146	149	153	156	160
1,4	22	46	70	94	117	137	140	142	145	148	150	154
1,5	22	46	69	93	116	135	137	139	141	143	145	148

e_0	Значения $x_н$ м при v_0 м/с											
	50	100	150	200	250	300	350	400	450	500	550	600
	$\alpha=0,05$											
0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0
0,1	1	9	30	60	110	170	230	300	390	480	580	690
0,2	20	40	100	150	200	300	460	670	830	1100	1300	1600
0,3	60	100	170	250	340	490	700	930	1200	1550	2050	2420
0,4	100	150	230	340	450	630	860	1130	1410	1760	2250	2640
0,5	110	180	280	410	540	700	920	1170	1440	1770	2130	2500
0,6	120	210	320	450	580	740	930	1150	1390	1690	2020	2350
0,7	120	240	350	480	610	770	940	1140	1340	1620	1920	2220
0,8	130	260	380	510	640	800	950	1130	1300	1560	1830	2100
0,9	130	270	400	540	670	830	960	1120	1280	1510	1750	1990
1,0	140	280	420	560	690	850	970	1120	1270	1470	1680	1900
1,1	150	290	430	570	710	860	980	1120	1260	1440	1630	1820
1,2	150	300	440	580	720	870	990	1130	1260	1420	1580	1750
1,3	160	300	450	590	730	880	1000	1140	1260	1400	1540	1680
1,4	170	310	460	600	740	890	1020	1150	1270	1390	1500	1610
1,5	170	310	460	600	750	900	1040	1170	1280	1380	1480	1550

[redacted]

1126

e_0	Значения x_n м при v_0 м/с											
	50	100	150	200	250	300	350	400	450	500	550	600
						$\alpha=0{,}10$						
0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0
0,1	2	16	50	110	210	320	440	580	730	900	1090	1290
0,2	30	70	150	300	490	630	820	1070	1300	1760	2200	2700
0,3	90	180	330	520	730	950	1200	1600	1900	2500	3120	3620
0,4	140	270	450	670	890	1170	1450	1810	2250	2670	3150	3720
0,5	170	320	510	720	950	1220	1520	1850	2240	2620	3050	3580
0,6	170	340	530	740	970	1240	1520	1840	2200	2560	2950	3440
0,7	180	360	540	760	990	1250	1520	1830	2160	2510	2860	3310
0,8	180	370	550	770	1000	1260	1520	1820	2130	2450	2770	3180
0,9	180	380	560	780	1010	1270	1520	1800	2090	2390	2690	3060
1,0	180	380	570	790	1020	1280	1520	1780	2050	2330	2610	2950
1,1	180	380	570	800	1030	1280	1510	1760	2010	2270	2530	2840
1,2	180	390	580	810	1040	1290	1510	1740	1970	2210	2460	2740
1,3	180	400	600	820	1050	1290	1500	1720	1930	2150	2390	2640
1,4	190	410	620	840	1060	1300	1490	1700	1890	2100	2320	2540
1,5	210	430	650	860	1080	1300	1480	1680	1860	2060	2250	2450
						$\alpha=0{,}15$						
0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0
0,1	3	20	70	140	260	370	520	710	890	1140	1380	1700
0,2	40	110	230	360	550	700	1000	1300	1730	2070	2600	3550
0,3	120	250	430	630	830	1100	1440	1800	2600	3030	3530	4060
0,4	170	350	590	830	1090	1400	1720	2080	2520	2940	3400	3900
0,5	180	370	570	790	1060	1380	1680	2040	2440	2840	3290	3750
0,6	170	350	550	770	1040	1350	1640	1990	2330	2740	3160	3690
0,7	170	340	540	760	1020	1320	1610	1950	2290	2640	3030	3430
0,8	160	330	530	760	1010	1300	1590	1910	2220	2550	2910	3280
0,9	160	330	520	750	1000	1280	1570	1870	2160	2470	2800	3130
1,0	160	330	520	750	990	1270	1550	1830	2100	2390	2690	3000
1,1	170	320	530	760	1000	1260	1540	1790	2050	2320	2590	2880
1,2	180	340	550	770	1010	1260	1530	1760	2000	2250	2500	2760
1,3	[illegible]	370	[illegible]	790	[illegible]	1270	[illegible]	[illegible]	[illegible]	2190	[illegible]	[illegible]
1,4	[illegible]	410	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	2140	[illegible]	[illegible]
1,5	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1700	[illegible]	[illegible]	[illegible]	[illegible]

[redacted]

1137

e_0	Значения $x_н$ м при v_0 м/с											
	50	100	150	200	250	300	350	400	450	500	550	600
						$\alpha=0{,}20$						
0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0
0,1	3	20	80	170	290	440	640	880	1150	1420	1810	2270
0,2	50	130	270	450	700	1000	1320	1800	2500	3050	3800	4600
0,3	150	300	520	760	1000	1340	1700	2250	2900	3400	3920	4420
0,4	190	400	670	920	1200	1530	1970	2340	2770	3220	3700	4160
0,5	210	460	710	989	1250	1550	1870	2240	2640	3050	3500	3930
0,6	210	430	670	930	1190	1770	1770	2130	2510	2890	3310	3720
0,7	200	410	630	880	1130	1400	1690	2030	2390	2740	3130	3530
0,8	190	390	600	840	1080	1350	1630	1940	2280	2610	2970	3350
0,9	180	380	580	810	1050	1310	1580	1860	2180	2490	2830	3180
1,0	180	370	570	790	1030	1270	1540	1800	2090	2380	2700	3020
1,1	180	360	570	780	1020	1240	1500	1750	2020	2290	2590	2880
1,2	180	360	580	770	1010	1220	1480	1710	1970	2210	2480	2760
1,3	190	370	590	780	1020	1230	1470	1690	1940	2160	2390	2650
1,4	200	390	610	820	1050	1250	1490	1710	1930	2140	2330	2550
1,5	210	430	650	880	1100	1310	1530	1740	1940	2130	2300	2470
						$\alpha=0{,}25$						
0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0
0,1	3	20	70	150	300	450	680	980	1400	1870	2570	3660
0,2	60	120	250	430	750	1130	1600	2300	3000	3680	4540	5760
0,3	130	280	570	910	1250	1750	2150	2800	3250	3720	4340	5030
0,4	200	500	830	1110	1510	1840	2220	2600	3020	3440	3960	4440
0,5	220	520	790	1080	1400	1720	2080	2430	2810	3230	3660	4100
0,6	220	480	720	1010	1300	1610	1950	2290	2650	3040	3430	3850
0,7	210	440	670	950	1220	1510	1830	2160	2500	2860	3220	3620
0,8	190	410	630	890	1140	1420	1720	2040	2370	2700	3040	3400
0,9	180	390	600	840	1080	1340	1630	1930	2250	2560	2880	3210
1,0	180	370	570	790	1030	1270	1550	1840	2140	2440	2750	3050
1,1	180	360	560	770	990	1240	1500	1780	2060	2340	2630	2900
1,2	180	370	550	760	980	1220	1470	1740	2000	2260	2520	2770
1,3	180	380	560	770	990	1220	1460	1720	1970	2210	2440	2650
1,4	190	400	600	810	1000	1250	1480	1730	1960	2180	2390	2560
1,5	210	430	650	880	1100	1310	1540	1770	1980	2170	2350	2500

[redacted]

1128

c_0	Значения $x_н$ м при v_0 м/с											
	50	100	150	200	250	300	350	400	450	500	550	600
						$\alpha=0{,}30$						
0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0
0,1	4	30	80	180	350	610	880	1250	1690	2440	3460	4770
0,2	50	130	270	460	840	1150	1660	2400	3310	3970	4800	6100
0,3	130	260	540	850	1250	1650	2200	2660	3140	3840	4360	6060
0,4	210	430	770	1070	1420	1820	2120	2510	2930	3420	3860	4450
0,5	240	490	750	1030	1330	1670	1990	2350	2730	3140	3560	4000
0,6	220	460	700	970	1250	1560	1870	2220	2560	2930	3300	3710
0,7	210	430	650	910	1180	1470	1770	2090	2400	2740	3090	3460
0,8	190	400	610	860	1120	1390	1680	1970	2250	2570	2900	3250
0,9	180	380	590	820	1070	1320	1600	1860	2130	2420	2730	3070
1,0	180	370	570	790	1030	1270	1530	1780	2030	2300	2590	2900
1,1	180	360	560	780	1000	1230	1490	1730	1960	2220	2480	2750
1,2	180	360	560	770	990	1210	1460	1700	1930	2160	2400	2640
1,3	190	380	570	780	1000	1220	1450	1690	1920	2130	2350	2560
1,4	200	400	600	820	1040	1250	1470	1720	1930	2130	2330	2510
1,5	210	430	650	880	1100	1310	1530	1760	1970	2160	2340	2490

c_0	Значения $y_н$ м при v_0 м/с ,											
	50	100	150	200	250	300	350	400	450	500	550	600
						$\alpha=0{,}05$						
0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0
0,1	1	9	30	60	110	180	290	310	400	490	590	700
0,2	20	40	100	150	210	310	480	690	850	1130	1320	1620
0,3	60	100	170	260	360	510	720	960	1240	1580	2080	2450
0,4	100	150	240	350	470	660	890	1170	1470	1800	2290	2690
0,5	110	180	290	430	570	740	960	1220	1520	1810	2190	2570
0,6	120	220	330	480	610	790	980	1210	1480	1740	2100	2450
0,7	120	250	370	520	670	830	1000	1210	1420	1680	2020	2350
0,8	130	270	400	550	710	860	1020	1210	1400	1640	1950	2260
0,9	130	280	420	580	730	900	1030	1220	1400	1610	1890	2180
1,0	140	290	440	590	750	920	1050	1240	1410	1610	1840	2110
1,1	150	300	460	600	770	940	1070	1250	1410	1600	1810	2060
1,2	150	310	470	620	780	960	1090	1260	1410	1600	1780	2030
1,3	160	320	480	640	800	970	1110	1280	1420	1600	1770	1980
1,4	170	330	490	660	820	990	1140	1310	1470	1620	1760	1940
1,5	170	330	500	670	830	1000	1170	1340	1500	1640	1770	1900

[redacted]

1129

c_0	Значения y_m м при v_0 м/с											
	50	100	150	200	250	300	350	400	450	500	550	500
						$\alpha=0{,}10$						
0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0
0,1	2	16	50	110	220	340	460	600	760	930	1130	1340
0,2	50	120	200	340	520	750	930	1190	1470	1810	2300	2850
0,3	110	250	410	640	900	1120	1360	1720	2200	2600	3240	3820
0,4	160	360	560	800	1100	1360	1670	2100	2520	2970	3460	4030
0,5	180	400	650	900	1190	1500	1850	2270	2700	3190	3670	4250
0,6	190	430	680	940	1240	1570	1950	2370	2790	3240	3770	4330
0,7	200	440	710	980	1290	1630	2020	2440	2850	3340	3830	4380
0,8	210	460	740	1020	1330	1680	2070	2480	2890	3370	3840	4390
0,9	220	480	760	1050	1370	1720	2100	2500	2910	3370	3830	4360
1,0	230	500	780	1080	1400	1750	2120	2500	2900	3350	3800	4300
1,1	240	520	800	1110	1420	1770	2130	2500	2880	3300	3730	4220
1,2	250	540	820	1130	1440	1780	2140	2480	2840	3240	3640	4110
1,3	260	560	840	1150	1460	1750	2130	2460	2800	3170	3540	3960
1,4	270	570	860	1170	1470	1800	2120	2430	2750	3090	3430	3790
1,5	280	580	880	1190	1480	1800	2090	2390	2690	3000	3300	3600
						$\alpha=0{,}15$						
0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0
0,1	3	20	70	150	270	380	540	740	940	1200	1450	1790
0,2	40	120	240	380	570	720	1030	1340	1800	2280	2920	3620
0,3	120	280	440	650	860	1140	1520	2000	2730	3380	4200	5040
0,4	180	370	600	860	1130	1460	1880	2420	3060	3700	4340	5070
0,5	210	440	700	1000	1300	1650	2070	2600	3150	3730	4320	5000
0,6	230	490	770	1080	1400	1750	2180	2670	3160	3720	4290	4930
0,7	250	530	820	1140	1480	1820	2250	2710	3170	3710	4250	4850
0,8	260	560	860	1190	1530	1880	2300	2730	3180	3690	4200	4760
0,9	270	580	890	1230	1570	1920	2330	2740	3170	3650	4140	4660
1,0	280	600	920	1260	1600	1950	2350	2750	3150	3600	4070	4550
1,1	280	620	940	1280	1620	1970	2360	2750	3130	3540	3990	4440
1,2	290	630	950	1290	1630	1980	2360	2730	3090	3480	3900	4320
1,3	300	630	960	1300	1640	1980	2350	2700	3050	3420	3810	4200
1,4	[illegible]	630	[illegible]	[illegible]	1620	1970	[illegible]	2670	[illegible]	[illegible]	3710	4080
1,5	300	630	960	1300	1620	1950	[illegible]	[illegible]	2950	3280	3600	[illegible]

[redacted]

1130

c_0	Значения $y_в$ м при v_0 м/с											
	50	100	150	200	250	300	350	400	450	500	550	600
						$\alpha=0,20$						
0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0
0,1	3	20	80	180	300	460	670	940	1280	1510	1940	2430
0,2	70	150	280	480	730	1070	1400	1950	2600	3230	4270	6100
0,3	160	320	540	800	1080	1440	1820	2450	3140	4000	5120	6300
0,4	200	420	700	970	1300	1690	2110	2720	3400	4250	5150	6100
0,5	220	480	760	1080	1440	1850	2300	2880	3540	4260	5000	5800
0,6	220	490	790	1130	1520	1970	2430	2970	3570	4210	4850	5530
0,7	230	510	820	1180	1590	2060	2520	3020	3580	4150	4720	5310
0,8	240	530	850	1220	1640	2110	2570	3050	3570	4080	4600	5120
0,9	250	550	870	1250	1680	2140	2590	3060	3540	4000	4480	4950
1,0	260	560	900	1280	1710	2150	2600	3040	3480	3920	4360	4800
1,1	270	570	920	1310	1720	2140	2580	3000	3430	3840	4250	4670
1,2	280	590	940	1330	1730	2130	2550	2950	3370	3750	4140	4540
1,3	290	610	960	1340	1720	2110	2510	2900	3290	3660	4040	4420
1,4	300	630	980	1350	1710	2090	2460	2830	3200	3560	3930	4310
1,5	310	660	1000	1360	1700	2050	2400	2750	3100	3460	3820	4200
						$\alpha=0,25$						
0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0
0,1	4	30	90	200	380	660	720	1150	1680	2540	3560	4870
0,2	80	160	300	560	860	940	1700	2480	3390	4300	5140	7100
0,3	160	350	590	950	1350	1800	2300	3000	4000	4840	6200	7850
0,4	230	480	800	1160	1450	1900	2460	3220	4070	4620	6070	7440
0,5	260	550	860	1220	1620	2050	2620	3280	4000	4700	5550	6600
0,6	270	570	910	1290	1740	2180	2700	3280	3930	4560	5260	6060
0,7	280	590	940	1350	1820	2270	2750	3270	3860	4440	5040	5680
0,8	290	610	960	1390	1870	2330	2770	3260	3790	4320	4860	5400
0,9	300	630	980	1420	1890	2350	2770	3240	3720	4210	4700	5180
1,0	310	640	1000	1440	1900	2350	2760	3200	3650	4100	4550	5000
1,1	320	650	1010	1450	1890	2330	2730	3150	3580	4000	4420	4850
1,2	320	660	1030	1460	1880	2300	2690	3100	3500	3910	4310	4720
1,3	[illegible]	670	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	4210	4610
1,4	340	[illegible]	[illegible]	[illegible]	[illegible]	2210	[illegible]	[illegible]	[illegible]	[illegible]	4110	[illegible]
1,5	[illegible]	700	[illegible]	[illegible]	[illegible]	2150	[illegible]	2900	[illegible]	3640	[illegible]	[illegible]

1131

ε_0	Значения $y_к$ м при v_0 м/с											
	50	100	150	200	250	300	350	400	450	500	550	600
						$\alpha=0,30$						
0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0
0,1	4	30	90	190	370	650	950	1360	1860	2740	3990	5670
0,2	80	180	340	500	890	1280	1800	2550	3500	5080	6800	8300
0,3	180	430	680	1000	1360	1800	2440	3350	4460	5600	6940	8440
0,4	270	570	880	1240	1680	2160	2860	3680	4540	5480	6610	7960
0,5	300	610	950	1350	1800	2300	2940	3680	4460	5280	6200	7350
0,6	310	630	980	1390	1850	2350	2940	3650	4360	5090	5890	6810
0,7	320	640	1000	1420	1880	2380	2930	3610	4260	4920	5610	6370
0,8	330	650	1020	1440	1900	2390	2930	3560	4160	4750	5370	6010
0,9	340	670	1040	1460	1920	2400	2920	3500	4050	4600	5170	5730
1,0	350	680	1060	1480	1930	2400	2900	3430	3940	4460	4990	5500
1,1	350	690	1070	1490	1930	2390	2870	3360	3840	4320	4820	5300
1,2	360	710	1080	1500	1920	2370	2820	3290	3740	4190	4650	5120
1,3	370	720	1100	1500	1910	2340	2760	3210	3640	4060	4490	4940
1,4	370	730	1110	1500	1900	2300	2700	3120	3540	3930	4340	4770
1,5	380	750	1130	1500	1880	2250	2630	3030	3430	3820	4200	4600

Key: (1). Values. (2). with. (3). m/s. (4). m with.

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